FLUIDS

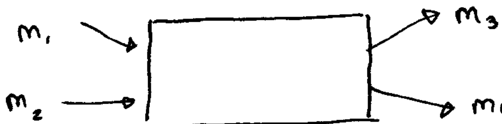
Oct. 15/18

Relative velocity : $\vec{V}_r = \vec{V} - \vec{V}_{cs}$

RTT, nonfixed CV : $\dfrac{dB_{sys}}{dt} = \dfrac{d}{dt}\displaystyle\int_{cv} \rho b \, dV + \int_{cs} \rho b \, d\vec{V}_r \cdot \vec{n} \, dA$

RTT, steady flow : $\dfrac{dB_{sys}}{dt} = \displaystyle\int_{cs} \rho b \vec{V}_r \cdot \vec{n} \, dA$

$$\underbrace{\frac{d}{dt}\int_{cv} \rho \, dV}_{\text{control volume boundary}} + \underbrace{\int_{cs} \rho(\vec{v} \cdot \vec{n}) \, dA}_{\text{Fluxes}} = 0 \qquad (B = m, \; b = B/m, \; b = 1)$$



$$\cancel{\frac{d}{dt}\int_{cv} \rho \, dV}^{\,0} + \sum \dot{m}_{in} - \sum \dot{m}_{out} = 0$$

Steady state : $\sum \dot{m}_{in} - \sum \dot{m}_{out} = 0$

$$\boxed{\sum \dot{m}_{in} = \sum \dot{m}_{out}}$$

$$\int_A \rho b \vec{V}_r \cdot \vec{n} \, dA \doteq b_{avg} \int_A \rho \vec{V}_r \cdot \vec{n} \, dA$$
$$= b_{avg} \dot{m}_r$$

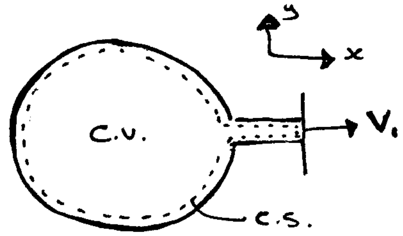
$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{c.v.} \rho b \, dV + \sum_{out} \underbrace{\dot{m}_r b_{avg}}_{\text{for each outlet}} - \sum_{in} \underbrace{\dot{m}_r b_{avg}}_{\text{for each inlet}}$$

$$\dot{m}_r \approx \rho_{avg} \dot{V}_r = \rho_{avg} V_{r,avg} A$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} \rho b \, dV + \sum_{out} \underbrace{\rho_{avg} b_{avg} V_{r\,avg} A}_{\text{for each outlet}} - \sum_{in} \underbrace{\rho_{avg} b_{avg} V_{r\,avg} A}_{\text{for each inlet}}$$

Example: A tank of $0.05\ m^3$, @ 800 kPa
15 °C

At $t = 0$, air escapes through valve with area $65\ mm^2$.
Air passing valve has speed 300 m/s, $\rho = 6\ kg/m^3$
Determine instantaneous rate of change of density @ $t = 0$



Assumptions :
- time dependent problem
- uniform flow
- properties are uniform

$$\frac{d}{dt}\int_{cv} \rho\, d\forall + \int_{cs} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

$$\frac{d}{dt}\, \rho_{cv} \int_{cv} d\forall + \int_{cs} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

$$\frac{d}{dt}\, \rho_{cv}\forall + \int_{cs} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

$$\underbrace{d/dt\, \rho_{cv}\forall}_{\text{Change in CV}} + \underbrace{\rho_1 V_1 A_1}_{\text{Fluxes}} = 0 \quad \Rightarrow \quad d/dt\, \rho_{cv}\forall = -\rho_1 V_1 A_1$$

$$\Rightarrow \frac{d\rho_{cv}}{dt} = -\frac{\rho_1 V_1 A_1}{\forall}$$

$$@\ t = 0\ ;\ \frac{d\rho_{cv}}{dt} \Rightarrow -(6)(300)(65)\left(\frac{1}{0.05}\right)\left(\frac{1}{10^6}\right) = -2.34\ \left(\frac{kg/m^3}{s}\right)$$

END.

The conservation of mass for a closed system undergoing a change is expressed as m_{sys} = constant

→ mass remains same during process

$\dot{m}_{in} - \dot{m}_{out} = dm_{cv}/dt$ → Conservation of mass

Linear momentum - Product of mass and velocity of body or "momentum of the body"

Linear momentum eq'n - Newton's second law

The conservation of energy principle : the net energy transfer to or from a system

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The conservation of energy principle (the energy balance):
The net energy transfer to/from a system is equal to the change in the energy content.

Conservation of mass: mass, like energy, is a conserved property.
mass flow rate: the amount of mass flowing through a cross-section per unit time.

The differential mass flow rate:

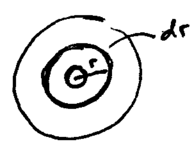
$\delta \dot{m} = \rho V_n dA_c \rightsquigarrow \dot{m} = \int_{Ac} \delta \dot{m} = \int_{Ac} \rho V_n dA_c$ (kg/s)

$V_{avg} = \frac{1}{A_c} \int_{Ac} V_n dA_c$ (Average velocity)

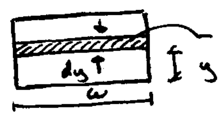
Mass flow rate:

$\dot{m} = \rho V_{avg} A_c$ (kg/s)

$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$



$dA = 2\pi r dr$



$dA = w\, dy$

Volume flow rate:

$\dot{V} = \int_{Ac} V_n dA_c = V_{avg} A_c = V A_c$ (m^3/s)

