OCT. 29TH/18

$$\dot{Q}_{in} + \dot{W}_{shaft,in} + \dot{W}_{pressure,\ net\ in} = \frac{dE_{sys}}{dt}$$

$$e = u + ke + pe = u + V^2/2 + gz$$

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{cv} e\rho\, dV + \int_{cs} e\rho(\bar{V}_i \cdot \bar{n})A$$

$$\dot{Q}_{net\ in} + \dot{W}_{shaft,in} + \dot{W}_{pressure,\ net,in} = d/dt \int_{cv} e\rho V + \int_{cs} e\rho(\bar{V}_i \cdot \bar{n})dA$$

(The net rate of energy transfer into a CV by heat + work transfer) = (The time rate of change of the energy content of the CV) + (The net flow rate of energy out of the control surface by mass flow)

Fixed C.V. : $\dot{Q}_{net,in} + \dot{W}_{shaft,\ net\ in} = d/dt \int_{cv} e\rho\, dV + \int_{cs} \left(\frac{P}{\rho}+e\right)\rho(\bar{V}_r \cdot \bar{n})dA$

$$\dot{m} = \int_{A_c} \rho(\bar{V}\cdot\bar{n})dA_c$$

$$\underbrace{\dot{Q}_{net\ in}}_{\text{heat added to system}} + \underbrace{\dot{W}_{shaft,\ net\ in}}_{\text{work added to system}} = d/dt \int_{cv} e\rho\, dV + \underbrace{\sum_{out}\dot{m}\left(\frac{P}{\rho}+e\right) - \sum_{in}\dot{m}\left(\frac{P}{\rho}+e\right)}_{\text{net energy flux}}$$

where $e = u + ke + pe$

$$\dot{Q}_{net,in} + \dot{W}_{shaft,\ net\ in} = d/dt \int_{cv} e\rho\, dV + \sum_{out}\dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right) - \sum_{in}\dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right)$$

where $h = u + Pv$

$$= u + P/\rho$$

Steady state :

$$\dot{Q}_{net,in} + \dot{W}_{shaft,\ net\ in} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_1 - z_2)\right)$$

$$q_{net,in} + w_{shaft,\ net\ in} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_1 - z_2)$$

$$\hookrightarrow h = u + P/\rho$$

$$w_{shaft,net\ in} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{net,in})$$

Ideal Flow (no mech. energy loss) : $q_{net,in} = u_2 - u_1$

Real Flow (w/ mech. energy loss) : $e_{mech,loss} = u_2 - u_1 - q_{net,in}$

$$e_{mech,in} = e_{mech,out} + e_{mech,loss}$$

Energy equation in terms of heads

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump,u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_L + h_{turbine,e}$$

$$h_{pump,u} = \frac{w_{pump,u}}{g} = \frac{\dot{W}_{pump,u}}{\dot{m} g} = \frac{\eta_{pump} \dot{W}_{pump}}{\dot{m} g}$$

$$h_{turbine,e} = \frac{w_{turbine,e}}{g} = \frac{\dot{W}_{turbine,e}}{\dot{m} g} = \frac{\dot{W}_{turbine}}{\eta_{turbine} \dot{m} g}$$

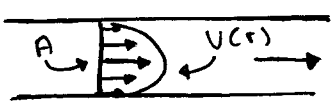
$$h_L = \frac{e_{mech,\ loss\ piping}}{g} = \frac{\dot{E}_{mech,\ loss\ piping}}{\dot{m} g}$$

Kinetic energy correction Factor, α

Eq by replacing $V^2/2$ by $\alpha V_{avg}^2/2$

$$w_{in} + \frac{P_1}{\rho_1} + \alpha_1 \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho_2} + \alpha_2 \frac{V_2^2}{2} + g z_2 + \text{losses}$$

Example Kinetic energy correction factor using actual velocity distribution.



$$\dot{KE}_{act} = \int ke\, \delta \dot{m} = \int_A \tfrac{1}{2}[V(r)^2][\rho V(r)\, dA] = \tfrac{1}{2}\rho \int_A [V(r)]^3\, dA$$

