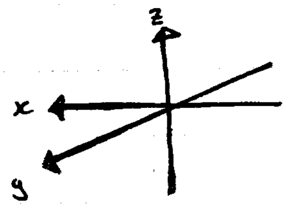
Degree of Freedom (DOF) of a body

↳ the DOF is equal to the number of independent coordinates needed to uniquely define its position.



DOF of a system:

number of actuators needed

(motors, hydraulic cylinders, solenoid)

Translation: All points on the body travel on Parallel paths, rectilinear and curvilinear.

Pure Rotation: Rotation about a Fixed axis

Complex motion: combination of rotation and translation

Rolling without slipping - 1 DOF

Rolling with slipping - 2 DOF

Link: a rigid body that has at least two nodes

Binary link has two nodes

Ternary : " " three nodes

Quaternary : " " Four nodes

↳ only counts as a node if it's being utilized.

Joint: connection between two or more links.

(1 DOF) Lower Pair: a joint with Surface contact (pin in hole, rotary contact)

(2 DOF) Higher Pair: a joint with a point or line in contact (pin in slot, link against plane)

Full Joints and Half Joints

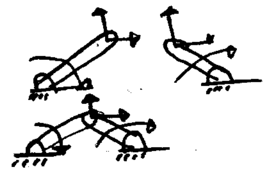
Full Joint: DOF = 1, may be lower pair (such as pin connection) or higher pair (such as disk rolling on plane without slipping)

Half-joint : DOF = 2 , It must be a higher pair (such as pin in slot)

Point or line

two unconnected links : 6

two connected links : 4



Kinematic Chains : assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

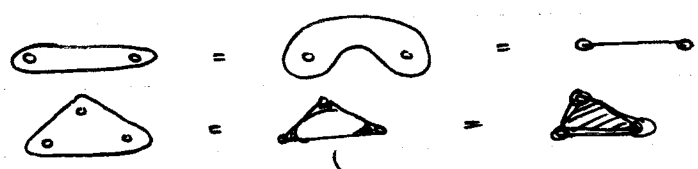
Conventional names :

crank - Full revolution

rocker - oscillation

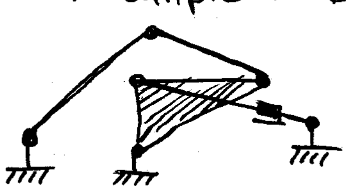
coupler - link in complex motion

ground - reference Frame

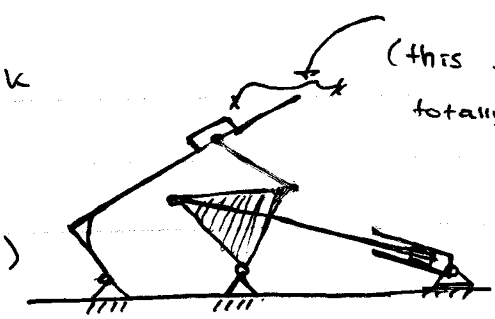


must be shaded (or its interpretted as 3 two node linkage)

Example : Dump Truck



(better drawn as...)



(this section isn't totally necessary)

DOF or Mobility of a linkage

DOF = No. of control inputs : actuators such as motor, hydraulic cylinder, solenoid

Dyad : a pair of binary linkages, Groebler's eq'n.

$$M = 3L - 2J_1 - J_2 - 3$$

$$\left(\text{or} \quad M = 3(L-1) - 2J_1 - J_2\right)$$

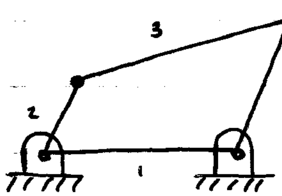
Consider → $M = 3L - 3 - 2J_1 - J_2$

↳ L = number of links (ground is fixed)

