Nov. 19TH/18

Change of total energy of a system :

$\Delta E = \Delta U + \Delta KE + \Delta PE$

Bernoulli eq'n : $P + (1/2)\rho V^2 + \rho g z + C$

Nondimensionalization of Equations : If we divide each term in the equation by a collection of variables and constants whose product has the same dimensions, the eq'n is rendered non-dimensional

Normalized equation : If the nondimensional terms in the equation are of order unity, the equation is called normalized.

Non-dimensional parameters : often a result From the process of nondimensionalizing an equation of motion (aka inspectional analysis)

Nondimensionalized bernoulli:

$$\frac{P}{P_\infty} + \frac{\rho V^2}{P_\infty} + \frac{\rho g z}{P_\infty} = \frac{C}{P_\infty}$$

Dimensional Variables : dimensional variables that change in the problem.

Nondimensional " " : " " but dimensionless

Dimensional constant : gravity, etc.

Pure constants : π, etc.

Scaling parameters - needed to non dimensionalize

$$\rightarrow \frac{d^2 z}{dt^2} = -g$$

velocity

Primary dimensions of all parameters : $\{z\} = \{L\}$ $\{t\} = \{t\}$ $\{z_o\} = \{L\}$ $\{\omega_o\} = \{L/t\}$ $\{g\} = \{L/t^2\}$

$$z^* = \frac{z}{z_o} \qquad t^* = \frac{\omega_o t}{z_o}$$

both initially have no value

Since $\{z_o\} = \{L\}$ $\qquad \{z\} = \{L\}$

$\{\omega_o\} = \{L/t\}$ $\qquad \{t\} = \{t\}$

$z_o \Rightarrow$ for length $\Rightarrow z^* = \frac{z}{z_o} \Rightarrow z = z^* \cdot z_o$

$$\left\{\frac{z}{\omega_o}\right\} \Rightarrow \left\{\frac{L}{L/t}\right\} = \{t\}$$

$\hookrightarrow$ scaling factor for time

$$t^* = \frac{t}{z_o/\omega_o} = \frac{t\,\omega_o}{z_o}$$

$$\frac{d^2 z}{dt^2} = \frac{d^2(z^* z_o)}{d\left(\frac{t^* z_o}{\omega_o}\right)^2} = -g \Rightarrow \frac{z_o\, d^2(z^*)}{\frac{z_o^2}{\omega_o^2}\, d(t^*)^2} = -g$$

$$\frac{d^2 z^*}{d(t^*)^2} = -\frac{g z_o^2}{\omega_o^2}$$

$$\left\{\frac{g\, z_o}{\omega_o^2}\right\} = \left\{\frac{L}{t^2} \times \frac{L}{(L/t)^2}\right\} = \{1\}$$

$\rightarrow$ Dimensional analysis is usually done on a scale model

The principle of similarity

(1) Geometric similarity $\rightarrow$ model must be same shape

(2) Kinematic similarity $\rightarrow$ velocity must be proportional

(3) Dynamic similarity $\rightarrow$ when all forces scale by a constant factor

In a general flow field, complete similarity is only achieved when there is dynamic, geometric, and kinematic similarity.

Π - non dimensional parameter (uppercase pi)

$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4, \ldots)$

dependent (C_D) drag force — independent (Re) reynolds #

If $\Pi_{2,\text{model}} = \Pi_{2,\text{prototype}}$, $\Pi_{3,m} = \Pi_{3,p}$ ... $\Pi_{k,m} = \Pi_{k,p}$

Then $\Pi_{1,m} = \Pi_{1,p}$

example 7.5

$\boxed{\text{Example}}$

$$\Pi_{2,m} = \Pi_{2,p}$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$T = 25°C$ $\left\{ \begin{array}{l} \rho = 1.184 \text{ kg/m}^3 \\ \mu = 1.849 \times 10^{-5} \text{ kg/m·s} \end{array} \right.$

$T = 5°C$ $\left\{ \begin{array}{l} \rho = 1.269 \text{ kg/m}^3 \\ \mu = 1.754 \times 10^{-5} \text{ kg/m·s} \end{array} \right.$

$$\to V_m = V_p \left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)$$

$$\Rightarrow (50)\left(\frac{1.754 \times 10^{-5}}{1.849 \times 10^{-5}}\right)\left(\frac{1.184}{1.269}\right)(5) = 221 \text{ m/h}$$

velocity of the wind tunnel

Nov. 21/18

7.6

Example

$$\left(F_D/\rho V^2L^2\right)_{model} = \left(F_D/\rho V^2L^2\right)_{prototype}$$

$$F_{D,Prototype} = \frac{(F_{D,molel})(\rho V^2L^2)_{prototype}}{(\rho V^2L^2)_{model}}$$

$$= (21.2)\left(\frac{1.184}{1.269}\right)\left(\frac{50}{221}\right)(5)^2 = 25.3 \text{ lb}_f$$

