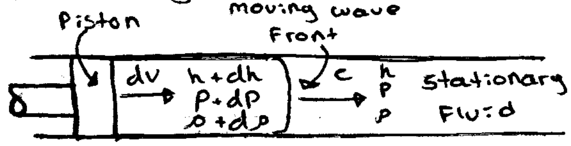
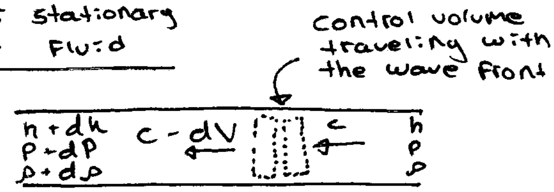
Sept. 17/18

Speed of Sound (sonic speed): The speed at which infinitesimally small pressure waves travel through a medium



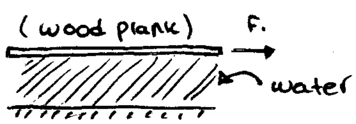


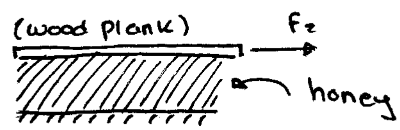
$$C^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T \qquad C = \sqrt{KRT}$$

For any fluid — For ideal gas

if $Ma = V/c \leq 0.3$ Flow is incompressible

Viscosity: A property that represents the internal resistance of a fluid to motion or "the fluidity"

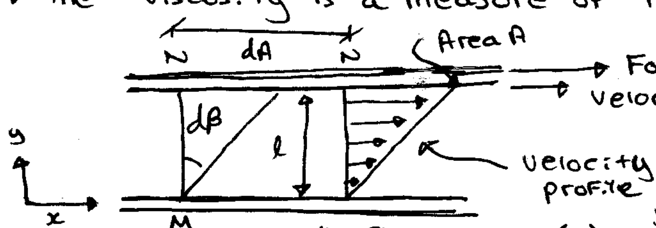




To have same velocity for top plate
$F_2 > F_1$

Drag Force: The force a flowing fluid exerts on a body in the flow direction, the magnitude depends, in part, on viscosity.

→ the viscosity is a measure of its resistance to deformation



$u(y) = \frac{y}{l} V$

$u(y) = \frac{y}{l} V$ and $\frac{du}{dy} = \frac{V}{l}$

$d\beta \approx \tan d\beta = da/l = Vdt/l$

$\Rightarrow du/dy \, dt$

$\tau \propto d\beta/dt$ or $\tau \propto du/dy$

$\tau = \mu \frac{du}{dy}$ (N/m^2) Shear stress

Newtonian Fluids: Fluids for which the rate of deformation is proportional to the shear stress

→ μ is Dynamic viscosity (kg/m·s)

1 poise = 0.1 Pa·s

Shear Force : $F = \tau / A \Rightarrow \mu A \, du/dy$ (N)

The rate of deformation of a newtonian fluid is proportional to shear stress – constant of proportionality is the viscosity

Kinematic viscosity

$\nu = \mu / \rho$ m^2/s or stoke

1 stoke = 1 cm^2/s

For liquids : both dynamic and kinematic are indep. of pressure.

For gases : dynamic viscosity doesn't change (at low to moderate pressure), but not for kinematic – density is proportional to pressure.

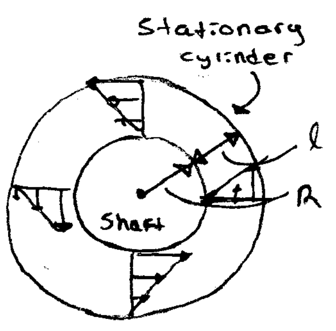
Air @ 20°C and 1 atm	Air @ 20°C and 4 atm
$\mu = 1.83 \times 10^{5}$ kg/m-s	$\mu = 1.83 \times 10^{5}$ kg/m-s
$\nu = 1.52 \times 10^{-5}$ m^2/s	$\nu = 0.38 \times 10^{-5}$ m^2/s

