Nov. 26/18

- 5 problems
- No m/c (or maybe just a few)
- Tutorials → Assignments
  ↳ review Practice problems

→ For final

$1\ Pa = \frac{1N}{m^2} \Rightarrow \frac{kg}{m \cdot s^2}$

$N = 1 \frac{kg \cdot m}{s^2}$

$\boxed{\text{Example:}}$ $\Delta P = f\{R, \sigma\}$ $n = 3$

| $\Delta P$ | $R$ | $\sigma_s$ |
|---|---|---|
| $\{mL^{-1}t^{-2}\}$ | $\{L\}$ | $\{mt^{-2}\}$ |

$\cancel{j = 3}$

$j = 2$

$k = n - j = \emptyset$

then $k = 3 - 2 = 1$

$\Pi_1 = \Delta P^1\, R^1\, \sigma_s^{-1}$

$\left(\frac{m}{Lt^2}\right)^1 (L)^1 \left(\frac{m}{t^2}\right)^{-1}$

$\left(\frac{m}{Lt^2}\right)(L)\left(t^2/m\right) = 1$

thus $\Pi_1 = \dfrac{\Delta P \cdot R}{\sigma_s}$

when there's just 1 dimensionless number, that number is equal to a constant.

$\boxed{\text{Example:}}$ $F_L = f(V, L_c, \rho, \mu, C, \alpha)$ $n = 7$

Speed of sound

| $F_L$ ✓ | $V$ ✓ | $L_c$ ✓ | $\rho$ ✓ | $\mu$ ✓ | $C$ ✓ | $\alpha$ |
|---|---|---|---|---|---|---|
| $\{kg \cdot m/s^2\}$ | $\{m/s\}$ | $\{m\}$ | $\{kg/m^3\}$ | $\{\frac{kg}{m \cdot s}\}$ | $\{m/s\}$ | $\{1\}$ |

↳ then $j = 3$

$7 - 3 = 4$, then rep. var. = $V, L_c, \rho$

then $\Pi_1 = F_L \cdot V \cdot L_c \cdot \rho$

$\left(\frac{kg \cdot m}{s^2}\right)\left(\frac{m}{s}\right)^{-2}(m)\left(kg/m^3\right)^{-1}$

$\left(\frac{kg \cdot m}{s^2}\right)\left(\frac{s^2}{m^2}\right)\left(m\right)^{-2}\left(\frac{m^3}{kg}\right)$

$\left(\frac{\cancel{kg \cdot m}}{\cancel{s^2}}\right)\left(\frac{\cancel{s^2}}{\cancel{m^2}}\right)\left(\frac{1}{\cancel{m^2}}\right)\left(\frac{\cancel{m^3}}{\cancel{kg}}\right) = 1$

then $\Pi_1 = \dfrac{F_L}{V^2 \cdot L_c^2 \cdot \rho}$

Then $\Pi_{1,\,modified} = \dfrac{F_L}{(1/2)V^2 A \rho} = C_L$ since $L \times L = A$

$(1/2)$ is constant

$$\Pi_2 = \mu \cdot V \cdot L_c \cdot \rho$$

$$= \left(\frac{kg}{m \cdot s}\right)\left(\frac{m}{s}\right)^{-1}(m)^{-1}\left(\frac{kg}{m^3}\right)^{-1}$$

$$= \left(\frac{\cancel{kg}}{\cancel{m \cdot s}}\right)\left(\frac{\cancel{s}}{\cancel{m}}\right)\left(\frac{1}{\cancel{m}}\right)\left(\frac{\cancel{m^3}}{\cancel{kg}}\right) = 1$$

then $\Pi_2 = \dfrac{\mu}{V \cdot L_c \cdot \rho}$

$\Pi_{2,\,modified} = Re$ Reynolds number, inverse

$$\Pi_3 = C \cdot V \cdot L_c \cdot \rho$$

$$= \left(\frac{m}{s}\right)\left(\frac{m}{s}\right)^{-1}(m)^{0}\left(\frac{kg}{m^3}\right)^{0}$$

$$= \left(\frac{m}{s}\right)\left(\frac{s}{m}\right)(1)(1) = 1$$

$$\Pi_3 = \frac{C}{V}$$

then $\Pi_{3,\,modified} = \dfrac{V}{C}$ mach number, inverse

$$\Pi_4 = \alpha \cdot V \cdot L_c \cdot \rho$$

$$= (1)\left(\frac{m}{s}\right)^{0}(m)^{0}(kg/m^3)^{0}$$

$$\Pi_4 = \alpha$$

$$\boxed{\text{thus, } C_L = f(Re, Ma, \alpha)}$$

Complete set of experiments (called Full Factorial test matrix)

