Let B represent any extensive property

Let $b = B/m$ intensive property

$$B_{sys,t} = B_{cv,t} \quad \text{(the system and CV coincide at time } t\text{)}$$

$$B_{sys,t} = B_{cv,t+\Delta t} - B_{I,t+\Delta t} + B_{II,t+\Delta t}$$

$$\frac{B_{sys,t+\Delta t}}{\Delta t} = \frac{B_{cv,t+\Delta t} - B_{cv,t}}{\Delta t} - \frac{B_{I,t+\Delta t}}{\Delta t} + \frac{B_{II,t+\Delta t}}{\Delta t}$$

since $B_{I,t+\Delta t} = b_1 m_{I,t+\Delta t} = b_1 \rho_1 V_{I,t+\Delta t} = b_1 \rho_1 V_1 \Delta t A_1$

$B_{II,t+\Delta t} = b_2 m_{II,t+\Delta t} = b_2 \rho_2 V_{II,t+\Delta t} = b_2 \rho_2 V_{II} \Delta t A_2$

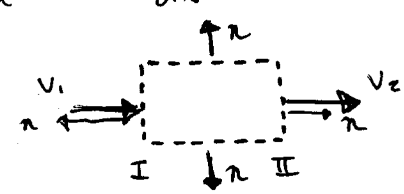
$$b = B/m \Rightarrow \dot{B}_{I,t+\Delta t} = b_I \dot{m}_{I,t+\Delta t}$$

$$\rho = \frac{m}{V} \qquad \dot{B}_{I,t+\Delta t} = b_I \rho_I \dot{V} = \underline{b_I \rho_I V_I A_I}$$

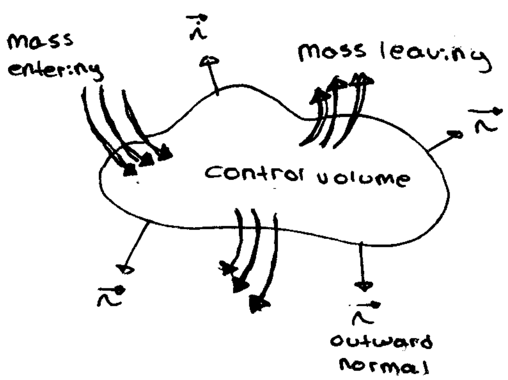
$$\dot{B}_{II,t+\Delta t} = \underline{b_{II} \rho_{II} V_{II} A_{II}}$$

$$\frac{dB_{sys}}{dt} = \frac{dB_{cv}}{dt} - b_1 \rho_1 V_1 A_1 + b_2 \rho_2 V_2 A_2$$

$$\int_{cs} (\bar{V}_1 \cdot \bar{n})$$



$$B_{cv} = \int_{cv} \rho b \, dV$$



$$\dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in} = \int_{cs} \rho b \vec{V} \cdot \vec{n} \, dA$$

$$\vec{V} \cdot \vec{n} = |\vec{V}||\vec{n}|\cos\theta = V\cos\theta$$

IF $\theta < 90°$, then $\cos\theta > 0$ (outflow)

IF $\theta > 90°$, then $\cos\theta < 0$ (inflow)

IF $\theta = 90°$, then $\cos\theta = 0$ (no flow)

RTT, Fixed CV: $\dfrac{dB_{sys}}{dt} = \dfrac{d}{dt}\int_{cv} \rho b \, dV + \int_{cs} \rho b \vec{V} \cdot \vec{n} \, dA$

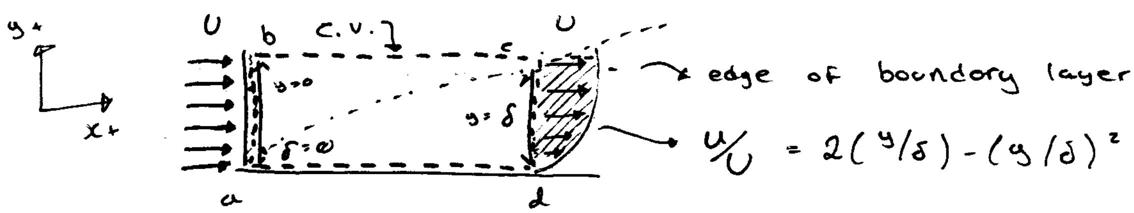
equate as zero

where $B = m$
$b = m/m = 1$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\int_{cv} \rho \, d\forall + \int_{cs} \rho (\bar{v}\cdot\bar{n})\, dA = 0$$

- conservation of mass, by conservation eq'n.

