DON

1

JAN. 28/19

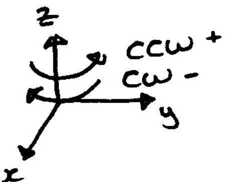
Position Analysis

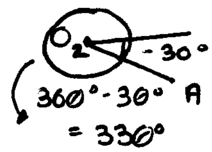
GCS(x y) - global coordinate system

LNCS(x = y) - local non-rotating coord. system

↳ ccw (+ve)

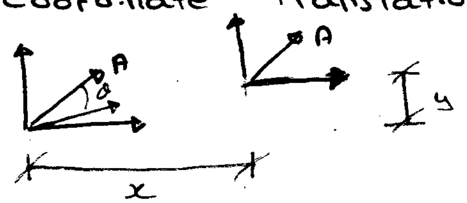
cw (-ve)





Vector { magnitude or length
angle

Coordinate translation, rotation:



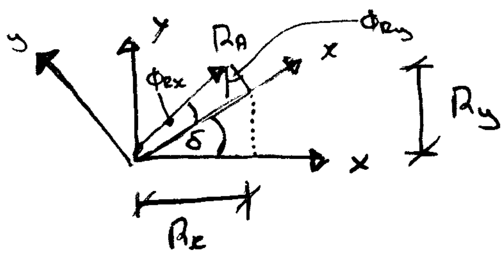
$R_x = |\hat{R}_A| \cos(\phi + \delta)$

$R_y = |\hat{R}_A| \sin(\phi + \delta)$

where:

$= |\hat{R}_A| (\cos\phi \cos\delta - \sin\phi \sin\delta) = R_x \cos\delta - R_y \sin\delta$

$= |\hat{R}_A| (\sin\phi \sin\delta - \cos\phi \cos\delta) = R_x \sin\delta - R_y \cos\delta$



$L_1 = 8cm$, $L_2 = 4$, $L_3 = 6$, $L_4 = 8$

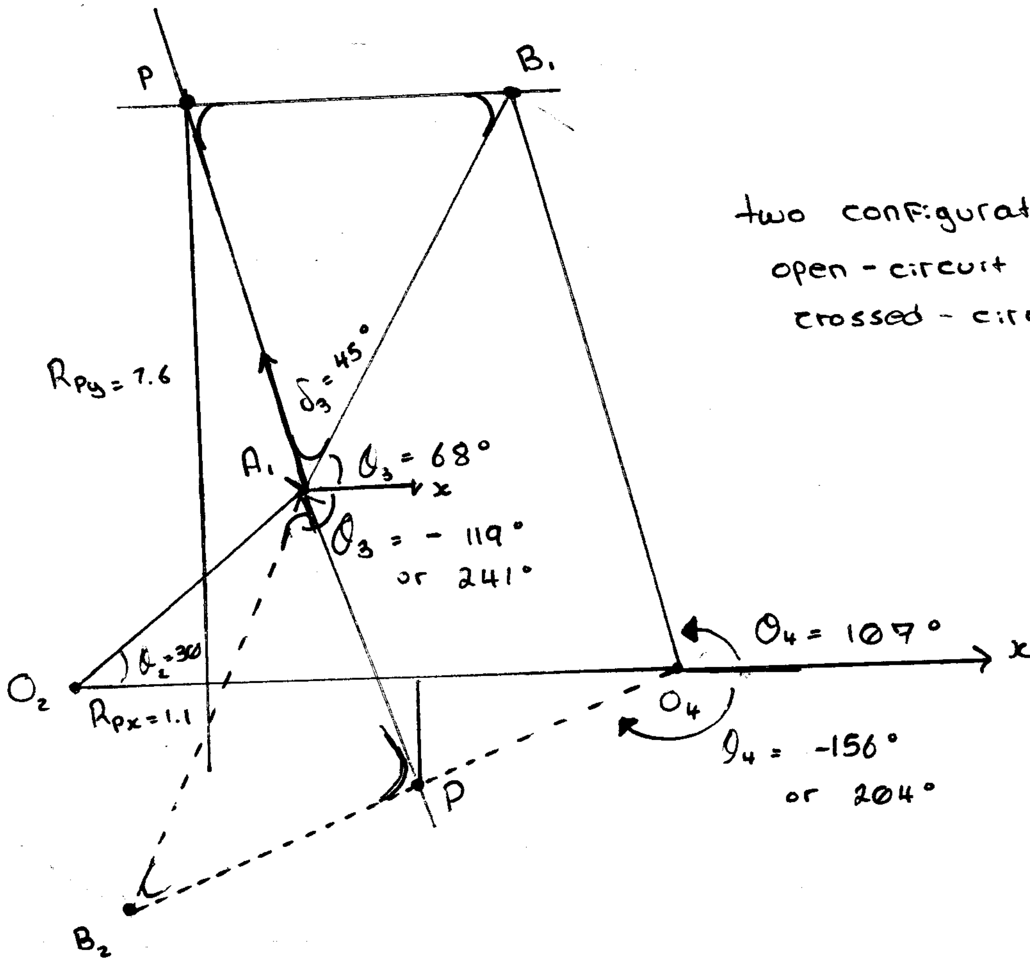
Example:

P is point on coupler

$\overline{AP} = 6$ cm, $\delta_3 = 45°$

Find θ_3, θ_4, R_{px}, R_{py} if $\theta_2 = 30°$

two configurations: open-circuit and crossed-circuit.