If 5 dependent variables initially, complete set would be $5^4 = 625$

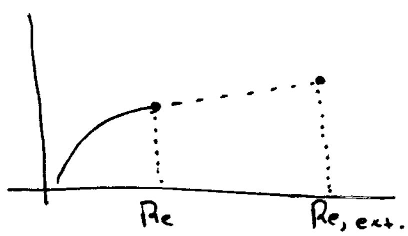
Using non-dimensional analysis, $K = 5 - 3 = 2$

then dependent variable $= 1$, thus new compete set $5^1 = 5$

- allows fewer experiments to return the same resolution.

- Not always possible to match all the models $\pi$ to $\pi$'s of Prototype : called incomplete similarity

↳ extrapolating results : result needs more testing



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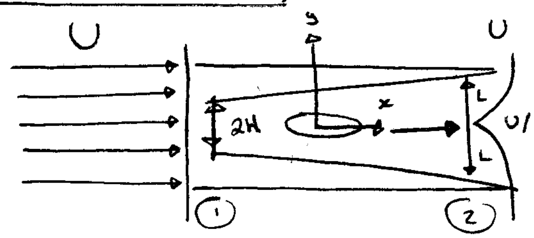
$\boxed{\text{Example:}}$




$$@\ y=0,\ u=U/2 \quad @\ y=L,\ u=U \qquad u=\frac{U}{2}\left(1+\frac{y}{L}\right)$$

$$\partial/\partial t \cancel{\int_{cv} \rho\, d\forall}^{\,0} + \int_{cs} \rho(\bar{V}\cdot\bar{n})\, dA = 0$$

$$\rightarrow \int_{sec2} \rho u\, dA - \int_{sec1} \rho u\, dA = 0$$

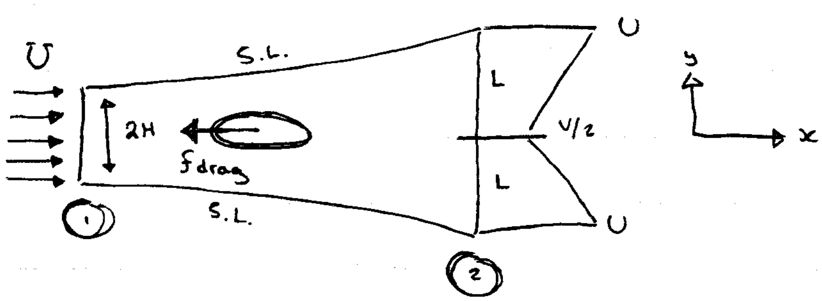
$$2\int_0^L \rho\frac{U}{2}(1+y/L)b\,dy - \int_0^H 2\rho U b\,dy = 0$$

$$2\rho\frac{U}{2}b\int_0^L(1+y/L)\,dy - 2\rho U bH$$

$$2\rho\frac{U}{2}b\left[y + y^2/2L\Big|_{y=0}^{y=L}\right] - 2\rho UbH = 0$$

$$2\rho\frac{U}{2}b\left[L + L^2/2L\right] - 2\rho UbH = 0$$

$$H = 3L/4$$




$$-f_{drag} = \partial/\partial t \cancel{\int_{cv} \rho V\, d\forall}^{\,0} + \int_{cs} \rho V_A(\bar{V}\cdot\bar{n})\, dA$$

$$-f_{drag} = \int_{sec2} u\rho u\, dA - \int_{sec1} u\rho u\, dA$$

$$-f_{drag} = 2\int_0^L \frac{U}{2}(1+y/L)\rho\frac{U}{2}(1+y/L)\,b\,dy - 2\int_0^H U\rho U b\,dy$$

$$-f_{drag} = 2\frac{U^2}{4}\rho b\int_0^L(1+y/L)^2 dy - 2U^2\rho bH$$

$$\int_0^L\left(1+\frac{2y}{L}+\frac{y^2}{L^2}\right)dy = y + \frac{2y^2}{2L} + \frac{y^2}{3L^2}\Big|_0^L$$

$$L + \frac{2L^2}{2L}\ \ldots \quad 7/3\,L\ ?$$

Tarokh got confused and stopped.

