Motor : $\eta_{motor} = \dfrac{\text{Mechanical power output}}{\text{Electrical power input}} = \dfrac{\dot{W}_{shaft,out}}{\dot{W}_{elec,in}}$

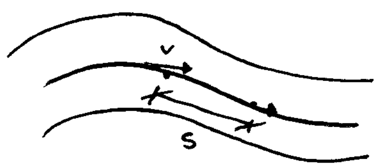
Generator : $\eta_{gen} = \dfrac{\text{Electric power output}}{\text{Mech. power input}} = \dfrac{\dot{W}_{elec,out}}{\dot{W}_{shaft,in}}$

Pump-motor ; overall efficiency

$$\eta_{pump-motor} = \eta_{pump}\eta_{motor} = \frac{\dot{W}_{pump,in}}{\dot{W}_{elec,in}} = \frac{\Delta E_{mech,fluid}}{\dot{W}_{elec,in}}$$

Turbine generator, overall efficiency

$$\eta_{turbine-gen} = \eta_{turbine}\eta_{gen} = \frac{\dot{W}_{elec,out}}{\dot{W}_{turb,e}} = \frac{\dot{W}_{elec,out}}{\Delta E_{mech,fluid}}$$



$$dV = \frac{\partial V}{\partial s}ds + \frac{\partial V}{\partial t}dt \quad : V(t,s)$$

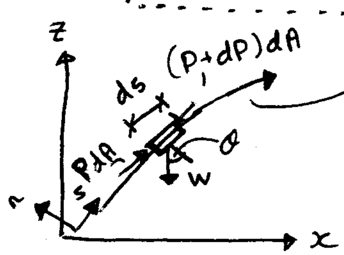
$$\phi(a,b,c) \Rightarrow d\phi = \frac{\partial\phi}{\partial a}da + \frac{\partial\phi}{\partial b}db + \frac{\partial\phi}{\partial c}dc$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$\left.\begin{array}{l} V = V(s) \\ \partial V/\partial t = 0 \end{array}\right\} \text{steady state}$$

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s}V = V\frac{dV}{ds} \Rightarrow a_s = V\frac{dV}{ds}$$

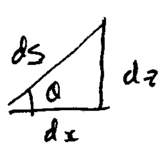
$$\boxed{V = \frac{ds}{dt}}$$



steady flow along streamline

$$\Sigma F_s = ma_s = mV\left(\frac{dV}{ds}\right)$$

$$PdA - (P+dP)dA - W(\sin\theta) = mV\left(\frac{dV}{ds}\right)$$



$$W = \rho \forall g = \rho g\, ds\, dA$$
$$(\text{or } m = \rho\forall = \rho\, ds\, dA)$$

$$\sin\theta = \frac{dz}{ds} \qquad -dP\,dA - \rho g\, dA\, ds\frac{dz}{ds} = \rho\, dA\, ds\, V\frac{dV}{ds}$$

$$-dP - \rho g\, dz = \rho V dV$$
$$VdV = (1/2)\, d(V^2)$$

$$\frac{dP}{\rho} + \left(\frac{1}{2}\right)d(V^2) + g\,dz = 0$$

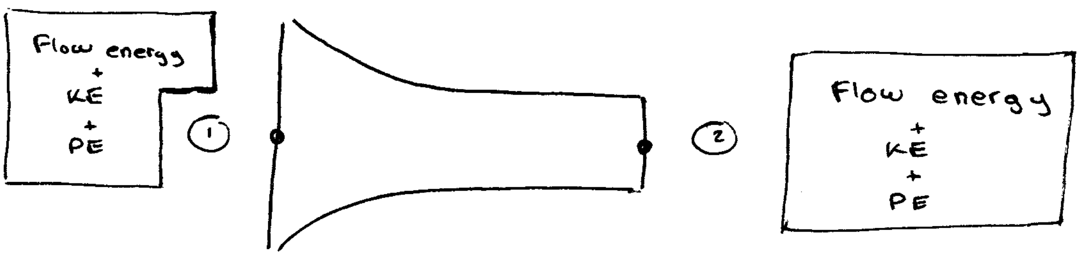
Steady Flow

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

Steady, incompressible Flow

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

flow energy — kinetic energy — potential energy



where $E_1 = E_2$, but individual values are not necessarily the same.