Conservation of mass principle for control volume:

$$\begin{pmatrix} \text{Total mass} \\ \text{entering CV} \\ \text{during } \Delta t \end{pmatrix} - \begin{pmatrix} \text{Total mass} \\ \text{leaving CV} \\ \text{during } \Delta t \end{pmatrix} = \begin{pmatrix} \text{Net change of} \\ \text{mass within the} \\ \text{CV during } \Delta t \end{pmatrix}$$

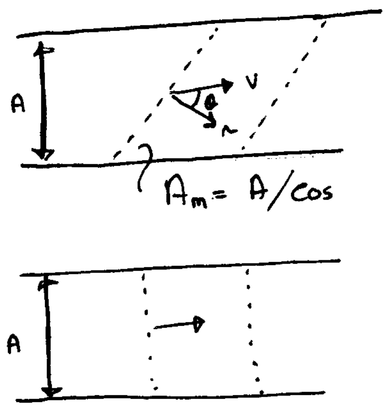
General Conservation of mass:

$d/dt \int_{CV} \rho dV + \int_{CS} \rho (\bar{V} \cdot \bar{n}) dA = 0$ (integration form)

The time rate of change of mass within the C.V. plus net mass flow rate through the C.S. equal to zero.

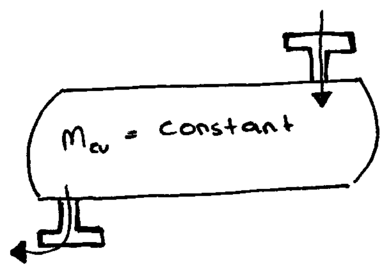
$d/dt \int_{CV} \rho dV + \sum_{out} \rho |V_n| A - \sum_{in} \rho |V_n| A = 0$

$d/dt \int_{CV} \rho dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$ or $\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$



$$\dot{m} = \rho V_n \cdot A_n$$
$$\dot{m} = \rho(V\cos\theta)\left(\frac{A}{\cos\theta}\right)$$
$$\dot{m} = \rho V A$$
$$\Downarrow$$
$$\dot{m} = \rho V A$$

↳ doesn't matter what is selected for CV (but inlet area, velocity would be different)



then $d/dt\, M_{cv} = 0$

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$
(single stream)

Special case: incompressible flow:

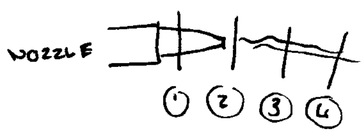
$$\sum_{in} \dot{V} = \sum_{out} \dot{V} \quad (m^3/s)$$ Steady, incompressible flow

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2 \quad \text{(single stream)}$$

Example: A garden hose attached with a nozzle is used to fill a 10-gal bucket. Inner diameter of hose is 2cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket w/ water, determine (a) volume and mass flow rate of water through the hose and (b) the average velocity @ nozzle exit

$$\dot{V} = \frac{10 \text{ gallon}}{50 \text{ sec.}}\left(\frac{3.7854 \text{ L}}{1 \text{ gallon}}\right) \Rightarrow \dot{V} = 0.757 \text{ L/s}$$

$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = 0.757 \text{ kg/s}$$



$$\dot{V}_1 = \dot{V}_2 = \dot{V}_3 = \dot{V}_4$$
$$\dot{V} = V_{avg} A_{nozzle}$$
$$A_{nozzle} = \pi(0.4)^2 \Rightarrow 0.5027 \times 10^{-4}\ m^2$$

$$\rightarrow V_{avg} = \dot{V}/A_{nozzle}$$

where $A = 0.5027 \times 10^{-4}\ m^2$

$$V_{avg} = \frac{\dot{V}}{A_{nozzle}} = \frac{0.757 \times \left(\frac{1}{1000}\right)}{0.5027 \times 10^{-4}} = 15.1\ m/s$$

$\boxed{\text{Example}}$ Cylindrical tank, 4-ft high, 3 ft diameter

Top is open to atmosphere, initially filled w/ water

Discharge plug pulled out

Water jet (0.5 in) streams out

Average velocity $V = \sqrt{2gh}$ where h = height of water in tank

How long until tank is 2 ft from bottom?

g = gravity

$$\frac{dm_{cv}}{dt} = \cancel{\dot{m}_i}^{\,0} - \dot{m}_{out}$$

$$m_{cv} = \rho V = \rho\left(\frac{\pi D^2}{4} \times h\right)$$

$$\dot{m}_{out} = \rho V A_{jet} = \rho\sqrt{2gh}\,A_{jet}$$

$$d/dt\left(\rho\frac{\pi D^2}{4} \times h\right) = -\rho\sqrt{2gh}\,A_{jet}$$

$$\cancel{\rho}\left(\frac{\pi D^2}{4}\right) d/dt\, h = -\cancel{\rho}\sqrt{2gh}\,A_{jet}$$

$$\int_{h_1=4}^{h_2=2} \frac{dh}{\sqrt{2gh}} = \frac{-4A_{jet}}{\pi D^2}\int_0^t dt \rightarrow t = 757\ \text{sec}$$

Mechanical energy = Flow energy + Kinetic energy + pot. energy

$$e_{mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz \quad \text{(intensive)}$$

Mechanical energy change:

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad (kJ/kg)$$

in Bernoulli's $\Delta e_{mech} = 0$

$$\eta_{pump} = \frac{\text{mech. power increase}}{\text{mech. power output}} = \frac{\Delta \dot{E}_{mech,fluid}}{W_{shaft}} = \frac{\dot{W}_{pump,in}}{\dot{W}_{pump}}$$

$$\eta_{turbine} = \frac{\text{mech. power output}}{\text{mech power decrease}} = \frac{\dot{W}_{shaft,out}}{[\Delta \dot{E}_{mech,fluid}]} = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,e}}$$