Sept. 25 / 18

Clausius inequality : $\int dQ/T \leq 0$

Entropy : $ds = dq/T + ds_{gen}$; $ds_{gen} \geq 0$

Rate equation for entropy : $\dot{S}_{cm} = \sum \dot{Q}_{c.m.}/T + \dot{S}_{gen}$

Entropy equation : $m(s_2 - s_1) = \int_1^2 \frac{\delta Q}{T} + {}_1S_{2\,gen}$; ${}_1S_{2\,gen} \geq 0$

Total entropy change : $\Delta S_{net} = \Delta S_{cm} + \Delta S_{surr} = S_{gen} \geq 0$

Gibbs relations $\quad Tds = du + Pdv$

Solids, liquids $\quad Tds = dh - vdP$

$v = \text{constant}, \; dv = 0$

Change in s $\quad s_2 - s_1 = \int du/T = \int C\, dT/T \simeq C\ln\left(\frac{T_2}{T_1}\right)$

$$Tds = du + Pdv$$

Solid, liq. $\Rightarrow v = \text{const.} \Rightarrow dv = 0$

$Tds = du \Rightarrow ds = \frac{du}{T}$

C_p and C_v = const. $\quad \boxed{du = C_v dT}$

$\int_1^2 ds = \int_1^2 C_v \frac{dT}{T} \Rightarrow s_2 - s_1 = C_v \ln(T_2/T_1)$ — absolute temp.

Ideal gas $\quad$ standard entropy $\quad s_T^o = \int_{T_0}^{T} \frac{C_{p0}}{T}\, dT$

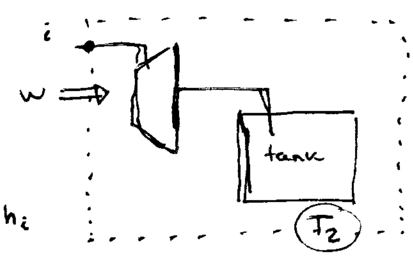
$$s_2 - s_1 = s_{T2}^o - s_{T1}^o - R\ln(P_2/P_1)$$

$$s_2 - s_1 = C_p \ln(T_2/T_1) - R\ln(P_2/P_1)$$

$$s_2 - s_1 = C_{v0} \ln(T_2/T_1) + R\ln(v_2/v_1)$$

(for constant C_p, C_v)

$\boxed{\text{Example 7.6}}$



(no exit)

I $\quad$ Continuity: $m_2 - m_1 = m_{in}$

II $\quad$ Energy: $m_2u_2 - m_1u_1 = \underbrace{\cancel{{}_1Q_2}}_{\text{adiabatic}}^{0} - {}_1W_2 + m_ih_i$

Second law :

III $\quad m_2s_2 - m_1s_1 = \underbrace{\cancel{\int \frac{dQ}{T}}}_{\text{adiabatic}}^{0} + \underbrace{\cancel{{}_1S_{gen\,2}}}_{\text{reversible}}^{0} + m_{in}s_i$

$s_1 = s_{in}$

From Eq. (III) $\Rightarrow$ $m_2 s_2 - m_1 s_1 = m_i s_i$

$(m_1 + m_i) s_1 = m_2 s_2$

From (I) $m_1 + m_i = m_2$ $\longrightarrow$ $s_1 = s_2$

Air is an ideal gas:

$$s_2 - s_1 = s^\circ_{T_2} - s^\circ_{T_1} - R\ln(P_2/P_1) \Rightarrow$$

$$s^\circ_{T_2} = s^\circ_{T_1} + R\ln(P_2/P_1) \Rightarrow s^\circ_{T_2} = 6.83521 + 0.287\ln\left(\frac{1000}{100}\right)$$

From table ($T = 290K$) $\quad s^\circ_{T_2} = 7.49605$ kJ/kg·K

(From interpolation) 555.7 → 540 / 560 $\quad$ 7.46642 / 7.50422 ← 7.49606

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{100 \times 0.04}{(0.287)(290)} = 0.04806 \text{ kg}$$

$$m_2 = \frac{P_2 V_2}{R T_2} = \frac{1000 \times 0.04}{(0.287)(555.7)} = 0.2508 \text{ kg}$$

From eq. (I) $\Rightarrow$ $m_i = m_2 - m_1 = 0.2027$ kg

$T_1 = 290K \begin{cases} u_1 = 207.19 \\ h_i = 290.43 \end{cases}$

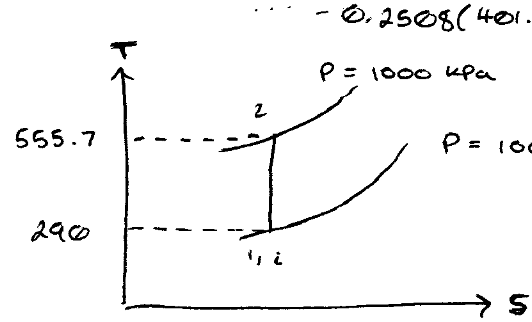
$T_2 = 555.7 \begin{cases} u_2 = 401.49 \text{ kJ/kg} \end{cases}$