$$-f_{drag} = 2\frac{U^2}{4}\rho b(7/3)L - 2U^2\rho b(3/4)L$$

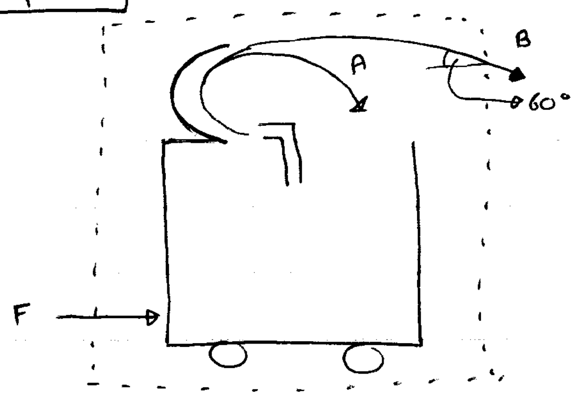
$$\boxed{F = (1/3)\rho U^2 Lb} \rightsquigarrow C_D = \frac{F_{drag}}{(1/2)\rho U^2 Lb} \rightsquigarrow C_D = \frac{(1/3)}{(1/2)} = 2/3$$

b) $F_{drag} = (1/3)\,\rho U^2 L b$

$F_{drag} = (1/3)(998)(4)^2(0.8)(1) = 4260\ N$

$C_D$ is still $2/3$ (doesn't change)

**Example:**




For A) $\sum F_x = \partial/\partial t \int_{cv} \cancel{\rho v\, d\forall} + \underbrace{\int_{cs} \rho v(\bar{v}\cdot\bar{n})\, dA}_{0}$

$\sum F_x = 0 = f$

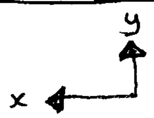
For B) $\sum F_x = \partial/\partial t\, \cancel{\int_{cv} \rho v\, d\forall} + \int_{cs} \rho v(\bar{v}\cdot\bar{n})\, dA$

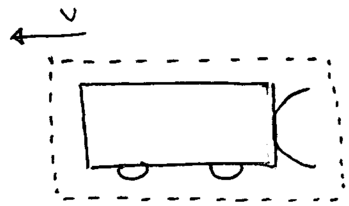
$F = 0 + (\dot{m}\, V_{x,out} - 0)$

$F = (1.94\ slug/ft^3)(200/448\ ft^3/s)(45 \cos 60°)$

$\boxed{F = 19.5\ lb_f}$

**Example**




$$\sum F_x = \partial/\partial t \int_{cv} \rho v \, d\forall + \int_{cs} \rho V_x (\bar{V}\cdot\bar{n})\, dA$$

$$0 = \partial/\partial t \int_{cv} v\, dm + \int_{cs} \rho V_x (\bar{V}\cdot\bar{n})\, dA$$

$$0 = \partial/\partial t \; V \int_{cv} dm + \dot{m}_{out} U_{out} - \dot{m}_{in} U_{in}$$

$$0 = \partial V/\partial t \; m_{car} - 2\dot{m}(V_j - V) \qquad (V_j = \text{velocity of jet})$$

$$= \partial V/\partial t \; m_{car} - 2\rho A_j (V_j - V)^2$$

$$\frac{dV}{dt} m_{car} = 2\rho A_j (V_j - V)^2$$

$$\frac{dV}{dt} = \frac{2\rho A_j}{m_{car}} (V_j - V)^2 = k (V_j - V)^2$$

$$k = \frac{2\rho A_j}{m_{car}}$$

$$\frac{dV}{dt} = k (V_j - V)^2$$

$$\int \frac{dV}{(V_j - V)^2} = \int k\, dt \longrightarrow \frac{1}{V_j - V} + C = kt$$

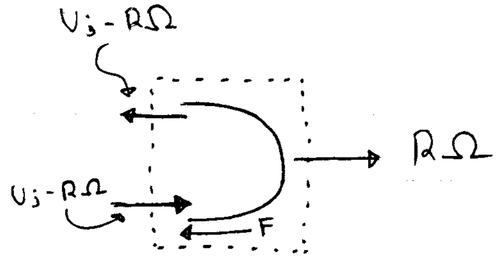
$$\begin{matrix} t = 0 \\ V = 0 \end{matrix} \Rightarrow \frac{1}{V_j - 0} + C = 0 \Rightarrow C = \frac{-1}{V_j}$$

$$\frac{1}{V_j - V} - \frac{1}{V_j} = kt \rightsquigarrow \boxed{V = \frac{V_j^2 kt}{1 + V_j kt}}$$

$$k = \frac{2\rho A_j}{m_{car}} = \frac{2(998)(\pi/4)(0.01)^2}{(12/9.81)} \Rightarrow k = 0.0905\ m^{-1}$$

