Nov. 27/18

Example: Velocity of sound @ 300 K

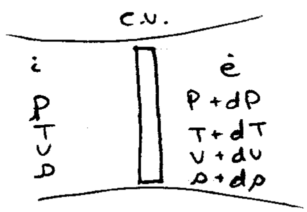
$$C = \sqrt{kRT} \rightsquigarrow C = \sqrt{(1.4)(0.287)(300\ K)(1000)}$$

In kJ (so multiply by 1000)

(for air): $k = 1.4$ $\quad C = 547.2\ m/s$

Reversible, adiabetic, one-dimensional flow of an ideal gas through a nozzle

↳ minimum cross-sectional area is the "throat"



$$\cancel{q} + h_i + V_i^2/2 + \cancel{gz_i} = \cancel{w} + h_e + V_e^2/2 + \cancel{gz_e}$$

$$\cancel{h} + \cancel{V^2/2} = (h + dh) + \frac{(V+dV)^2}{2} = (h + dh) + (1/2)(V^2 + 2dV\,V + dV^2)$$

$$0 = dh + VdV$$

Energy eq'n: $\quad dh + VdV = 0 \quad$ I

Property eq'n: $\quad Tds = dh - dP/\rho \quad$ II

Continuity eq'n: $\quad \dot{m} = \rho VA$

$$d\dot{m} = 0\ (d\rho)VA + \rho A dV + \rho V dA$$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad \text{III}$$

Combine I and II for isentropic process

$$dh = dP/\rho \Rightarrow \boxed{\frac{dP}{\rho} = -VdV}$$

$$\boxed{dV = -\frac{1}{\rho V}dP} \quad \text{IV}$$

From III: $\quad \frac{dA}{A} = \left(-\frac{d\rho}{\rho} - \frac{dV}{V}\right)$

Substitute IV in III $\Rightarrow$

$$\boxed{\frac{dA}{A} = -\frac{d\rho}{\rho}\left(\frac{dP}{dP}\right) + \left(\frac{1}{\rho V^2}\right)dP}$$

$$\frac{dA}{A} = \frac{-dP}{\rho}\left(\frac{d\rho}{dP} - \frac{1}{V^2}\right) = \frac{dP}{\rho}\left(-\frac{1}{(dP/d\rho)} + \frac{1}{V^2}\right)$$

$$dP/d\rho = c^2 = \left(\frac{V}{M_a}\right)^2$$

$$\boxed{\frac{dA}{A} = \frac{dP}{\rho V^2}\left(1 - M_a^2\right)}$$

For a nozzle, $dP < 0$:

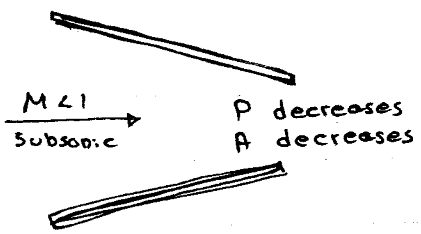
Subsonic nozzle: $M < 1$, $dA < 0$: nozzle converging

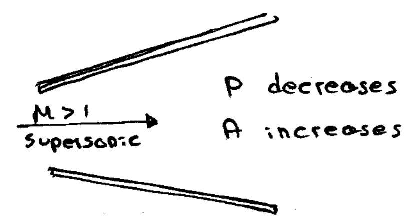
Supersonic nozzle: $M > 1$, $dA > 0$: nozzle diverging

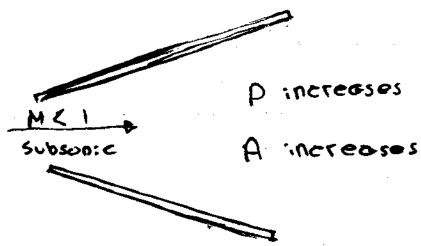
For a diffuser, $dP > 0$:

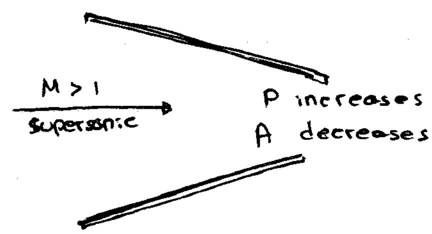
Subsonic diffuser: $M < 1$, $dA > 0$: diffuser diverging

Supersonic diffuser: $M > 1$, $dA < 0$: diffuser converging









$$h + \frac{V^2}{2} = h_o$$

Ideal gas $\Rightarrow$ $C_{p_o} T + \frac{V^2}{2} = C_{p_o} T_o$

$$V^2 = 2 C_{p_o} (T_o - T)$$

$C_p - C_v = R$

$C_p / C_v = k$

$\longrightarrow$ $C_p = \frac{Rk}{k-1}$ (*)