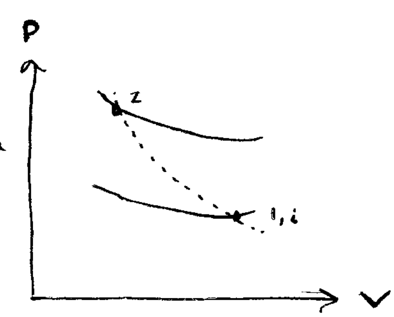
$$m_2 u_2 - m_1 u_1 = -W_{1\to2} + m_i h_i$$

$$W_{1\to2} = m_i h_i + m_1 u_1 - m_2 u_2 = 0.2027(290.43) + 0.04806(207.19) \cdots$$

$$\cdots - 0.2508(401.49)$$

$W_{1-2} = -31.9$ kJ





The energy equation:

$$q + h_i + \tfrac{1}{2} V_i^2 + g z_i = h_e + \tfrac{1}{2} V_e^2 + g z_e + w$$

The second law:

$$s_i + s_{gen} + \int \frac{\delta q}{T} = s_e$$

Differential Form

$$\delta s_{gen} + \delta q / T = ds \longrightarrow \delta q = T ds - T \delta s_{gen}$$

Thermodynamic property relation (Gibbs Relation)

$$T ds = du + P dv$$

$$T ds = dh - v dP$$

$$\hookrightarrow h = u + Pv$$

$$\left.\begin{aligned} dh &= du + P dv + v dP \\ du &= dh - P dv - v dP \end{aligned}\right\} \longrightarrow T ds = du + P dv$$

$$T ds = dh - \cancel{P dv} - v dP + \cancel{P dv}$$

$$T ds = dh - v dP$$

$$\delta q = T ds - T \delta s_{gen} = dh - v dP - T \delta s_{gen}$$

$$q = \int_i^e \delta q = \int_i^e dh - \int_i^e v dp - \int_i^e T \delta s_{gen} = h_e - h_i - \int_i^e v dP - \int_i^e T \delta s_{gen}$$

The energy equation:

$$w = q + h_i - h_e + \tfrac{1}{2}(V_i^2 + V_e^2) + g(z_i - z_e)$$

$$= h_e - h_i - \int_i^e v dP - \int_i^e T \delta s_{gen} + h_i - h_e + \tfrac{1}{2}(V_i^2 + V_e^2) + g(z_i - z_e)$$

(simplified) $w = -\int_i^e v dP + \tfrac{1}{2}(V_i^2 + V_e^2) + g(z_i - z_e) - \int_i^e T \delta s_{gen}$

1 → the maximum work for a reversible process $\delta s_{gen} = 0$

2 → For a reversible process, the shaft work is associated with changes in pressure, kinetic energy, potential energy either individually or in combination.

↳ when pressure increases, work negative, shaft work in (pump, comp.)

↳ " " decreases, work positive, shaft work out (turbine)

P5.67 Solution :

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = \underline{0.6}$$

$$\eta_{thermal} = \frac{W}{Q_H} = \frac{180}{300} = \underline{0.6}$$

∴ Same as carnot, so the process is reversible

if it was lower, there would be some irreversibility

if higher, impossible.

P5.55 Solution:

$$\int_1^2 dQ_H = \int_{T_i}^{T_F} -m\, C_p\, dT_H$$

$$Q_H = -mC_p \int_{T_i}^{T_F} dT = -mC_p(T_F - T_i)$$

$$Q_H = -10000\,(1.5)(300-560) = \boxed{3900 \text{ MJ}}$$

(η_{HE} = heat engine efficiency = η_{carnot})

$$dW = \eta_{carnot}\, dQ_H$$

$$dW = \left(1 - \frac{T_L}{T_H}\right)(-m\, C_p\, dT_H)$$

$$W_{1\to2} = -m\int_1^2 \left(1 - \frac{T_L}{T_H}\right) C_p\, dT_H = -mC_p\int dT_H + mC_p\int \frac{T_L}{T_H}\, dT_H$$

$$W_{1\to2} = -m\, C_p\,(T_{H_2} - T_{H_1}) + mC_p\, T_c \ln(T_{H2}/T_{H1})$$

$$\Rightarrow -10000(1.5)(300-560) + (10000)(1.5)(300)\ln\left(\frac{572}{833}\right)$$

$$\boxed{W_{1-2} = 2216 \text{ MJ}}$$

P5.38 Solution :