In a liquid, viscosity decreases as temp. increases

In a gas, viscosity increases as temp. increases

$\mu = a 10^{b/(T-c)}$ For liquids

$\mu = a T^{1/2} / (1 + b/T)$ for gases



$T = F \times R = (\tau A) \times R$

$= (\mu \, du/dy \, A) \times R = (\mu \frac{\Delta u}{\Delta y} 2\pi R L) \times R$

$= (\mu \frac{V}{l} 2\pi R L) \times R = (\mu \frac{R\omega}{l} 2\pi R L) \times R$

$T = FR = \dfrac{\mu 2\pi R^3 \omega L}{l} = \dfrac{\mu 4\pi^2 R^3 \dot{n} L}{l}$

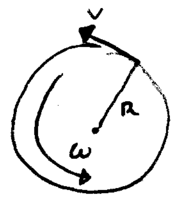
$\dot{n} = 300$ rpm

L = length of cylinder

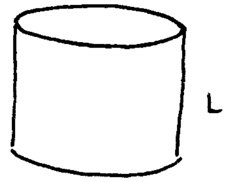
$\dot{n}$ = number of rev per unit time

$\omega = 2\pi \dot{n}$

(can be used to find viscosity by measuring torque)

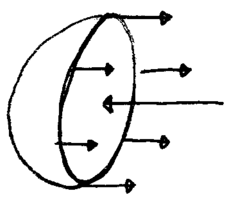


$V = R\omega$



$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{u_{inc} - u_{outc}}{l}$$

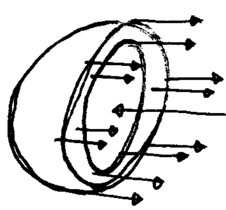
Surface tension (coefficient of surface tension)



$(\pi R^2)\Delta P_{droplet}$

Droplet or air bubble : $(2\pi R)\sigma_s = (\pi R^2)\Delta P_{droplet}$

$\rightarrow \Delta P_{droplet} = P_i - P_o = 2\sigma_s / R$



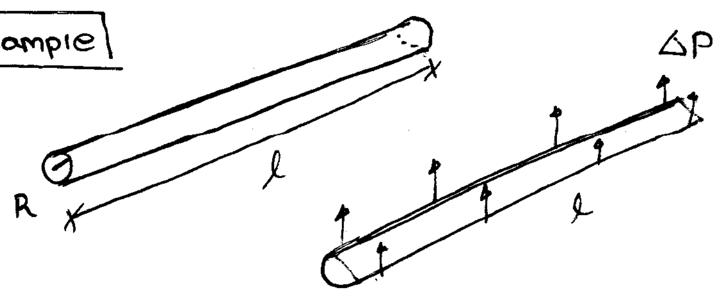
$(\pi R^2)\Delta P_{bubble}$

Soap bubble : $2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{bubble}$

$\rightarrow \Delta P_{bubble} = P_i - P_o = 4\sigma_s / R$

$\boxed{\text{Example}}$

$\Delta P \propto \sigma_s$?



side view

$$\frac{2\sigma_s L}{}$$

$\Delta P\, 2RL = 2\sigma_s L$

$\Rightarrow \Delta P = \sigma_s / R$

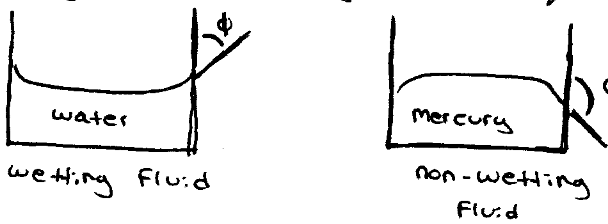
Sept. 19/18

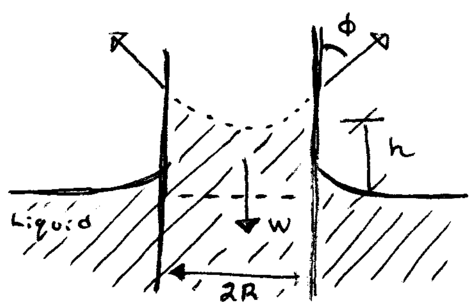
Capillary effect: the rise or fall of a liquid in a small-diameter tube inserted into the liquid

