Sept. 24/18

Midterm exam: Review assignment Q's, Practice Q's

2 problems, 15-20 multiple choice

No Formula sheet, but Formulas provided

Example – 4.1 from textbook

$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$

→ Stagnation point : point in the flow field where the velocity is practically (basically?) zero

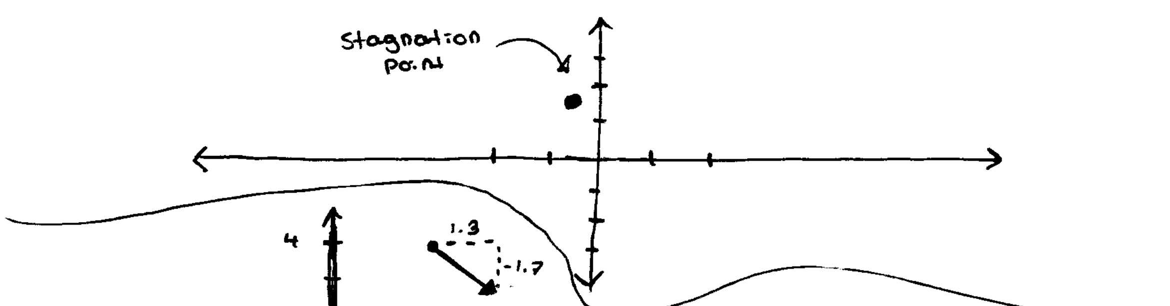
a) where are stagnation points, if they exist.

b) Sketch velocity vectors between $x = -2$ to $2m$; $y = 0$ to $5m$

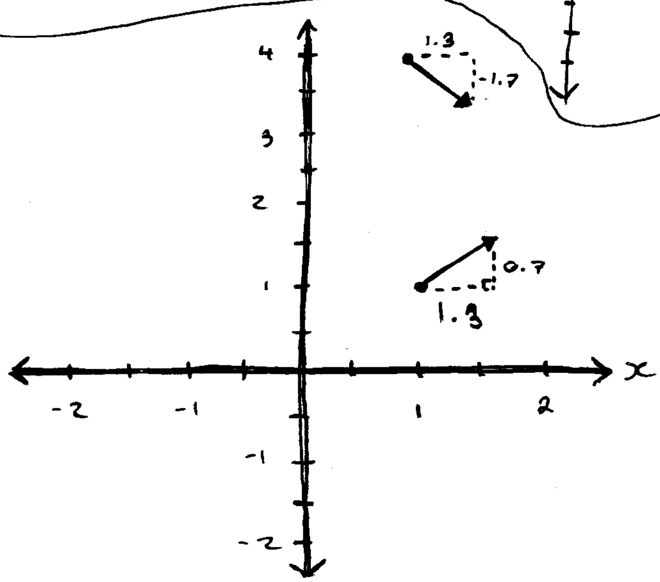
a) $\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$

$u = 0 \Rightarrow 0.5 + 0.8x = 0 \Rightarrow x = -0.625\,m$
$v = 0 \Rightarrow 1.5 - 0.8y = 0 \Rightarrow y = 1.875\,m$

} stagnation point $(-0.625, 1.875)$



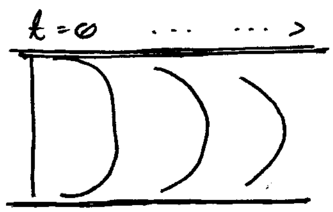
b)



$x = 1$, $y = 1$ $\Rightarrow$ $u = 1.3$, $v = 0.7$

$x = 1$, $y = 4$ $\Rightarrow$ $u = 1.3$, $v = -1.7$

Timeline : a set of adjacent fluid particles that were marked at an earlier time.



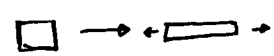
Plots of Fluid Flow Data

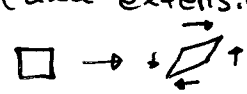
- Profile plot : indicates how value of a scalar property varies along some desired direction
  e.g. velocity profile plot
- Vector plot : arrows indicating magnitude and direction of vector property at time instant
- Contour plot (isocontour) : Curves of constant values of a scalar property (or magnitude of vector prop.) at an instant in time
  e.g. pressure, temp., velocity mag.

