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Example

$$\underbrace{\partial/\partial t \int_{cv} \rho \, d\forall}_{1} + \underbrace{\int_{cs} \rho (\bar{V}\cdot\bar{n})\, dA}_{2} = 0 \quad \text{I}$$

1 $$d/dt \int_{cv} \rho\, d\forall = d/dt(\rho_w A_t h) + d/dt(\rho_a A_t (H-h))$$
$$= \rho_w A_t (dh/dt) \quad \text{II}$$

2 $$\int_{cs} \rho(\bar{V}\cdot\bar{n})\, dA = -\rho_w V_1 A_1 - \rho_w V_2 A_2 \quad \text{III}$$

IF substitute II and III in I =>

$$\rho_w A_t (dh/dt) - \rho_w \underbrace{\boxed{V_1 A_1}}_{Q_1} - \rho_w \underbrace{\boxed{V_2 A_2}}_{Q_2} = 0 \quad \Rightarrow \quad dh/dt = \frac{Q_1 - Q_2}{A_t}$$

$$dh/dt = 1/A_t \left[V_1 \frac{D_1^2 \pi}{4} + V_2 \frac{D_2^2 \pi}{4} \right]$$

where
$D_1 = 1\text{ in}$
$D_2 = 3\text{ in}$
$V_1 = 3\text{ ft/s}$
$V_2 = 2\text{ ft/s}$
$A_t = 2\text{ ft}^2$

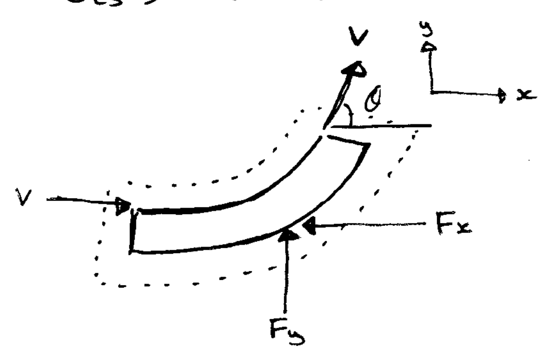
$$\frac{dh}{dt} = \frac{1}{2}\left[3\left(\frac{\pi}{4}\right)\left(\frac{1}{12}\right)^2 + (2)\left(\frac{\pi}{4}\right)\left(\frac{2}{12}\right)^2 \right]$$

Example

$$\Sigma \bar{F} = \partial/\partial t \int_{cv} \rho \bar{V}\cdot d\forall + \int_{cs} \rho \bar{V}(\bar{V}\cdot\bar{n})\, dA$$

$$x: \quad \Sigma F_x = \partial/\partial t \int_{cv} \cancel{\rho u \, d\forall} + \int_{cs} \rho u (\bar{V}\cdot\bar{n})\, dA$$

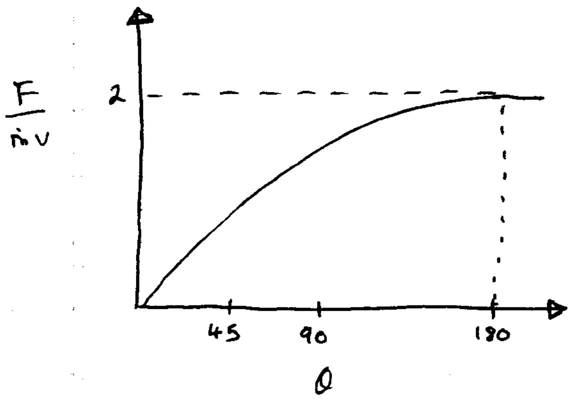
$$y: \quad \Sigma F_y = \partial/\partial t \int_{cv} \cancel{\rho v\, d\forall} + \int_{cs} \rho v (\bar{V}\cdot\bar{n})\, dA$$