Capillaries: Such narrow tubes or confined flow channels

Meniscus: Free curved surface in a capillary tube

↳ strength of capillary effect quantified by contact angle
↳ (or wetting)





$$2\pi R\sigma_s \cos(\phi) = W = \rho V g = \rho A h g$$
$$= \rho \pi R^2 h g$$

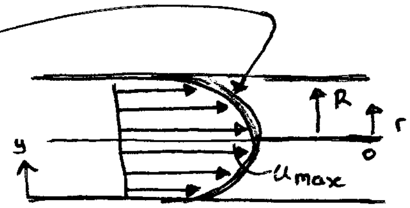
Capillary rise: $h = \dfrac{2\sigma_s \cos\phi}{\rho g R}$

(R = constant)

inversly proportional to radius of tube and density of liquid.

| Problem 2.75 | (From textbook)

$u(r) = u_{max}(1 - r^n/R^n)$



$y = R - r$

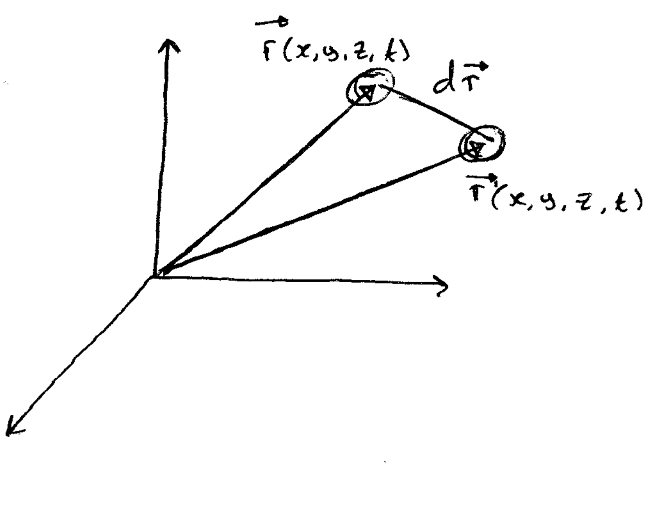
$\tau_w = \mu \, du/dy = -\mu \, du/dr \big|_{r=R}$

$\tau_w = -\mu \, d/dr\,(u_{max}(1 - r^n/R^n)) = -\mu u_{max}\left(\dfrac{-n r^{n-1}}{R^n}\right)\Big|_{r=R}$

$\boxed{\tau_w = \dfrac{n \mu u_{max}}{R}}$

$F = \tau_w A = \dfrac{n \mu u_{max}}{R} 2\pi R L$

$\Rightarrow \boxed{F/L = 2 n \pi \mu u_{max}}$



Lagrangian and Eulerian descriptions

Kinematics : The study of motion

Fluid kinematics : The study of how fluids flow, and how to describe fluid motion.

Lagrangian desc. : Follow the path of individual object
↳ requires us to track the position and velocity of each individual fluid parcel

In Eularian description of fluid flow, a finite volume called flow domain or control volume is defined, through which fluid flows in and out.