Types of motion or deformation :

(a) translation 

(b) rotation 

(c) linear strain (aka extensional strain) 

(d) Shear strain 

Velocity (rate of translation)

angular velocity (rate of rotation)

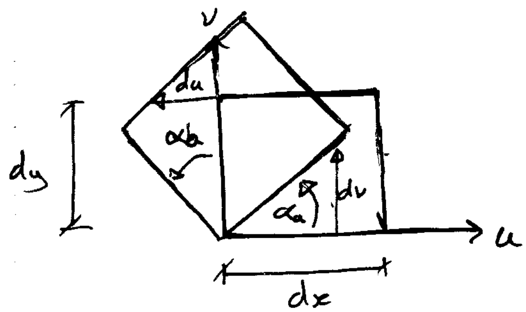
linear strain rate (rate of linear strain)

shear strain rate (rate of shear strain)

Rate of translation vector in Cartesian coordinates:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

Rate of rotation



$$\alpha_a \approx \tan \alpha_a = \frac{\partial v}{\partial x}$$

$$\alpha_{bs} - \partial u/\partial y$$

$$\omega_s \; d/dt\left(\frac{\alpha_a}{2} + \frac{\alpha_b}{2}\right)$$

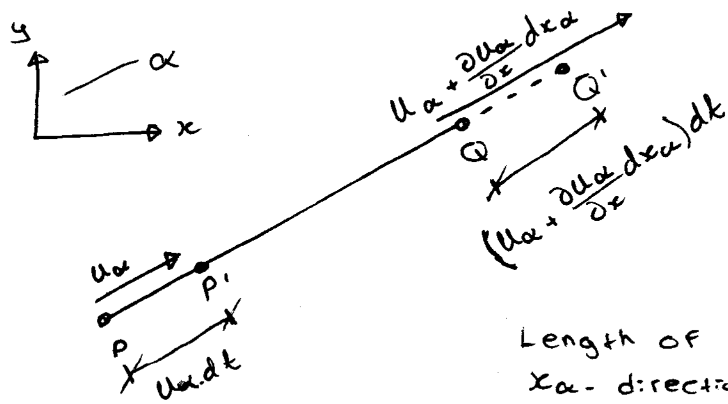
$$\omega_s = 1/2\left(\partial v/\partial x + \partial u/\partial y\right)$$

General rate of rotation vector in Cart. coord:

$$\vec{\omega} = \left(\tfrac{1}{2}\right)\left(\partial w/\partial y - \partial v/\partial z\right)\vec{i} + \left(\tfrac{1}{2}\right)\left(\partial u/\partial z - \partial w/\partial x\right)\vec{j} + \left(1/2\right)\left(\partial v/\partial x - \partial u/\partial y\right)\vec{k}$$

Linear strain rate in Cart. coord:

$$\varepsilon_{xx} = \partial u/\partial x \quad ; \quad \varepsilon_{yy} = \partial v/\partial y \quad ; \quad \varepsilon_{zz} = \partial w/\partial z$$



$$\varepsilon_{\alpha\alpha} = d/dt\left(\frac{P'Q' - PQ}{PQ}\right)$$

$$\cong \left(\frac{\overbrace{\left(u_\alpha + \frac{\partial u_\alpha}{\partial x_\alpha} dx_\alpha\right) dt + dx_\alpha - u_\alpha dt}^{\text{Length of } P'Q' \text{ in } x_\alpha\text{- direction}} \; - \; \overbrace{d\alpha}^{\text{Length of PQ in } x\text{-direction}}}{\underbrace{dx_\alpha}_{\text{Length of PQ in } x_\alpha \text{ direction}}} \right) = \frac{\partial u_\alpha}{\partial x_\alpha}$$

Volumetric strain rate in Cartesian Coordinates:

$$\frac{1}{V}\frac{DV}{Dt} = \frac{1}{V}\frac{dV}{dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

zero in incompressible flow

→ SN1015 and RB2048 are midterm rooms

Rate of translation vector in Cartesian coordinates

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

Shear strain rate

$\varepsilon_{xy} = \left(-\frac{1}{2}\right) \frac{d}{dt}\, \alpha_{a\to b} = \left(\frac{1}{2}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

$\varepsilon_{yz} = \left(\frac{1}{2}\right)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$

$\varepsilon_{xz} = \left(\frac{1}{2}\right)\left(\frac{\partial u}{\partial w} + \frac{\partial w}{\partial x}\right)$

→ Shear strain rate

$$\varepsilon = \begin{Bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{Bmatrix} \Rightarrow \begin{Bmatrix} \frac{du}{dx} & \left(\frac{1}{2}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \left(\frac{1}{2}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \left(\frac{1}{2}\right)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{dv}{dy} & \left(\frac{1}{2}\right)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \left(\frac{1}{2}\right)\left(\frac{dw}{\partial x} + \frac{\partial u}{\partial z}\right) & \left(\frac{1}{2}\right)\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{Bmatrix}$$

→ Shear strainrate → linear strain rate

$\boxed{\text{Example}}$ – From textbook, 4-6.