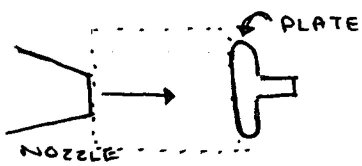
$$-F_x = -\rho V V A_1 + \rho (V\cos\theta) V A_2$$
$$-F_x = -\rho V^2 A + \rho V^2 A \cos\theta$$
$$\boxed{F_x = \rho V^2 A (1 - \cos\theta)}$$

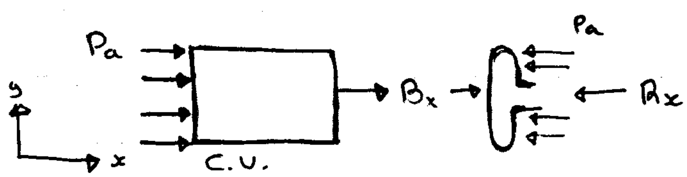
$$F_y = 0 + \rho (V \sin\theta) V A_2$$
$$\boxed{F_y = \rho V^2 A \sin\theta}$$

b) $$F = \sqrt{F_x^2 + F_y^2} = \left[\rho^2 V^4 A^2 (1-\cos\theta)^2 + \rho^2 V^4 A^2 \sin^2\theta \right]^{1/2}$$
$$= \rho V^2 A \left[(1-\cos\theta)^2 + (\sin\theta)^2 \right]^{1/2}$$
$$= 2\rho V^2 A \sin(\theta/2)$$
$$\rightarrow F = 2\dot{m} V \sin(\theta/2)$$



Example





$\Sigma F_x = 0$

$$x: \quad \Sigma F_x = \partial/\partial t \int_{cv} \cancel{\rho \bar{V}\, d\forall}^{\,0} + \int_{cs} \rho \bar{V} (\bar{V}\cdot\bar{n})\, dA$$

$$B_x + \cancel{P_a}^{\,0} A = \rho V_N V_N A_N$$

$$B_x = -\rho V_N^2 A_N = -(1000)(15)^2(0.01) = (-2.25 \text{ kN})$$

$$\Sigma F_x = 0 \quad ; \quad B_x + R_x = 0$$

$$R_x = -B_x = 2.25 \text{ kN}$$

Special cases:

Steady flow: $\Sigma \vec{F} = \int_{cs} \rho \bar{V} (\bar{V}\cdot\bar{n})\, dA$

Mass flow across inlet/outlet: $\dot{m} = \int_{Ac} \rho (\bar{V}\cdot\bar{n})\, dA_c = \rho V_{avg} A_c$

Momentum-Flux Correction Factor, β

$$\Sigma \bar{F} = d/dt \int_{cv} \rho \bar{V}\, d\forall + \sum_{out} \beta \dot{m} \bar{V}_{avg} - \sum_{in} \beta \dot{m} \bar{V}_{avg}$$

$$\beta = \frac{\int_{Ac} \rho \bar{V} (\bar{V}\cdot\bar{n})\, dA_c}{\dot{m} V_{avg}}$$

(β always ≥ 1)

(β close to 1 for turbulent, not close for laminar)

$$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c$$ (momentum - Flux correction factor)

Example

$$V = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{Ac}}\right)^2 dA_c = \frac{4}{\pi R^2}\int_0^R \left(1 - r^2/R^2\right)^2 2\pi r\,dr$$

$$\rightarrow \beta = \frac{4}{\pi R^2}\int_{A_c}\left(\frac{V_{avg}(1-r^2/R^2)}{V_{avg}}\right)^2 2\pi r\,dr$$

$$8/R^2\int_0^R \left(1 - r^2/R^2\right) r\,dr = 8/R^2\int 1 - \frac{2r^3}{R^2} + r^5/R^4\,)\,r\,dr$$