→ function of pos'n, time

Field variable at a particular location, at a particular time

Pressure field is scalar field variable

Velocity field is vector field variable

Pressure Field : $P = P(x, y, z, t)$

Vector Field $\vec{V} = \vec{V}(x, y, z, t)$

Accel. Field $\vec{a} = \vec{a}(x, y, z, t)$

These and others form flow field

$$\vec{V} = (u, v, w) = u(x,y,z,t)\vec{i} + v(x,y,z,t)\vec{j} + w(x,y,z,t)\vec{k}$$

Acceleration Field

↳ define the particle's location in space in terms of a material pos'n vector

$x_{particle}(t)$, $y_{particle}(t)$, $z_{particle}(t)$

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}}{dt} = \frac{dV(x_p, y_p, z_p, t)}{dt}$$

$$= \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x}\frac{dx_p}{dt} + \frac{\partial V}{\partial y_p}\frac{dy_p}{dt} + \frac{\partial \vec{V}}{\partial z_p}\frac{dz_p}{dt}$$

$$\vec{a}_{particle}(x,y,z,t) = \frac{d\vec{V}}{dt} = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial t} + v\frac{\partial V}{\partial t} + w\frac{\partial V}{\partial t}$$

$F_{part} = m_{part} + \vec{a}_{part}$

Particle: $\frac{\partial \vec{V}}{\partial t}$ Local accel.

Flow Field: $(\vec{V} \cdot \vec{\nabla})\vec{V}$ advective acceleration

$$\vec{a}(x,y,z,t) = \frac{\partial \vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

Gradient or del operation

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

Components of acceleration vector in cartesian coordinates:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Material Derivative

$$a(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

Material derivative : $\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V} \cdot \vec{\nabla})$

Material acceleration : $\vec{a}(x, y, z, t) = \frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$

Material derivative of pressure : $\frac{DP}{Dt} = \frac{dP}{dt} = \frac{\partial P}{\partial t} + (\vec{V} \cdot \vec{\nabla})P$

Streamline: A curve that is everywhere tangent to the instantaneous local velocity vector

↳ useful as instantaneous indicators of fluid motion

Equation for streamline : $\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

Streamtube - bundle of streamlines much like a communications cable consists of a bundle of fibre-optic cables.

↳ Streamlines are everywhere parallel to local velocity
Fluid cannot cross streamline

Pathline - actual path traveled by an individual particle over some time period

Streakline - a locus of fluid particles that have passed sequentially through a prescribed point in the flow

Streaklines, streamlines, pathlines are the same in steady flow, different in unsteady flow.

Sept. 21 /18

Problem 2.60 (Tutorial 1)

$M_1 = \frac{V_1}{C_1}$ $\quad V_1 = 50 \text{ m/s}$

(idealgas) $C_1 = \sqrt{K_1 R T_1} \Rightarrow \sqrt{(1.288)(0.1889)(1200)\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}$

(table) $\begin{cases} R = 0.1889 \text{ kJ/kg·K} \\ K = 1.288 \end{cases}$ $\quad C_1 = 540.3 \text{ m/s}$

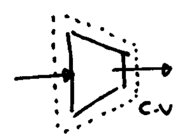
$C_p = 0.8439 \text{ kJ/kg·K}$

$M_1 = \frac{50}{540.3} = \boxed{0.0925}$

b) $C_2 = \sqrt{K_2 R T_2} = \sqrt{(1.288)(0.1889)(400)\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 312 \text{ m/s}$

$M_2 = V_2 / C_2$

$\left.\begin{array}{l} w = 0 \\ q = 0 \\ \Delta Pe = 0 \end{array}\right\}$ (energy equation) $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$



$V_2^2 = \left(h_1 - h_2 + \frac{V_1^2}{2}\right) 2$

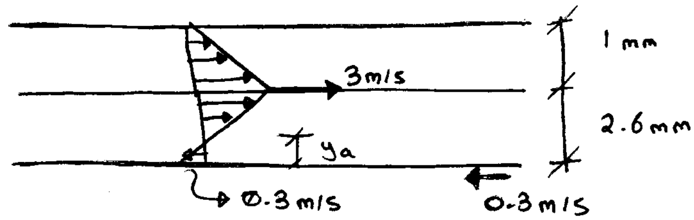
$\boxed{V_2^2 = C_p (T_1 - T_2) + V_1^2}$