Force Balance across streamlines

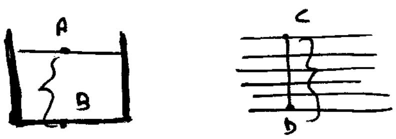
$$P/\rho + \int \frac{V^2}{R}\,dn + gz = \text{constant} \qquad \text{(across streamlines)}$$

For a flow along a straight line, $R \to \infty$

reducing equation to: $P/\rho + gz = \text{constant}$

$P = -\rho g z + \text{constant}$

(expression for variation of hydrostatic pressure w/ vert. distance for a stationary fluid)



where $P_A - P_B = P_C - P_D$

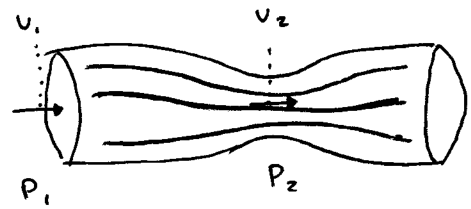
Bernoulli for unsteady, compressible Flow:

$$\int \frac{dP}{\rho} + \boxed{\int \frac{\partial V}{\partial t}\,ds + \frac{V^2}{2}} + gz = \text{constant}$$

$$dV = \frac{\partial V}{\partial s}\,ds + \frac{\partial V}{\partial t}\,dt$$

Compressible, unsteady :
Incompressible, unsteady :
Compressible, steady :
Incompressible, steady :

Example :



By Continuity :

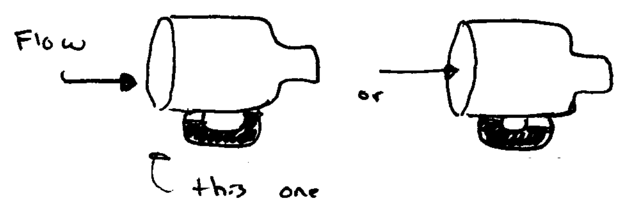
$A_1 V_1 = A_2 V_2 \rightarrow A_1 > A_2 \rightarrow V_1 < V_2$

By Bernouli

$$\frac{P_1}{\rho} + \tfrac{1}{2} V_1^2 + g\cancel{z_1}^{0} = \frac{P_2}{\rho} + \tfrac{1}{2} V_2^2 + g\cancel{z_2}^{0}$$

thus $P_2 < P_1 \rightarrow P_1 > P_2$

Example :



Example 10-cm fire hoze with 3-cm nozzle discharges 0.025 m3/s (Frictionless flow)

where $Q = AU$

→ Finding V_1, V_2

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1}^{0} = \cancel{\frac{P_2}{\rho}}^{0} + \frac{V_2^2}{2} + g\cancel{z_2}^{0}$$

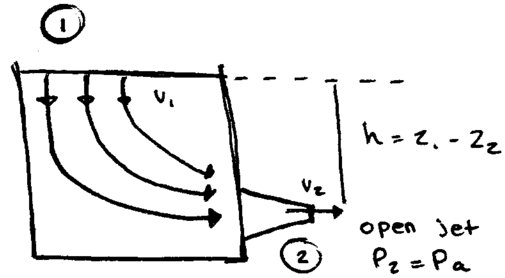
→ open to atmosphere (0 gage)

$Q = 0.025\,m^3/s \rightarrow V_1 = \dfrac{(0.025)}{[\pi/4(0.10)^2]}$; $V_1 = 3.2$ m/s

$V_2 = 35.4$ m/s

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{V_2^2}{2}$$ (where $\rho = 1000$ kg/m³)

Example:



→ Cons. of Mass:

$A_1 V_1 = A_2 V_2 \rightarrow V_1 = \dfrac{A_2 V_2}{A_1}$

→ Bernoulli's:

$$\cancel{\frac{P_1}{\rho}} + \frac{V_1^2}{2} + z_1 g = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + z_2 g$$

$P_1 = P_2$

$$\rightarrow V_2^2 - V_1^2 = 2g(z_1 - z_2) = 2gh$$

Thus, $V_2^2 = \dfrac{2gh}{1 - (A_2^2/A_1^2)}$

For a large tank, $A_1 \gg A_2$ } but you can't always assume this.