$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$

$$\left.\begin{aligned} u &= 0.5 + 0.8x \\ v &= 1.5 - 0.8y \\ w &= 0 \end{aligned}\right\} \text{rate of translation}$$

rate of rotation $\begin{cases} \vec{\omega} = \left(\frac{1}{2}\right)\left(\cancel{\frac{\partial w}{\partial y}}^{0} - \cancel{\frac{\partial v}{\partial z}}^{0}\right)\vec{i} + \left(\frac{1}{2}\right)\left(\cancel{\frac{\partial u}{\partial z}}^{0} - \cancel{\frac{\partial w}{\partial x}}^{0}\right)\vec{j} + \left(\frac{1}{2}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \\ \vec{\omega} = \left(\frac{1}{2}\right)(0 - 0) \Rightarrow \vec{\omega} = 0\vec{i} + 0\vec{j} + 0\vec{k} \end{cases}$

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} \Rightarrow 0.8 \\ \varepsilon_{yy} &= \frac{\partial v}{\partial y} \Rightarrow -0.8 \\ \varepsilon_{zz} &= \frac{\partial w}{\partial w} = 0 \end{aligned}\right\} \text{linear strain rate}$$

$$\left.\begin{aligned} \varepsilon_{xy} &= \left(\tfrac{1}{2}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0 \\ \varepsilon_{yz} &= \left(\tfrac{1}{2}\right)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 \\ \varepsilon_{xz} &= \left(\tfrac{1}{2}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0 \end{aligned}\right\} \text{Shear strain rate}$$

$$\varepsilon_{ij} = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underbrace{\frac{1}{\underset{\text{volume}}{V}} \frac{DV}{Dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}}_{\text{Volumetric flow rate}} = 0.8 + (-0.8) + 0 = 0$$

$\Rightarrow$ flow is incompressible

Vorticity and Rotationality

$\vec{\zeta} = \vec{\nabla} \times \vec{V} = \text{Curl}(\vec{V})$ — vorticity vector

where $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$\text{Curl}(\vec{V}) = \vec{\nabla} \times \vec{V} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (u, v, w)$

Clockwise = negative

(*) counter-clockwise = positive

$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{V} = \frac{1}{2}\,\text{curl}(\vec{V}) = \frac{\vec{\zeta}}{2}$ — rate of rotation vector

$\vec{\zeta}$

$\vec{\omega}$

$\Big\}$ vorticity is equal to twice the rotation

If vorticity is non-zero, flow in the region is <u>rotational</u>

If vorticity is zero, flow is <u>irrotational</u>

→ Fluid particles rotate end over end as they move along flow

vorticity vector in cart. coord $\Big\}$ $\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$

$\vec{\zeta} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$ $\Big\}$ two-dimensional flow (in x-y plane)

$\boxed{\text{Example}}$ From textbook, 4-8

$\vec{V} = (u, v) = x^2\vec{i} + (-2xy - 1)\vec{j}$

$u = x^2$
$v = -2xy - 1$ $\Big\}$ $\zeta_k = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$

$\zeta = (-2y - 0)\vec{k} \Rightarrow \zeta = -2y$

$\boxed{\therefore \text{ non-zero, flow is rotational}}$

vorticity vector in cylindrical coordinates $\Big\}$ $\vec{\zeta} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \left(\frac{1}{r}\right)\left(\frac{\partial(r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z$

$\vec{\zeta} = \frac{1}{r}\left(\frac{\partial(r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{k}$ $\Big\}$ two dimensional flow

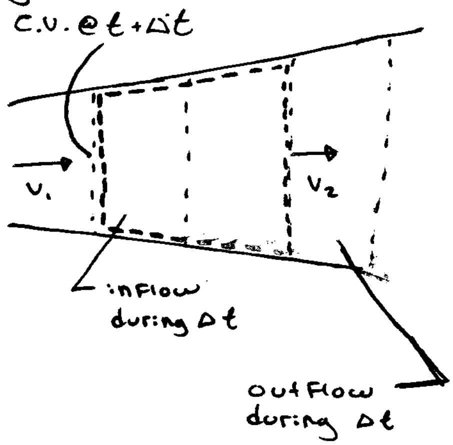
$\boxed{\text{Example}}$ from textbook, 4-9

$$U_r = \frac{\dot{V}}{2\pi L}\left(\frac{1}{r}\right) \qquad U_\theta = 0$$

$$\vec{\zeta} = \left(\frac{1}{r}\right)\left(\frac{\partial(r\cancel{U_\theta})^{0}}{\partial r} - \frac{\partial \cancel{U_r}^{0}}{\cancel{\partial\theta}}\right)\vec{k} = 0$$

$\boxed{\therefore \text{ zero, so flow is irrotational}}$

The Reynolds transport theorem (RTT)



$B_{sys,t} = B_{c.v.,t}$ (the system and CV coincide @ time t)