$$8/R^2\left[r^2/2 - (2/R^2)(r^4/4) + (r^6/6R^4)\right]\Big|_0^R$$

$$\Rightarrow 8/R^2\left[R^2/2 - (2/R^2)(R^4/4) + (R^6/6R^4)\right]$$

$$= 8(1/2 - 1/2 + 1/6) = \boxed{1.333}$$

Steady linear momentum

$$\sum \vec{F} = \sum_{out}\beta\dot{m}\vec{V} - \sum_{in}\beta\dot{m}\vec{V}$$

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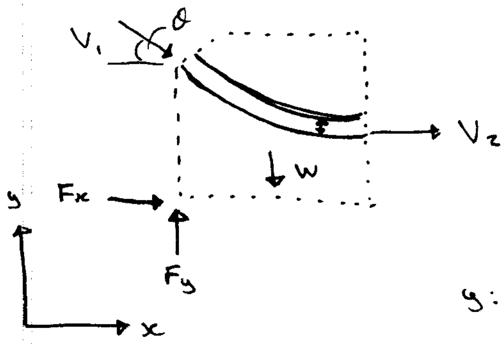
One inlet and outlet: $\sum \vec{F} = \dot{m}(\beta_2 \vec{V}_2 - \beta_1 \vec{V}_1)$

along x-axis: $\sum \vec{F}_x = \dot{m}(\beta_2 V_{2x} - \beta_1 V_{1x})$



where $V_1 A_1 = V_2 A_2 \rightarrow V_1 = V_2 = V$

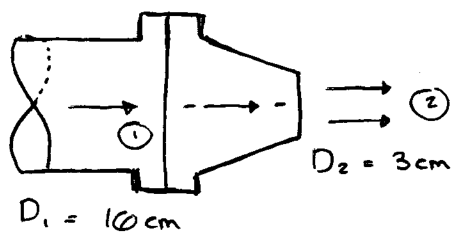
$$F_x = \dot{m}(\beta_2 V_2 - \beta_1 V_1 \cos\theta)$$

$$F_x = \dot{m} V(\beta_2 - \beta_1 \cos\theta)$$

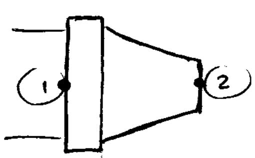
y: $$F_y - W = \dot{m}\beta_1 V_1 \sin\theta$$

Example:

$\rho = 1000\ kg/m^3$



$Q = 1.5\ m^3/min = 0.025\ m^3/s$



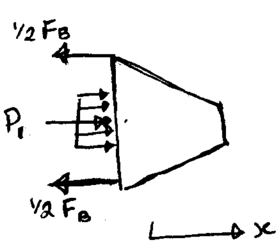
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}$$

$$P_1 = \cancel{P_2}^{\,0} + (1/2)\rho(V_2^2 - V_1^2)$$

$$V_1 = \frac{Q}{A_1} = \frac{0.025}{(\pi/4)(10\times10^{-2})^2} = 3.2\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.025}{(\pi/4)(3\times10^{-2})^2} = 35.4\ m/s$$

$$P_1 = (1/2)(1000)(35.4^2 - 3.2^2) = 620\ 000\ Pa\ (gage)$$
$$= 620\ kPa\ (gage)$$



$$\sum F_x = -1/2 F_B - 1/2 F_B + P_1 A_1$$

$$\sum F_x = \dot{m}(V_2 - V_1)$$

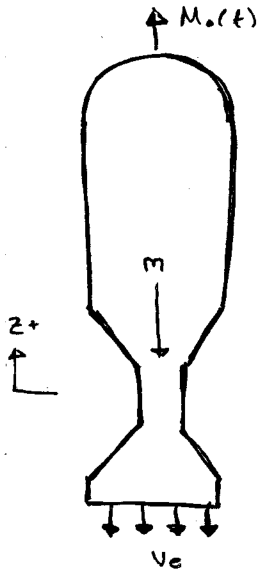
$$\boxed{-F_B + P_1 A_1 = \dot{m}(V_2 - V_1)}$$

$$F_B = 620000((\pi/4)(10\times10^{-2})^2) - (1000)(0.025)(35.4 - 3.2)$$

$$F_B = 4067\ N$$

No external Forces: $\emptyset = \frac{d(m\bar{V})}{dt} + \sum_{OUT} \beta \dot{m} \bar{V} - \sum_{IN} \beta \dot{m} \bar{V}$