Analytical Position Analysis

$\hat{R}_a = 2 @ \angle 0°$ polar form

$\hat{R}_b = 1.5 @ 60°$

$\hat{R}_c = \hat{R}_A + \hat{R}_B$

$= 2e^{i0°} + 1.5e^{i60°}$

$= 2\cos(0) + j\,2\sin(0) + 1.5\cos(60°) + j\,1.5\sin(60°)$

$\hat{R}_c = 2.75 + j1.299$; $\hat{R}_c = \hat{R}_A - \hat{R}_B$

$\hat{R}_c = |\hat{R}_c| e^{j\theta_c} = 2.385e^{j33°}$

$|\hat{R}_c| = \sqrt{2.75^2 + 1.299^2} = 2.304$

$\theta_c = \tan^{-1}(1.299/2.75) = 25.3°$

$a = L_2$, $b = L_3$, $c = L_4$

$\hat{R}_2 + \hat{R}_3 - \hat{R}_1 - \hat{R}_4 = 0$

$ae^{j\theta_2} + be^{j\theta_3} - d - ce^{j\theta_4} = 0$

lengths a, b, c, d. Solve for θ_4

θ_2

Find θ_3, θ_4

$\hat{R}_B = \hat{R}_2 + \hat{R}_3 = \hat{R}_1 + \hat{R}_4$ (Analytical solution)

$ae^{j\theta_2} = a\cos\theta_2 + ja\sin\theta_2$

$\theta_1 = 0, \quad e^{j0} = 1$

→ Solve for θ_4

$$b\cos\theta_3 = d - a\cos\theta_2 + c\cos\theta_4 \quad (1)$$

$$b\sin\theta_3 = -a\sin\theta_2 + c\sin\theta_4 \quad (2)$$

$$(1)^2 + (2)^2 \quad b^2(\cos^2\theta_3 + \sin^2\theta_3) = (d - a\cos\theta_2 + c\cos\theta_4)^2 + \ldots$$
$$\ldots (-a\sin\theta_2 + c\sin\theta_4)^2$$

$$\Rightarrow K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos\theta_2\sin\theta_4 + \sin\theta_2\cos\theta_4$$

Half-angle identities:

$$\sin\theta_4 = \frac{2\tan(\theta_4/2)}{1+\tan^2(\theta_4/2)}, \quad \cos\theta_4 = \frac{1-\tan^2(\theta_4/2)}{1+\tan^2(\theta_4/2)}$$

$$A\tan^2(\theta_4/2) + B\tan(\theta_4/2) + C = 0$$

$$ax^2 + bx + c = 0 \rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Example: Find θ_3, θ_4

$d = 8, \ a = 4, b = 6, c = 8, \quad \theta_2 = 30°$

→ $K_1 = 8/4 = 2$

$K_2 = 8/8 = 1$

$$K_3 = \frac{4^2 - 6^2 + 8^2 + 8^2}{2(4)(8)} = 1.6875$$

$A = \cos 30° - 2 - (1)\cos(30°) + 1.6875 = -0.3125$

$B = -2\sin 30° = -1$

$C = 2 - (1+1)\cos(30°) + 1.6875 = 1.9555$

$$\theta_{4_{1,2}} = 2\tan^{-1}\frac{(-1) \pm \sqrt{(-1)^2 - (4)(-0.3125)(1.950)}}{2(-0.3125)}$$

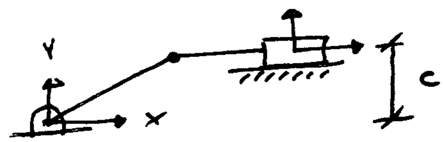
$$= 2\tan^{-1}\left(\frac{1 \pm 1.857}{-0.625}\right)$$

$$\theta_{4/1} = 2\tan^{-1}\left(\frac{2.857}{-0.625}\right) = 2(180° - 77.7°) = 360° - 155.4°$$
$$\rightarrow 204.6°$$

$$\theta_{4/2} = 2\tan^{-1}\left(\frac{-0.857}{-0.625}\right) = 2(180° + 53.9°)$$
$$\rightarrow 107.8°$$

in-line ; $c = 0$

off-line ; $c \neq 0$



- Given a, b, c, θ_2

  Find d, θ_3

  $\hat{R}_A = \hat{R}_2 = \hat{R}_1 + \hat{R}_4 + \hat{R}_3$

Example 4-2

(Graphical solution)

$a = 4$, $b = 12$, $c = -2$, $\theta_2 = 60°$

Find θ_3, d

1. Choose X, O_2
2. Draw $60°$ from O_2, with $a = 4$.
3. Draw arc from A, with $b = 12$
4. Draw line parallel with X and $C = -2$ to intersect with the arc B_1, B_2

(see diagram 4-2 from lecture slides)