* Keep a reference for speed of sound

Method of Repeating Variables

1 → list parameters, and count their total number

2 → list primary dimensions of each of the n-parameters

3 → set the reduction j as the number of primary dims

$k = n - j$

4 → choose j repeating parameters

5 → construct k Π's and manipulate

6 → check that Π are dimensionless

Equation of motion: $\dfrac{d^2z}{dt^2} = -g$

Step 1 : $z = f(t, w_0, z_0, g)$, $n = 5$

Step 2 : $\overset{z}{(L)}$ $\overset{t}{(t)}$ $\overset{w_0}{(L/t)}$ $\overset{z_0}{(L)}$ $\overset{g}{(L/t^2)}$

Step 3 : reduction : $j = 2$

Number of expected Π's $k = n - j = 3$

Step 4 : repeating parameters : w_0 and z_0

Step 5 : Dependent Π $\Pi_1 = z w_0^{a_1} z_0^{b_1}$

Dimensions of Π_1 : $\{L^0 t^0\} = \{z w_0^{a_1} z_0^{b_1}\} = \{L^1(L^1 t^{-1})^{a_1} L^{b_1}\}$

Time : $\{t^0\} = \{t^{-a_1}\}$ $-a_1 = 0$, $a_1 = 0$

Length $\{L^0\} = \{L^1 L^{a_1} L^{b_1}\}$ $0 = 1 + a_1 + b_1$ $\longrightarrow b_1 = -1 - a_1$

$b_1 = -1$

$\boxed{\Pi_1 = z/z_0}$ First dependent $\Pi_2 = t\omega_0^{a_2} z_0^{b_2}$

Dimension of Π_2 : $(\Pi)_2 = \{L^0 t^0\} = \{t\omega_0^{a_2} z_0^{b_2}\} = \{t(L^1 t^{-1})^{a_2} L^{b_2})$

Equating exponents,

Time : $\{t^0\} = \{t^1 t^{-a_2}\}$ $\quad 0 = 1 - a_2 \quad a_2 = 1$

Length : $\{L^0\} = \{L^{a_2} L^{b_2}\}$ $\quad 0 = a_2 + b_2$

$b_2 = -a_2$

$b_2 = -1$

$$\boxed{\Pi_2 = \frac{\omega_0 t}{z_0}}$$

Second dependent Π : $\Pi_3 = g\omega_0^{a_3} z_0^{b_3}$

Dimensions of Π_3 : $\{\Pi_3\} = \{L^0 t^0\} = \{g\omega_0^{a_3} - z_0^{b_3}\} = \{L^1 t^{-2} (L^1 t^{-1})^{a_3} L^{b_3}\}$

Equating exponents,

Time : $\{t^0\} = \{t^{-2} t^{-a_3}\}$ $\quad 0 = -2 - a_3 \quad a_3 = -2$

Length : $\{L^0\} = \{L^1 L^{a_3} L^{b_3}\}$ $\quad 0 = 1 + a_3 + b_3$

$b_3 = -1 - a_3 \quad b_3 = 1$

$$\boxed{\Pi_3 = \frac{g z_0}{\omega_0^2}} \rightarrow \text{modified } \Pi_{3,\text{mod}} = \left(\frac{g z_0}{\omega_0^2}\right)^{-1/2} = \boxed{\frac{\omega_0}{\sqrt{g z_0}} = F_r}$$

Step 6 : Relationship between Π's : $\Pi_1 = f(\Pi_2, \Pi_3)$

$$\rightarrow z/z_0 = f\left(\frac{\omega_0 t}{z_0}, \frac{\omega_0}{\sqrt{g z_0}}\right)$$

Final result of dimensional analysis

$$z^* = f(t^*, F_r)$$

Example

$$\Delta P = f(\rho, V, D, L, \mu, \varepsilon)$$

$$\underset{\rho}{\left(\frac{m}{L^3}\right)} \quad \underset{V}{\left(L/t\right)} \quad \underset{D}{\left(L\right)} \quad \underset{L}{\left(L\right)} \quad \underset{\mu}{\left(\frac{m}{Lt}\right)} \quad \underset{\varepsilon}{\left(L\right)} \quad \underset{\Delta P}{\left(\frac{m}{Lt^2}\right)} \qquad n = 7, \; j = 3$$