- Finding maximum power → Find Carnot

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \left(\frac{750}{1500}\right) = 0.5$$

$$\eta_{car} = \frac{\dot W}{\dot Q_H} = \dot W = \eta_{car}\,\dot Q_H = \eta_{car}(\dot m_{Fuel} \times Q_{HV})$$

$$\dot W = 0.5\,(5 \text{ g/s} \times 40000 \text{ kJ/kg})$$

$$(0.5)\left(\frac{5}{1000} \text{ kg/s}\right)(40000 \times \cancel{1000} \text{ J/kg}) \Rightarrow 100 \text{ kW}$$

P5.85 Solution :

$$\beta = 0.6\,\beta_{car} \qquad \beta = \frac{Q_L}{W}$$

$$① \quad \frac{\dot Q_L}{\dot W} = 0.6\left(\frac{T_L}{T_H - T_L}\right) \qquad \beta = \frac{Q_L}{Q_H - Q_L}$$

$$② \quad \dot Q_L = 0.6\,(T_H - T_L) \qquad \beta_{car} = \frac{T_L}{T_H - T_L}$$

From ① and ② $\Rightarrow$ $$\frac{0.6\,(T_H - T_L)}{\dot W} = 0.6\left(\frac{T_L}{T_H - T_L}\right)$$

$$\frac{\cancel{0.6}\,(T_H - T_L)^2}{\dot{W}} = \cancel{0.6}\,T_L$$

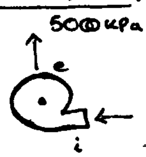
$$\Rightarrow \quad (T_H - T_L)^2 = \dot{W}\,T_L = 1.2(20 + 273)$$

$$\Rightarrow \quad 351.78\ K^2$$

$$T_H - T_L = 18.76 \quad \Rightarrow \quad T_H = 38.76\ {}^\circ C$$

Sept. 27/18

$\boxed{\text{Example}}$ From textbook 7.7 (8th edition)



100kPa, T=30°C } From table $v = 0.001004 \ m^3/kg$

Energy eq'n: $\cancel{q}^{0} + h_i + \cancel{V_i^2/2}^{0} + \cancel{gZ_i}^{0} = w + h_e + \cancel{V_e^2/2}^{0} + \cancel{gZ_e}^{0}$

$$h_i = w + h_e$$

Second law: $S_i = S_e$

Solution (I): we can use table to find h_e and h_i

$$\Rightarrow \quad w = h_i - h_e$$

Solution (II): $w = -\int_i^e v\,dP = -v\int_i^e dP$

$$w = -v(P_e - P_i) = -0.001004(5000-100)$$
$$= -4.92 \ kJ/kg$$

$h_e = ?$

$h_i = h_f = 125.77$ $\qquad S_i = 0.4369$

If the process is isentropic, $S_i = S_e = 0.4369$

$$h_e = h_i - w = 125.77 + 4.92 = 130.69 \ kJ/kg$$

$w = -\int_i^e v\,dP - \int_i^e T\,\delta S_{gen}$ irreversible

---

(For reversible flow of an incompressible fluid, v = const.)

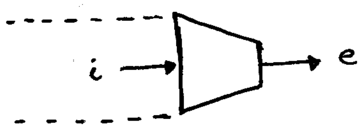
extended Bernoulli equation:

$$w = -v(P_e - P_i) + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e)$$

for the zero work term:

$$vP_i + \frac{1}{2}V_i^2 + gZ_i = vP_e + \frac{1}{2}V_e^2 + gZ_e$$

Example



6. - P_e = atmospheric pressure
5. - $V_i = 0$ (connected to large reservoir)
4. - $\Delta PE \simeq 0$

1. - incompressible Flow
2. - $W = 0$
3. - Ideal Flow (reversible, adiabatic)

$$v P_i + \cancel{V_i^2/2}^{\,0} + \cancel{g z_i}^{\,0} = v P_e + V_e^2/2 + \cancel{g z_e}^{\,0}$$

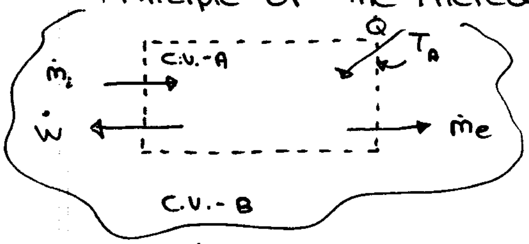
$$V_e^2/2 = v_i(P_i - P_e) \qquad v_i\Big|_{T=20°C} = 0.001002 \text{ (Table B1)}$$

Pascal conv.

$$V_e^2/2 = (0.001002)(300-101.3) \times 1000$$

$$V_e = 19.95 \text{ m/s}$$

Principle of the Increase of Entropy



$$\frac{dS_{c.v.A.}}{dt} = \dot m_i s_i - \dot m_e s_e + \frac{\dot Q}{T_A} + \dot S_{gen\,A}$$

$$\frac{dS_{c.v.B}}{dt} = \dot m_i s_i + \dot m_e s_e - \frac{\dot Q}{T_A} + \dot S_{gen\,B}$$