$C_v = \frac{R}{k-1}$ (**)

Substitute (*) in (IV) $\Longrightarrow$ $V^2 = 2 \frac{kRT}{k-1} \left(\frac{T_o}{T} - 1 \right)$

$V^2 = 2 \frac{c^2}{k-1} \left(\frac{T_o}{T} - 1 \right)$ $\Rightarrow$ $\frac{V^2}{c^2} = \frac{2}{k-1} \left(\frac{T_o}{T} - 1 \right)$

$M^2 = \frac{2}{k-1} \left(\frac{T_o}{T} - 1 \right)$ $\Rightarrow$ $\boxed{\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2}$ (A)

$$\left(\frac{T_o}{T} \right)^{k/k-1} = P_o / P$$

$\boxed{\frac{P_o}{P} = \left[1 + \frac{k-1}{2} M^2 \right]^{(k/k-1)}}$ (B) (TABLE A.12)

$$\left(\frac{T_o}{T} \right)^{1/k-1} = \frac{\rho_o}{\rho}$$

$\boxed{\frac{\rho_o}{\rho} = \left[1 + \frac{k-1}{2} M^2 \right]^{(1/k-1)}}$ (C)

Throat condition $(*)$ $(M = 1)$

$T^*/T_o = 2/_{k+1}$ $\qquad \rho^*/\rho_o = \left(\frac{2}{k+1}\right)^{(1/k-1)}$

$P^*/P_o = \left(\frac{2}{k+1}\right)^{(k/k-1)}$

T^*, P^*, ρ^* are critical properties

$$\dot{m} = \rho V A \rightsquigarrow \dot{m}/A = \rho V = \frac{P}{RT} V \frac{\sqrt{kT_o}}{\sqrt{kT_o}} = \frac{PV}{\sqrt{kRT}} \sqrt{\frac{k}{R}} \sqrt{\frac{T_o}{T}} \sqrt{\frac{1}{T_o}}$$

$$\dot{m}/A = PM/\sqrt{T_o} \sqrt{k/R} \sqrt{1 + \frac{k-1}{2} M^2}$$

Use eqn (B) $\Rightarrow$

(bigger)

$$\dot{m}/A = P_o/\sqrt{T_o} \sqrt{k/R} \left(\frac{M}{(1 + \frac{k-1}{2} - M^2)^{k+1/2(k-1)}}\right)$$

$$\frac{\dot{m}}{A} = \frac{P_o}{\sqrt{T_o}} \left(\sqrt{\frac{k}{R}}\right) \left(\frac{M}{(1 + \frac{k-1}{2} - M^2)^{\frac{k+1}{2(k-1)}}}\right)$$

At the throat $\Rightarrow$ $M = 1$

$$\boxed{\frac{\dot{m}}{A^*} = \frac{P_o}{\sqrt{T_o}} \left(\sqrt{\frac{k}{R}}\right) \frac{1}{\left(\frac{k+1}{2}\right)^{k+1/2(k-1)}}}$$

$$A/A^* = 1/M \left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2} M^2\right)\right]^{\frac{k+1}{2(k-1)}}$$

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Optional tutorial Tuesday @ Same class time, next week.

Example: " Convergent nozzle has an exit area ... "

$K = 1.4$

$\hookrightarrow$ using table A.12

$T^*/T_o = 0.8333 \Rightarrow T^* = 300\ K$

$P^*/P_o = \left(\frac{2}{K+1}\right)^{K/K-1}$

$P^* = 0.528$

when $M_1 = 1 \Rightarrow V = c = \sqrt{KRT}$

$= \sqrt{(1.4)(0.287)(1000)(300)} = 347.2$

$P^* = 528 \longrightarrow$ ideal gas law $\quad P^* = \rho^* R T^*$

$\rho^* = P^*/RT^*$

$$\rho^* = \frac{528}{(0.287)(300)} = 6.1324$$

$\dot{m} = \dot{m}^* = \rho A V = (6.1324)(500\times10^{-6})(347.2)$

$= 1.0646\ kg/s$

If: $P_E = 800\ kPa$: $P_E/P_o = 0.8 \xrightarrow[\text{Table A.12}]{\text{From}} M_E = 0.573$