$B_{sys,t+\Delta t} = B_{c.v.,t+\Delta t} - B_{I,t+\Delta t} + B_{II,t+\Delta t}$

$$\frac{B_{sys,t+\Delta t} - B_{sys,t}}{\Delta t} = \frac{B_{cv,t+\Delta t} - B_{cv,t}}{\Delta t} - \frac{B_{I,t+\Delta t}}{\Delta t} + \frac{B_{II,t+\Delta t}}{\Delta t}$$

$$\frac{dB_{sys}}{dt} = \frac{dB_{cv}}{dt} - \dot{B}_{in} + \dot{B}_{out}$$

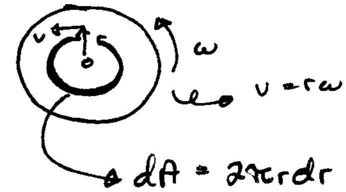
B represent <u>extensive</u>

$b = B/m$ <u>intensive</u>

30 M/C - most are definitions

2 calc. problems

Example From textbook, 2.128



$$\tau_\omega = \tau_y + \mu \, du/dr$$

$$= \tau_y + \mu \, \overset{\curvearrowright r\omega}{v}/h$$

$$\tau_\omega = \tau_y + \mu \frac{r\omega}{h}$$

$$dF = \tau_\omega dA \Rightarrow \left(\tau_y + \mu \frac{r\omega}{h}\right)(2\pi r\, dr)$$

$$dT = r\,dF$$

$$\int_0^T dT = \int_{R_1}^{R_2} r \left(\tau_y + \mu \frac{r\omega}{h}\right)(2\pi r\, dr)$$

$$T = 2\pi \left[\int_{R_1}^{R_2} r^2 \tau_y \, dr + \frac{\mu\omega}{h} \int_{R_1}^{R_2} r^3 dr \right]$$

$$T = 2\pi \left[\pi/3 \left(R_2^3 - R_1^3\right) + \frac{\mu\omega}{4h}\left(R_2^4 - R_1^4\right) \right]$$

$$T\Big|_{\text{For } N \text{ disks}} = 2\pi N \left[\frac{\tau_y}{3}\left(R_2^3 - R_1^3\right) + \frac{\mu\omega}{4h}\left(R_2^4 - R_1^4\right) \right]$$

NOTE: $\left(\left(\omega = 2\pi \dot{n} = 2\pi(2400) = \begin{array}{l} 15080 \text{ rad/min} \\ = 251.3 \text{ rad/s} \end{array} \right)\right)$

b) $$T = 2\pi(11) \left[\frac{2500}{3}\left(0.2^3 - 0.05^3\right) + \frac{0.1(251.3)}{4(1.2\times10^{-3})}\left(0.2^4 - 0.05^4\right) \right]$$

$$T = 2060 \text{ N}\cdot\text{m}$$

Example - Calculate location of stagnation point:

$$\vec{V} = (u, v) = (-0.781 - 3.25x)\vec{i} + (-3.54 + 3.25y)\vec{j} \quad (m/s)$$

$$u = 0 \Rightarrow -0.781 - 3.25x = 0 \Rightarrow x = -0.24003 \text{ m}$$

$$v = 0 \Rightarrow -3.54 + 3.25y = 0 \Rightarrow y = 1.089 \text{ m}$$

Example - Calculate acceleration Field:

$$u = 0.205 + 0.97x + 0.851y$$
$$v = -0.509 + 0.953x - 0.97y$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$a_y = \frac{\partial u}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

(*) $$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + \cancel{w\frac{\partial w}{\partial z}}^{\,0}$$

$$\vec{a}_x = 0 + (0.205 + 0.97x + 0.851y)(0.97) + (-0.509 + 0.953x - 0.97y)(0.851)$$

$$\vec{a}_y = 0 + (-0.509 + 0.953x - 0.97y)(0.953) + (-0.509 + 0.953x + 0.97y)(-0.97)$$

$$\left.\begin{matrix} x = 2 \\ y = 1.5 \end{matrix}\right\} \Rightarrow \begin{matrix} a_x = \ldots \\ a_y = \ldots \end{matrix}$$

Example - From textbook 4.34

$$\vec{V} = (u, v, w) = 0.6x\,\vec{i} + 0.2t\,\vec{j} - 1.4\,\vec{k} \quad (m/s)$$

$$T = (x, y, z) = 400 - 0.4y - 0.6z - 0.2(5-x)^2 \quad (°C)$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + (\nabla \cdot V)T$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

Solve 4.37

Steady Flow $\Rightarrow \frac{\partial T}{\partial t} = 0$

$$\frac{\partial T}{\partial x} = 0.4(5-x) \qquad \frac{\partial T}{\partial y} = -0.4 \qquad \frac{\partial T}{\partial z} = -0.6$$

$$DT/Dt = 0 + (0.6x)(0.4(5-x)) + (0.2t)(-0.4) + (1.4)(-0.6)$$

IF $x = 1,\ t = 10 \rightarrow 1\ °C/s$