D, ρ, V repeating variables

$$\Pi_1 = \Delta P \rho^{a_1} V^{b_1} D^{c_1}$$

$$= \left(\frac{m}{Lt^2}\right)\left(\frac{m}{L^3}\right)^{-1}\left(\frac{L}{t}\right)^{-2}(L)^0$$

$$\Rightarrow a_1 = -1, \quad b_1 = -2, \quad c_1 = 0$$

$$\Pi_1 = \frac{\Delta P}{\rho V^2}$$

For $\Pi_2 = \mu \rho^{a_2} V^{b_2} D^{c_2}$

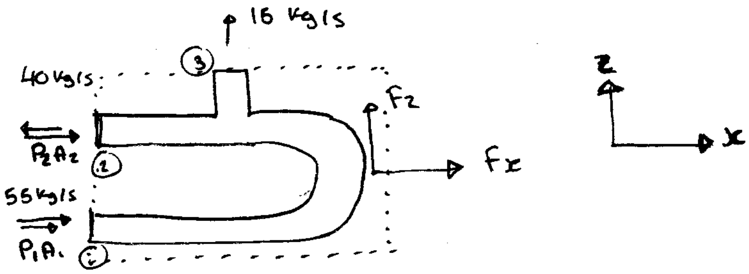
$$= \left(\frac{m}{Lt}\right)\left(\frac{m}{L^3}\right)^{-1}\left(\frac{L}{t}\right)^{-1}(L) \quad \ldots$$

$$= \left(\frac{\cancel{m}}{\cancel{L}\cancel{t}}\right)\left(\frac{L^3}{\cancel{m}}\right)\left(\frac{\cancel{t}}{\cancel{L}}\right)(L)^{-1}$$

then $a_2 = -1$, $b_2 = -1$, $c_2 = -1$

$$\boxed{\Pi_2 = \frac{\mu}{\rho V D}}$$

Problem 6 – Assignment 3 :



$\beta_1 = \beta_2 = \beta_3 = 1.03$

$$\sum \bar{F} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$

$$x: \sum F_x = \sum_{out} \beta \dot{m} V_x - \sum_{in} \beta \dot{m} V_x$$

$$P_1A_1 + P_2A_2 + F_x = \beta_2 \dot{m}_2(-V_2) - \beta_1 \dot{m}_1 (V_1) \qquad \text{I}$$

$$\dot{m}_1 = \rho_1 V_1 A_1 \Rightarrow V_1 = \frac{\dot{m}_1}{\rho_1 A_1} = \frac{(55 \text{ kg/s})}{(1000 \text{ kg/m}^3)(\pi/4)(0.05)^2} = 28.01 \text{ m/s}$$

$$\dot{m}_2 = \rho_2 V_2 A_2 = V_2 = \frac{\dot{m}_2}{\rho_2 A_2} = 5.093 \text{ m/s}$$

$$\rightarrow V_3 = \frac{\dot{m}_3}{\rho_3 A_3} =$$

From eq'n 1: $\Rightarrow \cancel{200}^{(200-100)}(\pi(0.05)^2/(4)) + \cancel{150}^{(150-100)}(\pi(0.1)^2/4) + F_x \ldots$

$\ldots = [(1.03)(40)(-5.093) - (1.03)(55)(28.01)](1/1000)$

$$F_x = -2.386 \text{ kN}$$

$$\Rightarrow F_x = 2.386 \text{ kN} \leftarrow$$

$$\sum F_z = \sum_{out} \beta_2 \dot{m}_2 V_{2z} - \cancel{\sum_{in} \beta_1 \dot{m} V_{1z}}^{0}$$

$$F_z = 0.328 \text{ kN}$$

b) $m = \rho V = \rho A L = \rho \frac{\pi D_{circ}^2}{4} L = 1000 \frac{\pi (0.075)^2}{4} (0.5) = 2.2 \text{ kg}$

$F_z - W = \beta_3 \dot{m}_3 V_3 \Rightarrow F_z = W + \beta_3 \dot{m}_3 V_3 = 2.2 \times 9.81 + 328$

$= 349.6 \text{ N}$

+ Sprinkler example ...