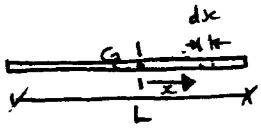


slender rod

$d = \emptyset$ where m/L = length density

total mass = m

$$dI_z = x^2\,dm = x^2(m/L)\,dx$$

$$I_z = \int dI_z = \int_{-L/2}^{L/2} x^2 (m/L)\,dx = (m/L)\int_{-L/2}^{L/2} x^2\,dx$$

$$= \left(\frac{m}{L}\right)\left(\frac{1}{3}\right)x^3\Big|_{-L/2}^{L/2} = \left(\frac{m}{L}\right)\left(\frac{1}{3}\right)\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= \left(\frac{m}{L}\right)\left(\frac{1}{3}\right)\left[\frac{L^3}{8} + \frac{L^3}{8}\right] = \left(\frac{1}{12}\right)mL^2$$

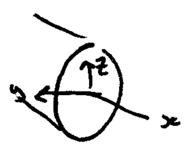
$$I_z = L^2 m$$

$$\Sigma M_G = I_G\alpha \longrightarrow \alpha = \frac{\Sigma M_G}{I_G}$$

$$\Sigma F = ma$$

$$(I_G)_1\omega_1 = (I_G)_2\omega_2 = \text{const.} \quad \text{(cons. of angular momentum)}$$

For a cylinder:



$$I_y = I_z = \frac{m}{12}(3r^2 + l^2) = \frac{m}{12}l^2\left(1 + \frac{3r^2}{l^2}\right) \approx \frac{1}{12}ml^2$$

if $r \ll l$ $(r/l)^2 \to \emptyset$

$\boxed{\text{Example :}}$ (slide 16, Ch. 10)

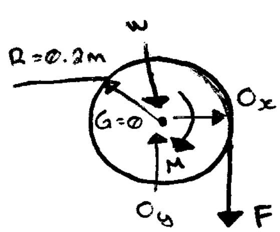
Known: $m = 30$ kg, $F = 10$ N, $M = 5$ N·m

Find: No. of rev. @ $\omega = 20$ rad/s

reactions at O

Solution:

1. FBD



(For a cylinder, $I_x = \frac{mr^2}{2}$)

$$I_G = \frac{(30)(0.2)^2}{2} = 0.6\ \text{kgm}^2$$

2. 2nd Law

$\Sigma F_x = ma_{Gx}$ $\qquad O_x = \emptyset$

$\Sigma F_y = ma_{Gy}$ $\qquad O_y - mg - F = \emptyset$

$\qquad O_y = 304$ N

$\Sigma M_G = I_G\alpha$ $\qquad (-5) - 10(0.2) = I_G\alpha$

$$\alpha = \frac{(-5) - 2}{0.6} = -11.67\ \text{rad/s}^2$$

then $\omega = -11.67t$

Since $\omega = -11.67t$

$$-20 = -11.67t$$

$$t = \frac{20}{11.67} = 1.7145$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(-11.67)(1.714)^2 = -17.142 \text{ rad}$$

$$\text{No. of revolution} = \frac{1.7142}{2\pi} = 2.73 \text{ rev}$$

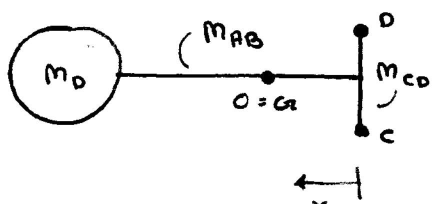
$\boxed{\text{Example:}}$ (slide 19, ch.10)