$V_2^2 = 2gh$

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant} \quad \text{(along a streamline)}$$

Oct. 24/18

Example: Derive the equation when compressibility effects are not negligible for an ideal gas undergoing
(a) an isothermal process
(b) isentropic process

a) $P = \rho RT \Rightarrow \rho = \frac{P}{RT}$

$$\int \frac{dP}{P/RT} + \frac{V^2}{2} + gz = \text{const.}$$

$$RT\int \frac{dP}{P} + V^2/2 + gz = \text{const.}$$

$$\boxed{RT \ln P + V^2/2 + gz = \text{const.}}$$

isothermal ideal gas

b) $Pv^k = \text{Const}$ (isentropic)

$\rho = \frac{1}{v} \Rightarrow P/\rho^k = \text{const.} = C$

$$\rho = C^{-1/k} P^{1/k}$$

$$\int \frac{dP}{C^{-1/k} P^{1/k}} + V^2/2 + gz = \text{const.}$$

$$\Rightarrow \frac{C^{1/k} P^{-1/k}}{1 - 1/k} = \left(\frac{P}{\rho^k}\right)^{1/k} \frac{P^{-1/k+1}}{1 - 1/k}$$

$$= \left(\frac{k}{k-1}\right)\left(P/\rho\right)$$

$$\Rightarrow \boxed{\left(\frac{k}{k-1}\right)\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{const.}}$$

isentropic ideal gas.

Limitations of use of Bernoulli's :

Steady flow: applicable

Frictionless flow: friction effects may / may not be negligible

No shaft work: Bernoulli can't be used when a device is : use energy eq'n instead.

Incompressible Flow: Density is taken constant, flow is incompressible for liquids and gases where $Ma < 0.3$

... etc.

Stagnation Pressure : The sum of the static and dynamic pressures. Represents the point where fluid is brought to a stop isentropically.

$$P_{stag} = P + \rho \frac{V^2}{2} \quad (\text{kPa})$$

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

Hydraulic Grade Line **(HGL)** and Energy Grade Line **(EGL)**

$$\underbrace{\frac{P}{\rho g}}_{\text{pressure head}} + \underbrace{\frac{V^2}{2g}}_{\text{velocity head}} + \underbrace{z}_{\text{elevation head}} = \underbrace{H}_{\text{total head}} = \text{constant} \quad (\text{along a streamline})$$

(dividing each term by g in Bernouli)

then

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad (\text{heights})$$

HGL : $\frac{P}{\rho g} + z$

EGL : $\frac{P}{\rho g} + \frac{V^2}{2g} + z$

Difference between the two: Dynamic Head : $\frac{V^2}{2g}$

5.5

Example: water is flowing from a hose attached to a water main at 400 kPa gage. A child places his thumb to cover up most of the outlet, causing a thin jet of high-speed water to emerge. If the hose is held upwards, what's the maximum amount it can achieve?

$$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}}^{0} + \cancel{g z_1}^{0} = \cancel{\frac{P_2}{\rho}}^{0} + \cancel{\frac{V_2^2}{2}}^{0} + g z_2$$

Since $\frac{V_1^2}{2} \ll 1 \rightarrow \frac{400\times10^3}{1000} = 9.81(z_2)$

$z_2 = 40.8\,m$

5.7

Example:

a) $\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}}^{0} + g z_1 = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + g z_2$

same

then $V_2^2 = 2g(z_1 - z_2)$

$V_2 = 3.84\,m/s$

then $\dot{V} = A_2 V_2$

$\dot{V} \Rightarrow (\pi/4)(4\times10^{-3})^2(3.84) = 7.53\times10^{-5}\,m^3/s$

then $\Delta t = \frac{V}{\dot{V}} = \boxed{53.1\text{ sec}}$

b) $\frac{P_3}{\rho} + \cancel{\frac{V_3^2}{2}} + g z_3 = \cancel{\frac{P_2}{\rho}}^{0} + \cancel{\frac{V_2^2}{2}} + g z_2$

then $P_3 = \rho g (z_2 - z_3)$

$= (750)(9.81)(0 - 2.75)$

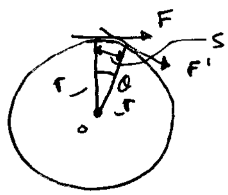
$= \boxed{-20.2\text{ kPa}}$

Energy Transfer by Heat, Q

Thermal Energy : The sensible and latent forms of internal energy.

Power: Rate of doing work.

