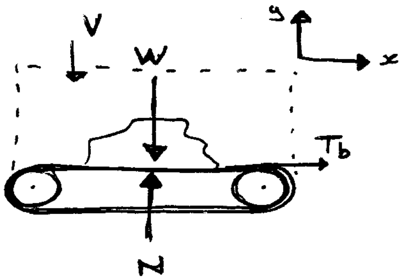
Nov. 12/18

$\boxed{\text{Example}}$ (horizontal conveyor ...)



$$\sum \vec{F} = \partial/\partial t \int_{cv} \rho \vec{V} d\forall + \int_{cs} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA$$

$$x: \quad \sum F_x = \partial/\partial t \int_{cv} \rho V_x d\forall + \int_{cs} \rho V_x (\vec{V} \cdot \vec{n}) dA$$

$$T_b = \partial/\partial t \int_{cv} \rho V_x d\forall + \dot{m} \cancel{V_{x_2}}^{0} - \dot{m} \cancel{V_{x_1}}^{0} \qquad (I)$$

$$T_b = \partial/\partial t \int_{cv} \rho V_x dA = \partial/\partial t \int_{cv} \rho V_{belt} d\forall = V_{belt}\, \partial/\partial t \int_{cv} \rho d\forall$$

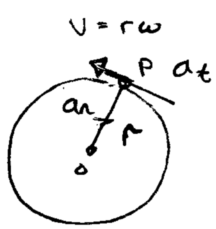
$$\partial/\partial t \int_{cv} \rho d\forall + \int_{cs} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = 0 \qquad (II)$$

$$\partial/\partial t \int_{cv} \rho d\forall = - \int_{cs} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = -(-\dot{m}_{sand}) = \dot{m}_{sand}$$

From (I) and (II) $\Rightarrow$ $T_b = V_{belt} \dot{m}_{sand}$

$$T_b = 3\ \text{ft/s} \times 600\ \text{lbm/s} \times 1\ \text{slug}/32.2\ \text{lbm} \cdots$$

$$\cdots \frac{1\ \text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = \boxed{46.6\ \text{lbf}}$$



$v = r\omega$

$a_n = r\omega^2$

$a_t = r\alpha$

$M_o = rF_t = rma_t = mr^2\alpha$

Magnitude of Torque: $M = \int_{mass} r^2 \alpha\, \delta m = \left[\int_{mass} r^2 \delta m\right] \alpha$

↳ (M)

$$M = I\alpha$$

Magnitude of Angular Momentum (H)

$$H = \int_{mass} r^2 \omega\, \delta m = \left[\int_{mass} r^2 \delta m\right] \omega = I\omega$$

$$H = I\omega$$

Angular momentum equation:

$$\vec{M} = I\vec{\alpha} = I \frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

$\dot{W}_{shaft} = FV = Fr\omega = M\omega \qquad (\omega = 2\pi\dot{n})$

Shaft power $= \dot{W}_{shaft} = \omega M = 2\pi \dot{n} M$

Rotational kinetic energy $\quad KE_r = \frac{1}{2} I \omega^2$

$a_r = \frac{V^2}{r} = r\omega^2$ $\qquad F_r = mV^2/r$

$\vec{a} = \vec{a_t} + \vec{a_r}$

Moment of a Force : $\quad \vec{M} = \vec{r} \times \vec{F}$

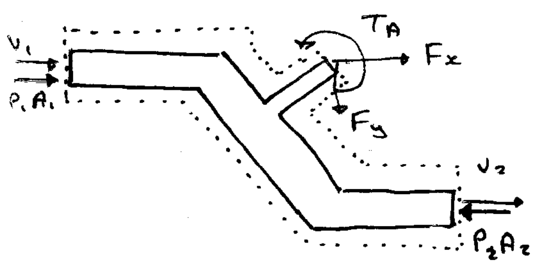
Moment of Momentum : $\quad \vec{H} = \vec{r} \times m\vec{V}$

$\Sigma \vec{M} = d\vec{H}_{sys}/dt \qquad \Sigma \vec{M} = \Sigma(\vec{r} \times \vec{F})$

$$\frac{d\vec{H}_{sys}}{dt} = d/dt \int_{cv} (\vec{r} \times \vec{V}) \rho \, dV + \int_{cs} (\vec{r} \times \vec{V}) \rho (\vec{V_r} \cdot \vec{n}) \, dA$$

Steady Flow $\quad \Sigma \vec{M} = \sum_{out} (\vec{r} \times \dot{m}\vec{V}) - \sum_{in} (\vec{r} \times \dot{m}\vec{V})$

$\boxed{\text{Example}}$



$\Sigma M = T_A + \vec{r_1} \times (-P_1A_1)\bar{n}_1 + \vec{r_2} \times (-P_2A_2)\bar{n}_2$

$\quad = T_A + P_1A_1h_1 - P_2A_2h_2$

Because:

$\Sigma M = \cancel{\partial/\partial t \int_{cv} \rho(\bar{r} \times \bar{v}) \, d\forall}^{\,0} + \int_{cs} \rho(\bar{r} \times \bar{v})(\bar{v} \cdot \bar{n}) \, dA$