Known: $m_D = 2$ kg

$m_{AB} = 1.3 \times 3 = 3.9$ kg

$M_{CO} = 3L$

Find L so that $O = G$



$$\bar{x} = \frac{M_{CO}(0) + M_{AB}\left(\frac{1.3}{2}\right) + M_D(1.3+0.2)}{3L + 3.9 + 2}$$

$$(0.5)(3L) + (0.5)(5.9) = (3.9)(0.65) + (2)(1.5)$$

$$L = \frac{5.535 - 2.95}{1.5} = 1.723 \text{ m}$$

$$I_o = (I_D)_o + (I_{AB})_o + (I_{CD})_o$$

$$= (1/2)(2)(0.2)^2 + (2)(1)^2 + (1/12)(3.9)(1.3)^2 + (3.9)((1.3/2) - 0.5)^2 \ldots$$

$$\ldots + (1/12)(5.169)(1.723)^2 + (5.169)(0.5)^2$$

$$= \underbrace{\left(\tfrac{1}{2}\right) mr^2 + mL^2}_{\text{cylinder}} + \underbrace{\left(\tfrac{1}{12}\right) M_{AB}\, l_{AB}^2 + M_{AB}\, d^2}_{\text{First slender bar}} + \underbrace{\left(\tfrac{1}{12}\right) M_{CD}\, l_{CD}^2 + M_{CD}\, d^2}_{\text{second slender bar}}$$

Question (From assignment - slide 20, ch.10)

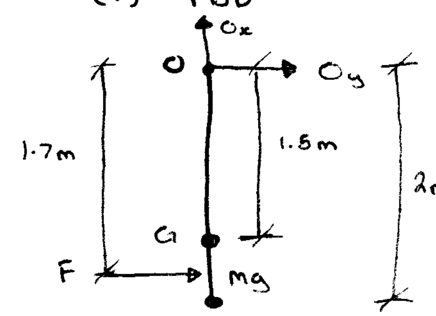
(a) $\bar{y} = 1.5\,m$

(b) $I_G = 2.333\ kg\cdot m^2$

(c) $I_O = 11.333\ kg\cdot m^2$

(d) pure rotation $O \neq G$

(1) FBD



(2) 2nd Law

$\Sigma F_x = m a_{Gx}$ $\qquad F + O_x = m a_{Gx}$ ①

$\Sigma F_y = m a_{Gy}$ $\qquad O_y - mg = 0$ ②

$\Sigma M_G = I_G \alpha$ $\qquad F(1.7 - 1.5) - O_x(1.5) = I_G \alpha$ ③

① and ③ $\quad O_x,\ a_{gx},\ \alpha$

① : $O_x = m\bar{y}\alpha - F$ & $a_G = \bar{y}\alpha$

③ : $F(1.7 - \bar{y}) - (m\bar{y}\alpha - F)\bar{y} = I_G \alpha$

③ $\quad \alpha = \dfrac{F\,1.7}{I_G + m\bar{y}^2} = \dfrac{(50)(1.7)}{(2.333) + (4)(1.5)^2} = 7.5\ rad/s^2$

$I_O = I_G + m\bar{y}^2$

In pure rotation: $\qquad \Sigma M_O = I_O \alpha$

(ONLY!)

$\rightarrow \qquad \alpha = \dfrac{FL}{I_O}$

Naming conventions:

F_{ij} — i: link the Force is acting on; j: link that applies the Force

Process: 1. FBD

2. Apply 2nd Law

$$\Sigma F_x = m a_{Gx} \longrightarrow \underline{F_{12x}} + F_{px} = m a_{Gx}$$

$$\Sigma F_y = m a_{Gy} \longrightarrow \underline{F_{12y}} + F_{py} = m a_{Gy}$$

$$\Sigma T_G = I_G \alpha \longrightarrow \underline{T_{12}} + (R_{12x}F_{12y} - R_{12y}F_{12x}) + (R_{px}F_{py} - R_{py}F_{px})$$

1. $F_{12x} = m a_{Gx} - F_{px}$
2. $F_{12y} = m a_{Gy} - F_{py}$
3. $-R_{12y}F_{12x} + R_{12x}F_{12y} + T_{12} = I_G\alpha - (R_{px}F_{py} - R_{py}F_{px})$

$$\underset{A}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{12y} & R_{12x} & 1 \end{bmatrix}} \underset{B}{\begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix}} = \underset{C}{\begin{bmatrix} \\ \\ \\ \end{bmatrix}} \qquad \begin{cases} AB = C \\ B = A^{-1}C \end{cases}$$