Shaft work: $T = Fr$, $F = T/r$



where $s = r\theta = r\,2\pi n$

$W = Fs = \frac{T}{r}\, r\, 2\pi n = 2\pi n T$

$\dot{W} = \frac{dW}{dt} = d/dt\,(2\pi n T)$

Work done by pressure forces

$\delta W_{boundary} = PA\,ds$

$\delta \dot{W}_{pressure} = \delta \dot{W}_{boundary} = PAV_{piston}$ $\quad V_{piston} = ds/dt$

$\delta \dot{W}_{pressure} = -P\,dA\,V_n = -P\,dA\,(\vec{V}\cdot\vec{n})$

$\dot{W}_{pressure,\,net\,in} = \int_A -P(\vec{V}\cdot\vec{n})\,dA = -\int_A \frac{P}{\rho}\rho\,(\bar{V}\cdot\vec{n})\,dA$

$\rightarrow \dot{W}_{net\,in} = \dot{W}_{shaft,\,in\,net} + \dot{W}_{pressure,\,in\,net} = \dot{W}_{shaft\,in\,net} - \int_A P(\bar{V}\cdot n)\,dA$

Oct. 26/18

5.52

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2$$

Ideal gas (incompressible) $\Rightarrow \rho_1 = \rho_2$

$z_2 - z_1 = 0.2\,m$

$P = \rho RT \rightarrow \rho_s = \frac{P}{RT} = \frac{105}{(0.287)(37+273)} = 1.18\ kg/m^3$

$\dot{V} = 65\ L/s$ or $0.065\ m^3/s$

$\dot{V} = A_1V_1 = A_2V_2 \qquad V_1 = \frac{\dot{V}}{A_1} = \frac{0.065}{(\pi/4)(0.06)^2}$

$V_1 = 22.99\ m/s$

$V_2 = \frac{\dot{V}}{A_2} = \frac{0.065}{(\pi/4)(0.04)^2}$

$V_2 = 51.73\ m/s$

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

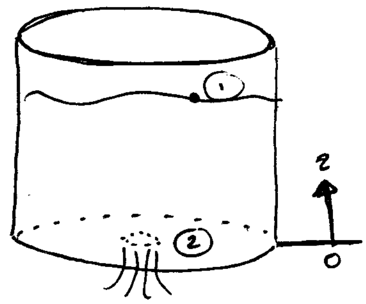
$$P_1 - P_2 = \rho\left[\left(\frac{51.73^2}{2} - \frac{22.99^2}{2}\right) + (9.81)(0.2)\right]$$

$$\boxed{P_1 - P_2 = 1268.96\ Pa}$$

$P_1 - P_2 = \rho_{water} gh \Rightarrow h = \frac{P_1 - P_2}{\rho_{water}\, g} = \frac{(1268.96)}{(1000)(9.81)} = 0.1293\ m$ or $\boxed{12.9\ cm}$

$$\boxed{5.61}$$

$$\cancel{\frac{P_1}{\rho}}^{0} + \cancel{\frac{V_1^2}{2}}^{0} + gZ_1 = \cancel{\frac{P_2}{\rho}}^{0} + \frac{V_2^2}{2} + \cancel{gZ_2}^{0}$$



$$gZ_1 = \frac{V_2^2}{2} \Rightarrow V_2 = \sqrt{2gZ_1}$$

$$\dot{m}_{out} = \rho V_2 A_2 = \rho\sqrt{2gZ_1} \quad \frac{\pi D_o^2}{4}$$

$$Z = h_{max}: \quad \dot{m}_{in} = \dot{m}_{out} \Rightarrow \dot{m}_{in} = \rho\sqrt{2gh_{max}}\left(\frac{\pi D_o^2}{4}\right)$$

$$\boxed{h_{max} = \frac{1}{2g}\left(\frac{4\dot{m}_{in}}{\rho\pi D_o^2}\right)^2}$$