$V_2^2 = 2(0.8439)(1200 - 400) + \frac{(50)^2}{1000}$

$V_2 = 1163 \text{ m/s}$

$\hookrightarrow M_2 = \frac{1163}{312} = \boxed{3.73}$ (no units)

Problem 2.77 (Tutorial 1)



$$\frac{2.6 - y_A}{y_A} = \frac{3}{0.3}$$

$$\Rightarrow y_A = 0.236\ \text{mm}$$

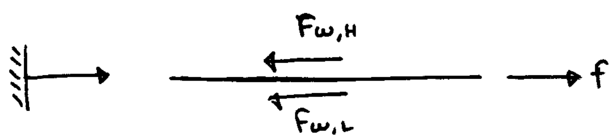
$$F = \tau_\omega A = \mu \, {du}/{dy} \, A$$

$$F_{\omega,H} = \mu \, {du}/{dy} \, A = \mu A \frac{\Delta u}{\Delta y} = \mu A \frac{3}{1\times10^{-3}}$$

$$F_{\omega,H} = 0.027\,(0.3 \times 0.3)\frac{3}{1\times10^{-3}} = \boxed{7.29\ \text{N}} = F_{\omega,H}$$

$$F_{\omega,L} = \mu \, {du}/{dy} \, A = \mu A \frac{\Delta u}{\Delta y} = (0.027)(0.3 \times 0.3)\left(\frac{3-(-0.3)}{2.6\times10^{-3}}\right)$$

$$F_{\omega,L} = \boxed{3.08\ \text{N}}$$

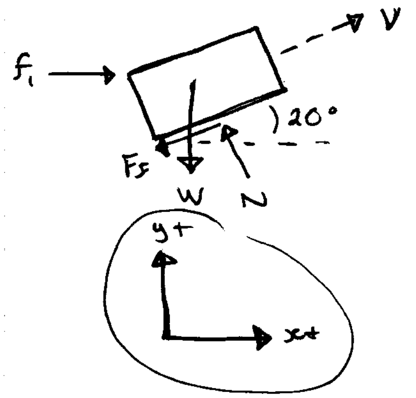


$$\Sigma F_x = 0 \quad \Rightarrow \quad F - F_{\omega,H} - F_{\omega,L} = 0$$

$$\Rightarrow \boxed{F = 10.4\ \text{N}}$$

velocity should be 1.1, not 0.8

Problem 2.81 (Tutorial 1)



constant velocity $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$

$\sum F_x = 0$

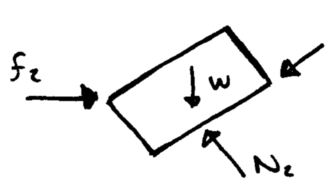
$F_1 - F_f \cos 20° - N \sin 20° = 0$ (I)

$\sum F_y = 0$

$N \cos 20° - F_f \sin 20° - W = 0$ (II)

$F_f = (0.27) N$ (III)

$N, F_f, F \xrightarrow{\text{(I)(II)(III)}} \boxed{F_1 = 105.5\ N}$



$F_{shear} = \tau_w A = \mu \left(\frac{du}{dy}\right) A$

$= (0.012)(0.5 \times 0.2)\left(\frac{1.1}{4 \times 10^{-4}}\right)$

$\boxed{F_{shear} = 3.3\ N}$

$\sum F_x = 0$

$F_2 - F_{shear} \cos 20° - N_2 \cos 20° = 0$

$\sum F_y = 0$

$N_2 \sin 20° - F_{shear} \sin 20° - w = 0$

$F_2, N_2 \xrightarrow{\text{(IV) (V)}} \boxed{F_2 = 57.2\ N}$

$\frac{F_1 - F_2}{F_1} \times 100 \Rightarrow \frac{105.5 - 57.7}{105.5} \times 100 = \boxed{45.3\ \%}$