$\Sigma M_z = \dot{m}h_2V_2 = \dot{m}h_1V_1$

$T_A + P_1A_1h_1 - P_2A_2h_2 = \dot{m}(h_2V_2 - h_1V_1)$

$$\boxed{T_A = h_2(P_2A_2 + \dot{m}V_2) - h_1(P_1A_1 + \dot{m}V_1)}$$

similar to Assignment 3, Q7

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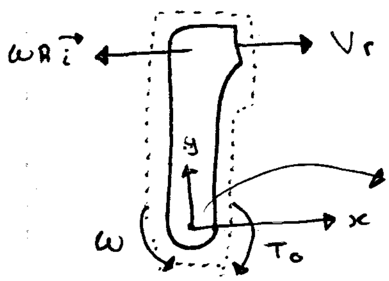
$\boxed{\text{Example}}$ Lawn Sprinkler arm from above:



$V_r = V_o\vec{i} - \omega R\vec{i}$

(this is $\dot{V}$)

$Q = V_o A_1 \rightarrow V_o = \frac{Q}{A}$

$$\Sigma \bar{M}_o = \partial/\partial t \int_{cv} \rho \cancel{(\bar{r} \times \bar{r})}^{\,0}\, d\forall + \int_{cs} \rho(\bar{r} \times \bar{v}_1)(\bar{U}\cdot\bar{n})\, dA$$

$$\Sigma M_o = -T_o\vec{k} = (r_2 \times V_2)\dot{m}_{out} - \cancel{(r_1 \times v_1)}\dot{m}_{in}$$

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} = \rho Q$$

$$-T_o\vec{k} = \left(R\vec{j} \times (V_o - \omega R)\vec{i}\right)\dot{m} = -R(V_o - \omega R)\dot{m}\,\vec{k}$$

$$T_o = R(V_o - \omega R)\rho Q$$

$$\boxed{\omega = \frac{V_o}{R} - \frac{T_o}{\rho Q R^2}}$$

Flow with no external moments

$$\emptyset = \frac{dH_{cv}}{dt} + \sum_{out}(\vec{r} \times \dot{m}\vec{V}) - \sum_{in}(\vec{r} \times \dot{m}\vec{V})$$

$$\vec{M}_{body} = I_{body}\vec{\alpha} = \sum_{in}(\bar{r} \times \dot{m}\vec{V}) - \sum_{out}(\vec{r} \times \dot{m}\vec{V})$$

Radial-Flow Devices: pumps, turbines, etc.
involve flow in the radial direction, normal to axis of rotation.

Axial-Flow device: linear momentum equation
Radial-Flow device: angular momentum equation

The conservation of mass eq'n for steady, incompressible flow.

$$\left.\begin{array}{l} \dot{V}_1 = \dot{V}_2 = \dot{V} \longrightarrow (2\pi r_1 b_1)V_{1,n} = (2\pi r_2 b_2)V_{2,n} \\ V_{1,n} = (\dot{V})/(2\pi r_1 b_1) \qquad V_{2,n} = (\dot{V})/(2\pi r_2 b_2) \end{array}\right\} \Sigma M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$$

Eulers turbine equation : $T_{shaft} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t})$

$$T_{shaft} = \dot{m}(r_2 V_2 \sin\alpha_2 - r_1 V_1 \sin\alpha_1)$$

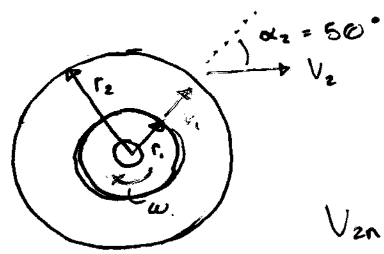
when $V_{1,t} = \omega r_1$ and $V_{2,t} = \omega r_2$

$$T_{shaft,ideal} = \dot{m}\omega(r_2^2 - r_1^2)$$

$$\dot{W}_{shaft} = \omega T_{shaft} = 2\pi \dot{n}\, T_{shaft}$$

6.58

Example



$$V_{1n} = \frac{Q}{2\pi r_1 b_1} = \frac{0.7}{2\pi(0.2)(0.082)}$$

$$V_{1n} = 6.793 \text{ m/s}$$

$$V_{2n} = \frac{Q}{2\pi r_2 b_2} = \frac{0.7}{2\pi(0.45)(0.056)} = 4.421 \text{ m/s}$$

$$V_{1t} = V_{1n}\tan\alpha_1 = V_{1n}(0) = 0$$

$$V_{2t} = V_{2n}\tan\alpha_2 = 4.421(\tan(50°)) = 5.269 \text{ m/s}$$

$$\dot{n} = 700 \text{ rpm} \Rightarrow \omega = 2\pi\dot{n} = 2\pi\left(\frac{700}{60}\right) = 73.3 \text{ rad/s}$$

$$\dot{m} = \rho Q = (1.25)(0.7) = 0.875 \text{ kg/s}$$

$$T_{shaft} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t})$$

$$T_{shaft} = (0.875)(0.45(5.269)) = 2.075 \text{ N}\cdot\text{m}$$

$$\dot{W} = \omega T_{shaft} = (73.3)(2.075) = \boxed{152 \text{ W}}$$

Example 6.8

Example