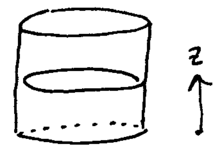
$$Z = f(t) = ?$$

$$dm_{out}/dt = \dot{m}_{out} \Rightarrow dm_{out} = \dot{m}_{out}\,dt$$

$$\Rightarrow dm_{out} = \rho\sqrt{2gz}\left(\frac{\pi D_o^2}{4}\right)dt$$

$$dm_{tank} = \rho A_{tank}\,dZ = \rho\frac{\pi D_1^2}{4}dz$$

$$\boxed{dm_{tank} = \dot{m}_{in}dt - \dot{m}_{out}dt}$$



$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{\rho\pi D_T^2}{4}dz = \dot{m}_{in}dt - \rho\sqrt{2gz}\,\frac{\pi D_o^2}{4}dt$$

$$\frac{\rho\pi D_1^2}{4}dz = \left(\dot{m}_{in} - \rho\sqrt{2gz}\left(\frac{\pi D_o^2}{4}\right)\right)dt$$

$$\int_{z=0}^{z}\frac{\left(\rho\frac{\pi D_1^2}{4}\right)}{\dot{m}_{in} - \rho\sqrt{2gz}\left(\frac{\pi D_o^2}{4}\right)}dz = \int_0^t dt = t$$

$$\frac{(1/2)\rho\pi D_T^2}{((1/4)\rho\pi D_o^2\sqrt{2g})^2}\left(1/4\,\rho\pi D_o^2\sqrt{2gz} - \dot{m}_{in}\ln\frac{\dot{m}_{in} - 1/4\,\pi D_o^2\sqrt{2gz}}{\dot{m}_{in}}\right) = t$$

$\boxed{5.55}$ where $P_a = 100$ kPa

$$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}}^{0} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}^{0}$$

Assume $D_T >> D_o \Rightarrow V_1 = 0$

$$\frac{V_2^2}{2} = \frac{P_1 - P_2}{\rho} + gz_1$$

$$V_2 = \sqrt{2\left(\frac{P_1 - P_2}{\rho}\right) + gz_1} = \sqrt{2\left(\frac{250-100}{1000}\right)(1000) + (9.81)(2.5)}$$

$$V_2 = 18.7 \text{ m/s}$$

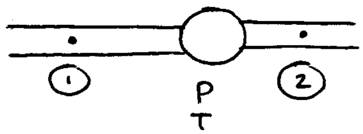
$$\dot{V} = ?$$

$$\dot{V} = A_2 V_2 = \frac{\pi(0.1)^2}{4}(18.7) = \boxed{0.147 \text{ m}^3/\text{s}}$$

5.76

$$\frac{P}{\rho} + \frac{V^2}{2} + gZ = \text{const.}$$

↳ intro...



pump head

turbine head

head loss

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 + g(h_P - h_T - h_{Loss}) = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2$$

general energy equation

large tank 0 — no turbine 0 — no losses mentioned 0 — at vertex

$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + gZ_1 + gh_P - \cancel{gh_T} - \cancel{gh_{loss}} = \cancel{\frac{P_2}{\rho}} + \cancel{\frac{V_2^2}{2}} + gZ_2$$

$$gZ_1 + gh_P = gZ_2$$

$$gh_P = g(Z_2 - Z_1)$$

$$h_P = [\cancel{(9.81)}(27-20)]/\cancel{(9.81)}$$

$$h_P = 7\,m$$

or

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_P - h_T - h_{loss} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2}$$

$$\Delta P = \rho g h \quad \Rightarrow \quad \Delta P_{pump} = \rho g h_P = 1000 \times 9.81 \times 7$$

$$\Delta P_{pump} = 68.7 \times 10^3 \text{ Pa}$$

$$\boxed{= 68.7 \text{ kPa}}$$

$$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}} + \cancel{Z_1} - h_{loss} - \underbrace{\cancel{h_T}}_{\text{no turb.}} + \underbrace{\cancel{h_P}}_{\text{no pump}} = \frac{P_2}{\rho} + \cancel{\frac{V_2^2}{2}} + \cancel{Z_2}$$

$$Z_1 = Z_2$$

$$\dot{V} = A_1 V_1 = A_2 V_2 \Rightarrow V_1 = V_2$$

$$\frac{P_1 - P_2}{\rho} = h_{loss} = \frac{2 \times 1000}{1000 \times 9.81} = \boxed{0.204\ m}$$

$$\dot{W}_{pump} = \dot{m} g h_{loss}$$

$$= \rho \dot{V} g h_{loss}$$

$$= (1000)(0.02\ m^3/s)(9.81)(0.204)$$

$$\dot{W}_{pump} = 40\ N \cdot m/s = 40\ J/s = \boxed{40\ W}$$