$\dot{m} = \rho V_{avg} A, \quad \rho = \text{const.}$

$$\dot{KE}_{avg} = \frac{1}{2}\dot{m} V_{avg}^2 = \frac{1}{2}\rho A V_{avg}^3$$

$$\alpha = \frac{KE_{act}}{KE_{avg}} = \frac{1}{A}\int \left(\frac{V(r)}{V_{avg}}\right)^3 dA$$

$\boxed{\text{Example}}$ Pump powered by 15 kW motor, $\eta_{motor} = 90\%$

$\dot{V} = 50$ L/s, $D_{in} = D_{out}$, $Z_1 \doteq Z_2$

$$\eta_{pump} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump,shaft}}$$

$$\dot{m} = \rho \dot{V} = (1000)(0.050) = 50 \text{ kg/s}$$

$$\dot{W}_u + \dot{m}\left(P_1/\rho_1 + \alpha_1 V_1^2/2 + gZ_1\right) = \dot{m}\left(P_2/\rho + \alpha_2 V_2^2/2 + gZ_2\right)$$

$$\dot{W}_u = \dot{m}\left(\frac{P_2 - P_1}{\rho} + \alpha_2 \frac{V_2^2}{2} - \alpha_1 \frac{V_1^2}{2} + g(Z_2 - Z_1)\right)$$

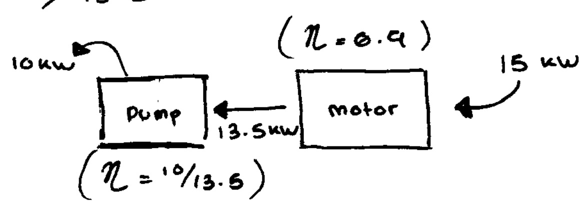
$$A_1 V_1 = A_2 V_2$$

$$A_1 = A_2 \rightarrow V_1 = V_2$$

$$\dot{W}_u = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = 50\left(\frac{300 - 100}{1000}\right) = 10 \text{ kW}$$

$$\dot{W}_{shaft} = \eta_{motor} \times \dot{W}_{motor} = (0.9)(15) = 13.5 \text{ kW}$$

$$\eta_{pump} = 10/13.5 = 0.741 \text{ or } 74.1\%$$



$$\dot{E}_{loss} = \dot{m} C \Delta T \Rightarrow \Delta T = \frac{\dot{E}_{loss}}{\dot{m} C}$$

$$\Delta T = \frac{(3.5)}{(50)(4.18)} = 0.017\ ^\circ C$$

6.13

$\boxed{\text{Example:}}$ $\dot{V} = 100\ m^3/s$, $z_1 = 120\ m$, $P_1 = P_2$, $V_1 \cong V_2 \cong 0$

$$\cancel{\frac{P_1}{\rho_1 g}}^{0} + \cancel{\frac{\alpha V_1^2}{2g}}^{0} + z_1 + \cancel{h_{pump}}^{0} - h_{turb} - h_{loss} = \cancel{\frac{P_2}{\rho_2 g}}^{0} + \cancel{\frac{\alpha V_2^2}{2g}}^{0} + \cancel{z_2}^{0}$$

$$\boxed{z_1 - h_{turb} - h_{loss} = 0} \Rightarrow h_{turb} = z_1 - h_L$$

$$= 120 - 35 = 85\ m$$

$$\dot{W}_{turb,ideal} = \dot{m} g h_{turb} = \rho \dot{V} g h_{turb} = (1000)(100)(9.81)(85)$$

$$\dot{W}_{turb,ideal} = 83400\ kW$$

$$\dot{W}_{turb,out} = \eta_{turgen} \dot{W}_{turb,ideal} = (0.8)(83400) = 66.7\ MW$$

Oct. 31/18

$\boxed{\text{Example}}$ "A Fan is to be selected to cool a computer case..."