$T_E/T_o = 0.9381$

$T_E = 337.2\ K$

$C_E = \sqrt{KRT} = \sqrt{(1.4)(0.287)(1000)(337.7)} = 368.4\ m/s$

$M_E = V_E/C_E \Rightarrow V_E = M_E \cdot C_E$

$\rightarrow V_E = (0.573)(368.4) = 211.1\ m/s$

$$\rho_E = \frac{P_E}{RT_E} = \frac{800}{(0.287)(337.7)} = 8.2542\ kg/m^3$$

$\dot{m} = (\rho V A)_E \quad (8.2542)(211.1)(500\times10^{-6}) = 0.8712\ kg/s$

Example: $A_E/A^* \Rightarrow$

$$\left.\begin{aligned} M_E &= 2.197 \\ P_E/P_o &= 0.0939 \\ T_E/T_o &= 0.5089 \end{aligned}\right\} \text{Interpolate}$$

$$P_E = 0.0939\,(1000) = 93.9 \text{ kPa}$$

$$T_E = 0.5089\,(360) = 183.2 \text{ K}$$

$$C_E = \sqrt{KRT_E} = \sqrt{(1.4)(0.287)(1000)(183.2)} = 271.3 \text{ m/s}$$

$$V_E = M_E C_E = 2.197\,(271.3) = 596.1 \text{ m/s}$$

b) $A_E/A^* = 2 \Rightarrow$
(subsonic)

$$\left.\begin{aligned} M_E &= 0.308 \\ P_E/P_o &= 0.936 \\ T_E/T_o &= 0.9812 \end{aligned}\right\} \text{Interpolate}$$

$$P_E = (0.936)(1000) = 936 \text{ kPa}$$

$$T_E = (0.9812)(360) = 353.3 \text{ K}$$

$$C_E = \sqrt{KRT_E} \Rightarrow \sqrt{(1.4)(0.287)(1000)(353.3)} = 376.8 \text{ m/s}$$

$$V_E = M_E C_E \Rightarrow (0.308)(376.8) = 116 \text{ m/s}$$

15.15 (from 8th)

Example: "Steam leaves a nozzle with a pressure of 500 kPa, temp of 350 °C, Velocity of ..."

$$h_o = h_1 + V^2/2$$

$P_1 = 500$ kPa

$T_1 = 350$ °C

from steam table:

$$h_1 = 3167.7 \text{ kJ/kg}$$

$$h_o = 3167.7 + \left(\frac{250^2}{2 \times 1000}\right) = 3198 \text{ kJ/kg}$$

$$S_o = S_1 = 7.6329 \text{ kJ/kg·K}$$

From steam table $\rightarrow$ $T_o = 365$ °C

$P_o = 556$ kPa

$\boxed{\text{Example:}}$ " Air leaves a compressor in a pipe with a stagnation ... "

$$h_o = h_1 + V^2/2$$

$$h_o - h_1 = V^2/2 \Rightarrow C_p(T_o - T_1) = V^2/2$$

$$(1.004)(150 - T_1) = \frac{125^2}{2000} \Rightarrow 142.2\ °C$$

$$T_1 = 142.2\ °C$$

$$P_1/P_o = (T_1/T_o)^{k/k-1} \Rightarrow P_1 = P_o(T_1/T_o)^{k/k-1}$$

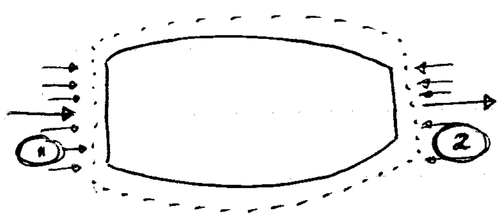
$$= (300)(415.4/432.2)^{1.4/0.4} = 281\ kPa$$

$$\dot{m} = \rho V A \longrightarrow (P_1/RT_1)\, V_1 A_1 = \left[\frac{281}{(0.287)(415.4)}\right](125)(0.02)$$

$$\dot{m} = 5.9\ kg/s$$

#27

$\boxed{\text{Example:}}$ " A jet engine at takeoff has air at 20°c ... "



$$A_1 = (\pi/4) D_1^2 = 0.7854\ m^2$$

$$A_2 = 0.1257\ m^2$$

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(293.2)}{100} = 0.8409\ m^3/kg$$

$$v_2 = 3.444\ m^3/kg$$

$$\dot{m}_1 = \dot{m}_2 \qquad 23.35\ ???$$

$$\dot{m}_1 = \rho_1 A_1 V_1 = \frac{A_1 V_1}{v_1} = \frac{(0.7854)(25)}{(0.8409)} = \cancel{48\ kg}/s$$

$$\dot{m}_2 = \rho_2 A_2 V_2 \longrightarrow V_2\ \frac{\dot{m} v_2}{A_2} \rightarrow \frac{(23.35)(3.444)}{(0.1257)}$$

$$V_2 = 641\ m/s$$

$$\Sigma F_x = (\dot{m}V)_{out} - (\dot{m}V)_{in}$$

$$-F - \cancel{(P_1 - P_a)A_1}^{\,0} + \cancel{(P_2 - P_a)A_2}^{\,0} = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

$$-F = \dot{m}(V_2 - V_1) \rightsquigarrow F = 14383\ N$$