$\boxed{\text{Example}}$: The fluid is in direct contact with a stationary solid boundary has zero velocity; there is no slip at the boundary, thus flow over the plate adheres to the plate surface and forms a boundary layer, as depicted below. The flow ahead of the plate is $U = 30$ m/s.
The velocity distribution within boundary layer $(0 < y < \delta)$ along cd is approximated as $u/U = 2(y/\delta) - (y/\delta)^2$



The boundary-layer thickness at location $d = \delta = 5$mm
The fluid is air with density $\rho = 1.24$ kg/m³.
Assuming the plate width (perpendicular to the paper) to be $w = 0.6$m, calculate the mass flow rate across surface bc of control volume abcd.

Assume: 1) steady flow
2) incompressible flow
3) in 2D flow

$$\partial/\partial t \int_{cv} \cancel{\rho \, d\forall}^{\,0} + \int_{cs} \rho(\bar{v}\cdot\bar{n})\, dA = 0$$

(volume doesn't change with time)

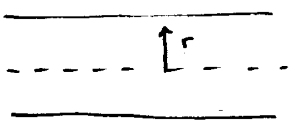
(sometimes denoted by $\rho \bar{v}\cdot d\bar{A}$)

P3:

$$U_{max}(1 - r^2/R^2) = U(r)$$

where $\tau = \mu \, du/dy \rightarrow \tau \mu \, du/dr |_{r=R}$

$$\tau = -\mu [U(r)] d/dr |_{r=R}$$



$$\tau = -\mu U_{max} (1 - r^2/R^2) d/dr |_{r=R}$$

$$-\mu U_{max} (-2r/R^2) |_{r=R}$$

$$\tau = -\mu U_{max} \; {-2}/R$$

$$= \frac{2 \mu U_{max}}{R}$$

$A = 2\pi r L$

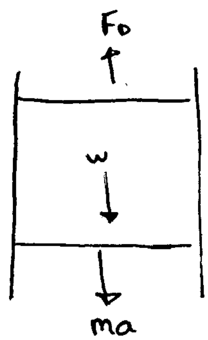
$$F = \left(\frac{2\mu U_{max}}{\cancel{R}} \right) \left(2\pi \cancel{R} L \right) \Rightarrow 4\pi\mu U_{max} L = F$$

$$F/L = 4\pi\mu U_{max}$$

→ use sign to show direction of vector

"Force per unit length, $L = 1$"

P4:



$$d(t) = \frac{at^2}{2} + vt + d_0$$

$$v(t) = at + v_0$$

$$a(t) = a$$

$$F_D = \tau \cdot A = \left[\mu \frac{du}{dy} \right] \left[A \right] \Rightarrow \mu \frac{v}{h} \cdot \pi D L$$

$$F_D = \frac{\mu \pi D L}{h} v = k v$$

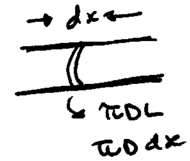
$$+\uparrow \sum F_y = F_D - W = ma \rightarrow \text{solve for } a$$

$$F_D - W = m(dv/dt) \Rightarrow F_D/m - g = dv/dt$$

→ Tarokh has no idea how to solve this question, so don't worry about it.

$$\rightarrow V(t) = \frac{mg}{k}(1 - e^{-(k/m)t})$$

IF $t \rightarrow \infty \Rightarrow V(\infty) = mg/k$



P6:

$$dF = \tau \, dA$$

$$\tau = \left(\mu \, du/dy\right)\left(\pi D dx\right)$$

$$\frac{h_1 - h_2}{0 - L}$$

$$h = h_1 - (h_1 - h_2)(x/L) \rightarrow y = mx + b \quad \rightarrow h_1$$

$$dF = \mu \frac{U}{(h_1 - (h_1 - h_2)x/L} \pi D \, dx$$

$$F = \mu U \pi D \int_0^L \frac{1}{(h_1 - (h_1 - h_2)\frac{x}{L}} dx$$

$$F = -\mu U \pi D \left. \frac{\ln\left(h_1 - (h_1 - h_2)\frac{x}{L}\right)}{(h_1 - h_2)/L} \right|_0^L$$

P1:

$$W = -\int P \, dV$$

$$\alpha = \frac{1}{K} = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\alpha = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T \Rightarrow \int dP = \int \frac{-dV}{V\alpha}$$

$$P - P_1 = \frac{-1}{\alpha} \ln V \Big|_{V_1}^{V_2}$$

$$P = P_1 - \frac{1}{\alpha} \ln\left(V/V_1\right)$$

$$W = -\int (P_1 - 1/\alpha \ln (V/V_1) \, dV$$

??? Tarokh just stopped.