$$\dot{m}\left(\cancel{P_1}/\rho + \alpha_1 \underbrace{\cancel{\frac{V_1^2}{2}}}_{\cong 0} + \cancel{gz_1}\right) + \dot{W}_{Fan} = \dot{m}\left(\cancel{P_2}/\rho + \alpha_2 \frac{V_2^2}{2} + \cancel{gz_2}\right) + \dot{E}_{losses}$$

$$\dot{m} = \rho \dot{V}$$

$$V_{case} = (12 \times 40 \times 40)(0.5) = 9600\ cm^3$$

$$\dot{V} = 9600/1s = 9600\ cm^3/s$$

$$= 9.6 \times 10^{-3}\ m^3/s$$

$$\dot{m} = (1.2)(9.6 \times 10^{-3}) = 0.0115\ kg/s$$

$$\dot{W}_{Fan,u} = \dot{W}_{Fan} - \dot{E}_{loss} = \dot{m}\,\alpha_2 \frac{V^2}{2}$$

For Fully developed turbulent flow $\alpha_2 = 1.11$

$$\dot{W}_{Fan,u} = 0.0115(1.11)\,V_2^2/2 \Rightarrow \dot{W}_{Fan,u} = 0.153\ W$$

$$A = (\pi/4)D^2 = \frac{\pi(0.05)^2}{4} = 1.96 \times 10^{-3}\ m^2$$

$$V_2 = \dot{V}/A = \frac{9.6 \times 10^{-3}}{1.96 \times 10^{-3}} = 4.9\ m/s$$

$$\eta_{th} = \frac{\dot{W}_{Fan,u}}{\dot{W}_{electricity}} = \dot{W}_{electricity} = \frac{\dot{W}_{Fan,u}}{\eta_{th}} = \frac{0.153}{0.3}$$

$$\rightarrow \dot{W}_{electricity} = \boxed{0.506\ W}$$

b) $$\dot{m}\left(P_3/\rho + \alpha_3 \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right) + \dot{W}_{Fan} = \dot{m}\left(P_4/\rho + \alpha_4 \cancel{V_4^2}/2 + \cancel{gz_4}\right) + \dot{E}_{loss}$$

$$\dot{W}_{Fan,u} = \dot{W}_{Fan} - \dot{E}_{loss} = 0.153\ W$$

Assuming $V_3 A_3 = V_4 A_4$

$$\left.\begin{array}{l} V_3 = V_4 \\ \alpha_3 = \alpha_4 \end{array}\right\}$$

$$\dot{W}_{Fan,u} = \dot{m}\,\frac{P_4 - P_3}{\rho} \Rightarrow P_4 - P_3 = \frac{\rho \dot{W}_{Fan,u}}{\dot{m}}$$

$$P_4 - P_3 = \frac{(1.2)(0.153)}{(0.0115)} = \boxed{15.8\ Pa}$$

Linear momentum:

$$\vec{F} = m\vec{a} = m\, d\vec{V}/dt = \frac{d(m\vec{V})}{dt}$$

Angular momentum:

$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

about x-axis:

$$M_x = I_x \frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

→ Total angular momentum of a rotating body remains constant when the net torque acting on it is zero.

Thus, anguar momentum is conserved.

Body Forces: gravity, electric, magnetic

Surface Forces: that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact)

→ only external forces are considered in analysis

Total Force acting on control volume: $\Sigma\vec{F} = \Sigma\vec{F}_{body} + \Sigma\vec{F}_{surface}$

Normal stresses: pressure and viscous stresses

Shear stresses: composed entirely of viscous stresses

Surface force acting on a differential surface element:

$$d\vec{F}_{surface} = \sigma_{ij} \cdot \vec{n}\, dA$$

General: $$\Sigma\vec{F} = \frac{d}{dt}\int_{cv} \rho\vec{V}\, dV + \int_{cs} \rho\vec{V}(\vec{V}\cdot\vec{n})\, dA$$

(The sum of all forces acting on CV) = (the time rate of change of lin. mom. of the contents of cv) + (net flow rate of linear mom. out of the cs by mass flow)