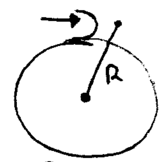
$$V = \frac{(509t)}{1 + 6.785t} \rightsquigarrow \left.\begin{matrix} x = 1 \\ t = ? \end{matrix}\right\} 0.072\ s \Rightarrow V = 24.6\ m/s$$

## Example




$$\Sigma F_x = \dot{m} u_{out} - \dot{m} u_{in}$$
$$= \dot{m}(-(V_j - R\Omega)) - \dot{m}(V_j - R\Omega)$$
$$-F = -2\dot{m}(V_j - R\Omega)$$
$$F = 2\dot{m}(V_j - R\Omega)$$

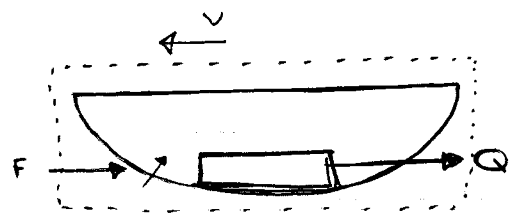
$$\mathbb{P} = T\Omega$$




$$\mathbb{P} = FR\Omega$$
$$\mathbb{P} = 2\dot{m}(V_j - R\Omega)R\Omega$$
$$\dot{m} = \rho A_j (V_j - R\Omega) \longrightarrow \mathbb{P} = 2\rho A_j R\Omega (V_j - R\Omega)^2$$

$$\frac{d\mathbb{P}}{d\Omega} = \emptyset = \frac{d}{d\Omega}\left(2\rho A_j R\Omega (V_j - R\Omega)^2\right) = \emptyset$$
$$2\rho A_j R \frac{d}{d\Omega}\left(\Omega (V_j - R\Omega)^2\right) = \emptyset$$
$$(V_j - R\Omega)^2 + 2\Omega(-R)(V_j - R\Omega) = \emptyset$$
$$(V_j - R\Omega)(V_j - R\Omega - 2R\Omega) = \emptyset$$
$$(V_j - R\Omega)(V_j - 3R\Omega) = \emptyset$$
$$\boxed{R\Omega = V_j/3}$$
$$\mathbb{P}_{max} = 2\rho A_j (V_j/3)(V_j - V_j/3)^2 = \underline{(8/27)\rho A_j V_j^3}$$

**Example:**




$$\sum F_x = \dot{m}u_{out} - \dot{m}u_{in}$$

$$KV^2 = \dot{m}_{pump}(V_j - V_{inlet} + V) = \rho Q (V_j - V_{inlet} + V)$$

$$V_{inlet} << V \ \& \ V_j$$

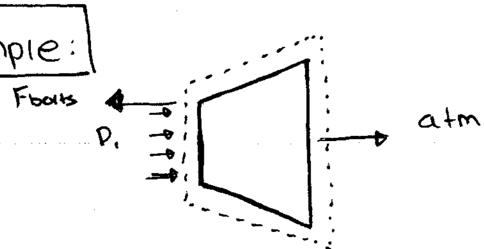
$$KV^2 = \rho Q (V_j + V)$$

$$V^2 = \frac{\rho Q}{K}(V_j + V) \Rightarrow \boxed{V^2 - \frac{\rho Q}{K}V - \frac{\rho Q}{K}V_j = 0}$$

$$\frac{\rho Q}{2K} = \alpha \quad \Rightarrow \quad \boxed{V = \alpha + (\alpha^2 + 2\alpha V_j)^{1/2}}$$

assume $\quad V << V_j \quad \Rightarrow \quad KV^2 = \rho Q V_j$

**Example:**




$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$V_1 = 0.1 / (\pi/4)(0.1)^2 = 12.73 \text{ m/s}$$

$$V_2 = 0.1 / (\pi/4)(0.2)^2 = 3.18 \text{ m/s}$$

From (*) $\Rightarrow$ $P_1 = (1/2)\rho(V_2^2 - V_1^2)$

$$= (1/2)(1000)(3.18^2 - 12.73^2)$$

$$\Rightarrow P_{1,gage} = 75970 \text{ Pa}$$

$$\sum F_x = \dot{m}u_{out} - \dot{m}u_{in}$$

$$P_1A_1 - F_{bolts} = \dot{m}(V_2 - V_1)$$

$$F_{bolts} = (75970)(\pi/4)(0.1)^2 - (1000)(0.1)(3.18 - 12.73)$$

$$F_{bolt} = 1552 \text{ N} \leftarrow$$