$\boxed{\text{Example}}$ — Slide 7 (slender rod w/ counterweight)

US customary system (lb, in, $g = 386$ in/s²)

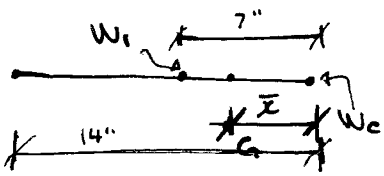
Solution

(1) Mass $\quad m_r = \dfrac{W_r}{g} = \dfrac{6}{386} = 0.0155$ blobs (lb/in/s²)

$$m_c = \frac{W_c}{g} = \frac{4}{386} = 0.0104 \text{ blobs}$$

$$M_w = 0.0156 + 0.0104 = 0.0259$$

(2)



$$\bar{x} = \frac{(6)(7) + (4)(0)}{(6)(4)} = 4.2 \text{ in}$$

(3) Moment of inertia:

$$I_G = \left(\tfrac{1}{12}\right)(0.0155)(14)^2 + (0.0155)(7-4.2)^2 + (0.0104)(4.2)^2 = 0.5953$$

(4) $\hat{a}_G = \hat{a}_G^t + \hat{a}_G^n$

$= j(2.2)(40)e^{j150°} - (2.2)(10)^2 e^{j150}$

$= \underbrace{146.52}_{a_{Gx}} - \underbrace{j186.21}_{a_{Gy}}$ in/s²

(5) $F_{px} = (60)(\cos 30°) = \underline{51.96}$, $F_{py} = (60)(\sin 30°) = \underline{30}$

(6) Position vectors:

$\hat{R}_{12} = 1.905 - j1.1 = -2.2e^{j150°}$

$\hat{R}_p = -8.487 + j4.9 = 9.8e^{j150}$

(7) Apply 2nd Law:

$\Sigma F_x = m a_{Gx} \Rightarrow F_{12x} = 0.0259(146.52) - 51.96 = -48.7$ lb

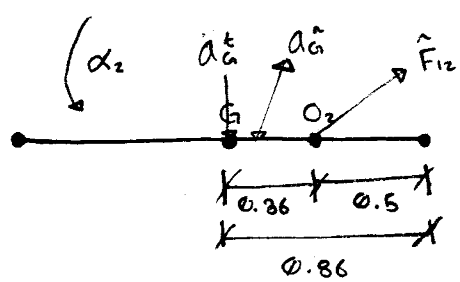
$\Sigma F_y = m a_{Gy} \Rightarrow F_{12y} = 0.0259(-186.21) - 30 = -34.82$ lb

$\Sigma T_G = I_G \alpha \Rightarrow -(-1.1)F_{12x} + 1.905F_{12y} + T_{12} = 0.5953(40) - [-8.487(\underbrace{30}_{F_{py}} - \ldots$

$\ldots 4.9(\underbrace{51.96}_{F_{px}})]$

$T_{12} = 652.3$ lb·in

$\bar{x} = 0.86$ m, $m = 7.2$, $\bar{I}_G = 3.04$ $\Big\}$ $\boxed{\text{Example}}$ – <u>Slide 8</u>



$\hat{a}_G = j0.36(100)e^{j180°} - (0.36)(-10)^2 e^{j180°}$

$= -36 - j36$

$\Sigma F_x = m a_{Gx}$

$\Rightarrow F_{12x} = 7.2(36) = 259.2$ N

$\Sigma F_y = m a_{Gy}$

$\Rightarrow F_{12y} = (7.2)(-36) = -259.2$

$\Sigma T_G = I_G \alpha \ \ldots$

$\Rightarrow T_{12} = 397.3$ (N·m)