$\boxed{\text{Example}}$



$$m = m(t) = M_0 - \dot{m}t$$

$$\sum F_z = \partial/\partial t \int_{cv} \rho v \, d\forall + \int_{cs} \rho v (\bar{v} \cdot \bar{n}) dA$$

$$\partial/\partial t \int \rho v \, d\forall = \int_{cv} \cancel{\partial\rho/\partial t}^{\,0} \rho v \, d\forall + \int_{cs} \partial v/\partial t \underbrace{\rho d\forall}_{dm}$$

$$-mg = \int_{cv} \frac{\partial v}{\partial t} dm - \dot{m} V_e$$

$$-mg = \frac{dV}{dt} m - \dot{m} V_e \rightarrow -mg - \frac{dV}{dt} m = -\dot{m} V_e$$

$$m(g + dV/dt) = \dot{m} V_e$$

$$m(t) = M_0 + \dot{m} t$$

$$(M_0 - \dot{m}t)(g + \frac{dv}{dt}) = \dot{m} V_e$$

$$g + \frac{dV}{dt} = \frac{\dot{m} V_e}{M_0 - \dot{m}t}$$

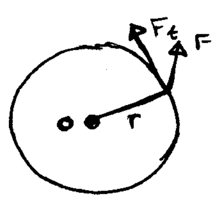
$$\Rightarrow dV/dt = \dot{m}V_e / M_0 - \dot{m}t - g$$

$$\int_{v_0}^{v} dv = \int_0^t \left(\frac{\dot{m} V_e}{M_0 - \dot{m}t} - g \right) dt$$

$$V = \int_0^t \frac{\dot{m} V_e}{M_0 - \dot{m}t} dt - \int_0^t g \, dt$$

$$\boxed{V = V_e \ln\left(\frac{M_0 - \dot{m}t}{M_0}\right) - gt}$$

$$M = rF_t = rma_t = mr^2\alpha$$



Magnitude of torque: $M = \int_{mass} r^2 \alpha \, \delta m$

$$\Rightarrow \left[\int_{mass} r^2 \delta m \right] \alpha = I\alpha$$

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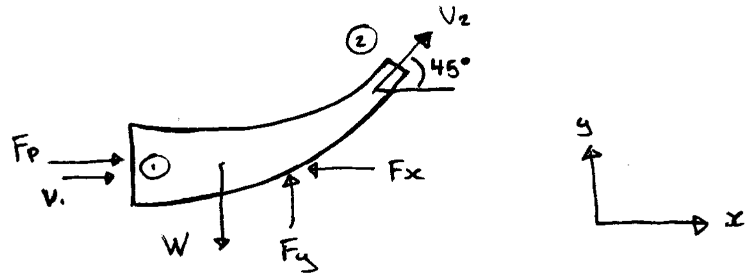

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{(30\ kg/s)}{(1000)(150\times10^{-4})} = 2\ m/s$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{(30\ kg/s)}{(1000)(25\times10^{-4})} = 12\ m/s$$

$$P_1 = \rho\left(g(z_2 - z_1) + \frac{V_2^2 - V_1^2}{2}\right) = 1000\left(9.81(40\times10^{-2}) + \frac{12^2-2^2}{2}\right)$$
$$= 73.9\times10^3\ Pa$$

x: $\sum F_x = (\beta_2 \dot{m} V_{x_2}) - (\beta_1 \dot{m} V_{x_1})$

y: $\sum F_y = (\beta_2 \dot{m} V_{y_2}) - (\beta_1 \dot{m} V_{y_1})$

$\sum F_x = F_{P_1} - F_x$

$F_{P_1} = P_1 A_1 = (73.9\times10^3)(150\times10^{-4})$

$(73.9\times10^3)(150\times10^{-4}) - F_x = (30)(1.03(V_2(\cos 45°)) - 1.03(V_1))$

$= 30(1.03)(12\cos 45° - 2)$

$\rightarrow F_x = 0.908\times10^3\ N$

$F_x = 0.908\ kN$

$\sum F_y = F_y - W$ . . .