J_1 = number of full joints

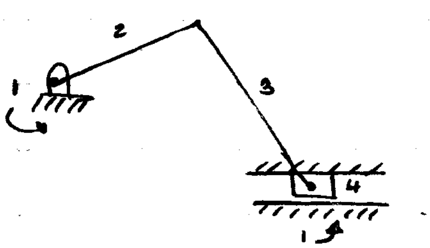
J_2 = number of half joints

Example

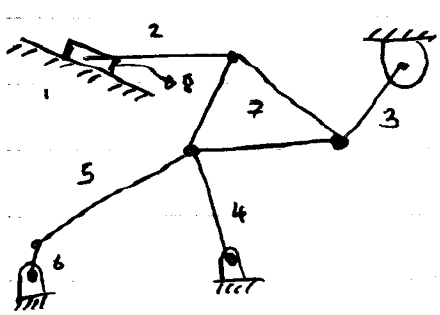


$$\left.\begin{array}{l} L = 4 \\ J_1 = 4 \\ J_2 = 0 \end{array}\right\} \quad M = 3(4) - 3 - 2(4) - 0$$

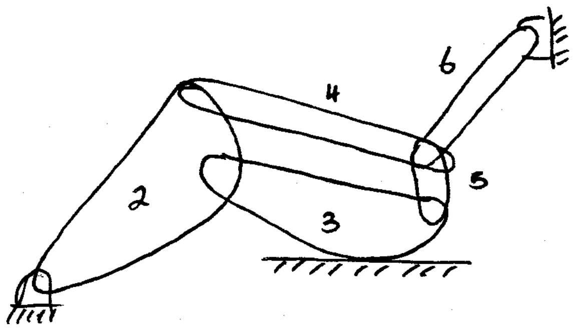
↳ $M = 1$



$$\left.\begin{array}{l} L = 4 \\ J_1 = 4 \\ J_2 = 0 \end{array}\right\} \quad M = 1$$



$$\left.\begin{array}{l} L = 8 \\ J_1 = 10 \\ J_2 = 0 \end{array}\right\} \quad M = 1$$



$L = 6$

$J_1 = 7$

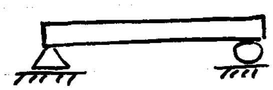
$J_2 = 1$

$$M = 3(5) - 2(7) - 1$$
$$= 15 - 14 - 1 = 0$$

Mechanism if DOF > 0

Structure if DOF = 0

Pre-loaded Structure if DOF < 0



(simply supported)

$L = 2$

$J_1 = 2$

$J_2 = 0$

$M = -1$

DOM

1

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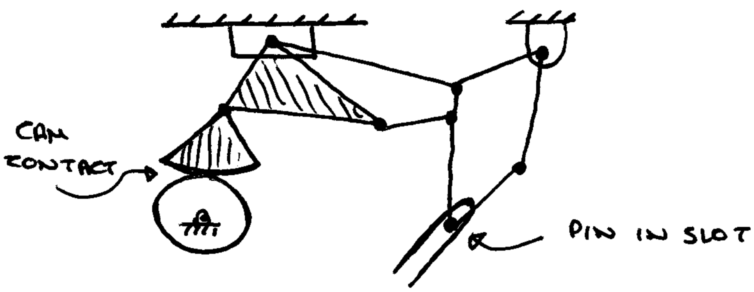
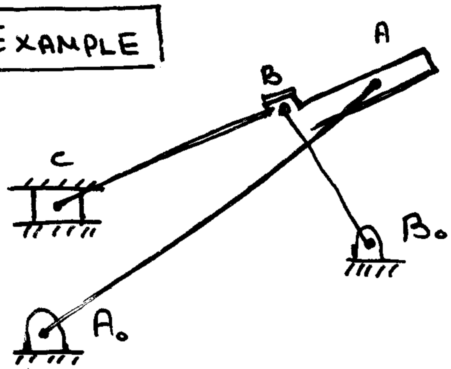
EXAMPLE

$$\left.\begin{aligned} L &= 12 \\ J_1 &= 13 \\ J_2 &= 2 \end{aligned}\right\} \quad \begin{aligned} M &= 3(12-1) - 2(13) - 2 \\ M &= 6 \end{aligned}$$

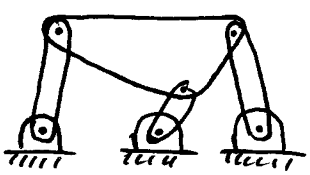


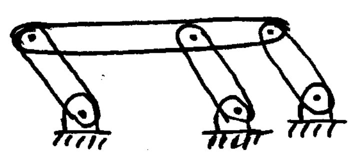
EXAMPLE



$$\left.\begin{aligned} L &= 5 \\ J_1 &= 5 \\ J_2 &= 1 \end{aligned}\right\} \quad \begin{aligned} M &= 3(5-1) - 2(5) - 1 \\ M &= 1 \end{aligned}$$