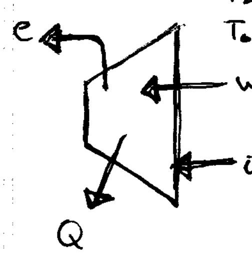
$$\frac{dS_{net}}{dt} = \frac{dS_{cvA}}{dt} + \frac{dS_{cvB}}{dt}$$

$$= \cancel{\dot m_i s_i}^{\,0} - \cancel{\dot m_e s_e}^{\,0} + \cancel{\dot Q/T_A}^{\,0} + \dot S_{gen\,A} - \cancel{\dot m_i s_i}^{\,0} + \cancel{\dot m_e s_e}^{\,0} - \cancel{\dot Q/T_A}^{\,0} + \dot S_{gen\,B}$$

$$= \dot S_{gen\,A} + \dot S_{gen\,B} \geq 0$$

- external irreversible process
- internal irreversible process

Example



$KE \simeq 0$
$PE \simeq 0$
$T_o = 30°C$

Continuity : $\dot m_i = \dot m_e = \dot m$ (I)

Energy eq'n : $\dot Q_{c.v.} + \dot m_i(h_i) = \dot m_e h_e + \dot W_{c.v.}$ (II)

Second Law : $0 = \dot m_i s_i - \dot m_e s_e + \dfrac{\dot Q_{c.v.}}{T} + \dot S_{gen}$ (III)

entropy eq.

From table B4: $h_i = 280.6$ kJ/kg $\quad S_i = 1.0272$ kJ/kg
$h_e = 307.8$ kJ/kg $\quad S_e = 1.014$ kJ/kg

From eq. (II) => $\dot Q_{c.v.} + 0.08(280.6) = 0.08(307.8) + (-3)$

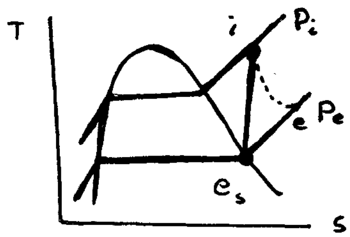
$$\boxed{\dot Q_{c.v.} = -0.824 \text{ kW}}$$

From eq. (III) => $\dot S_{gen} = -\dot m S_i + \dot m_e S_e + \dfrac{\dot Q_{c.v.}}{T}$

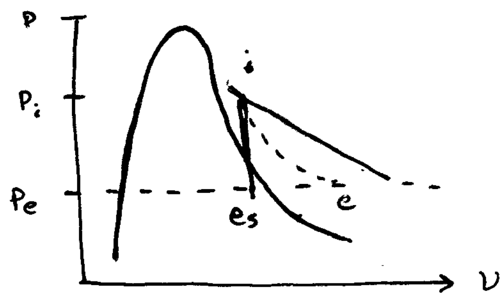
$$\dot S_{gen} = -(0.08)(1.0272) + (0.08)(1.014) + \frac{(-0.824)}{(30+273)}$$

$$\boxed{\dot S_{gen} = 0.00166 \text{ kW/K}}$$

Efficiency of the Turbine



$$\eta_{turbine} = \frac{w}{w_s} = \frac{h_i - h_e}{h_i - h_{es}}$$



$\boxed{\text{Example}}$ " Steam turbine recieves steam at a pressure "

$$\eta_{tur} = w_a / w_s = \frac{h_i - h_{ea}}{h_i - h_{es}}$$

Continuity eq'n: $\dot{m}_i = \dot{m}_e = \dot{m}$

Energy eq'n : $h_i = h_{es} + w_s$

Second law : $S_i = S_{es}$

From table $h_i = 3051.2 \ kJ/kg$ $\quad$ $S_i = 7.1228 \ kJ/kg.K$

$P_e = 15 \ kPa$ $\quad \Rightarrow \quad$ From table

$S_i = S_{es} = 7.1228$ $\quad$ @15 kPa $S_f = 0.7548$

$S_{fg} = 7.2536$

$S_{es} = S_f + x_{es} S_{fg} \Rightarrow x_{es} = 0.8779$

$h_{es} = h_f + x_{es} h_{fg} = 225.9 + 0.8779(2373.1) = \underline{2309 \ kJ/kg}$

$w_A = h_i - h_{ea} \Rightarrow h_{ea} = h_i - w_A$

$h_{ea} = 3051.2 - 600 = 2451.2 \ kJ/kg$

$$\eta_{tur} = \frac{w_a}{w_s} = \frac{3051.2 - 2451.2}{3051.2 - 2373.1} = 0.809 = 80.9 \%$$