EXAMPLE





$$\left.\begin{aligned} L &= 5 \\ J_1 &= 6 \\ J_2 &= 0 \end{aligned}\right\} \quad M = 0$$

$\therefore$ DOF $= 0$

(agrees with Gruebler)

$$\left.\begin{aligned} L &= 5 \\ J_1 &= 6 \\ J_2 &= 0 \end{aligned}\right\} \quad M = 0$$

according to Gruebler, but it will move Re: parallelogram.

$\therefore$ DOF $= 1$

(disagrees with Gruebler)

The Grashof Condition

→ determine whether there is a link that can make a full rotation in a fourbar linkage based on the link lengths only.

4 bar linkage :

S = length of shortest link

L = length of longest link

P, Q = length of the other two links

$\boxed{\text{EXAMPLE}}$ $L_1 = 20$, $L_2 = 10$, $L_3 = 16$, $L_4 = 16$

$$\left.\begin{array}{l} S = 10 \\ L = 20 \\ P = Q = 16 \end{array}\right\} \quad \left.\begin{array}{l} S+L = 30 \\ P+Q = 32 \end{array}\right\} \quad S+L < P+Q \to \underline{\underline{\text{CLASS I}}}$$

(*)

$$\left.\begin{array}{ll} \text{CLASS I :} & S+L < P+Q \\ \text{CLASS III :} & S+L = P+Q \end{array}\right\} \text{At least one link capable of making full rotation}$$

$$\text{CLASS II :} \quad S+L > P+Q \; \} \text{ No link capable of full rotation}$$

$\boxed{\text{EXAMPLE}}$ $L_1 = 20$, $L_2 = 10$, $L_3 = 14$, $L_4 = 14$

$$\left.\begin{array}{l} S+L = 30 \\ P+Q = 24 \end{array}\right\} \quad S+L > P+Q \to \underline{\underline{\text{CLASS II}}}$$

$\boxed{\text{EXAMPLE}}$ $L_1 = 20$, $L_2 = 10$, $L_3 = 14$, $L_4 = 16$

$$\left.\begin{array}{l} S+L = 30 \\ P+Q = 30 \end{array}\right\} \quad S+L = P+Q \to \underline{\underline{\text{CLASS III}}}$$

Crank : makes full rotation

Rocker : opposite of crank, oscillates

CLASS I {
- → Grounding shortest link results in double crank
- → Grounding opposite link to shortest results in double rocker

CLASS II :

All inversions will be triple-rocker in which no link can fully rotate.

CLASS III :

All inversions will be double-cranks or crank-rockers w/ have "change points" once or twice per revolution of the input crank where all links become colinear.

EXAMPLE (L_1)

No.	Ground	Link 2	Coupler	Link 4
A	7	4	2	6
B	6	4	7	6
C	6	8	6	5
D	4	6	6	6
E	8	3	6	6
F	6	4	6	4
G	8	3	6	4

	Crank Rocker	Double Crank	Double Rocker	Triple Rocker
A			✓	
B	✓			
C				✓
D		✓		
E	✓			
F		✓		
G				✓

(Ground is link 1, always).

A. $S+L = 2+7 = 9 < P+Q = 4+6 = 10$ (CLASS I)
↳ ground longest link opposite to S.
- double - rocker

B. $4+7 = 11 < 6+6 = 12$ (CLASS I)
- crank - rocker

C. $5+8 = 13 > 6+6 = 12$ (CLASS II)
- triple - rocker

D. $4+6 = 10 < 6+6 = 12$ (CLASS I)
- double - crank

Refer to previous rules

E. $3+8 = 11 < 6+6 = 12$ (CLASS I)
- crank - rocker

F. $4+6 = 10 = 4+6 = 10$ (CLASS III) (Parallelogram)
- double - crank

G. $8+3 = 11 > 6+4 = 10$ (CLASS II)
- triple - rocker

Three Types of Mechanisms

- Function Generation (correlation of input motion w/ output)
- Path Generation
- Motion Generation