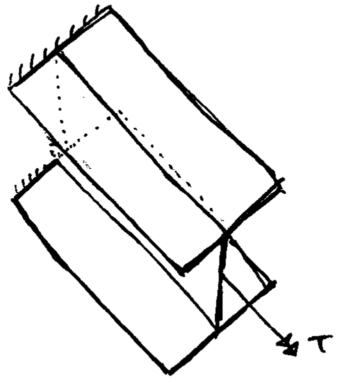
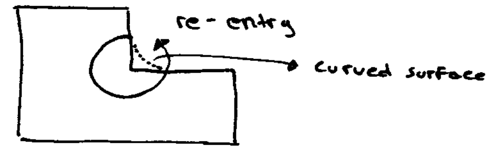
OCT. 30/18

Torsion member with restrained ends.

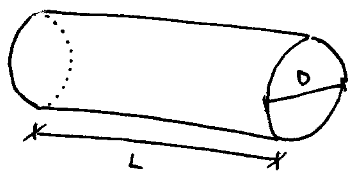


Shear concentration:



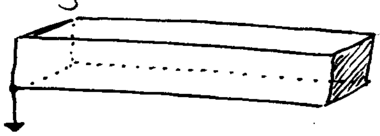
Ch. 7 - Bending of Straight Beams

7.1 - Fundamentals of beam bending



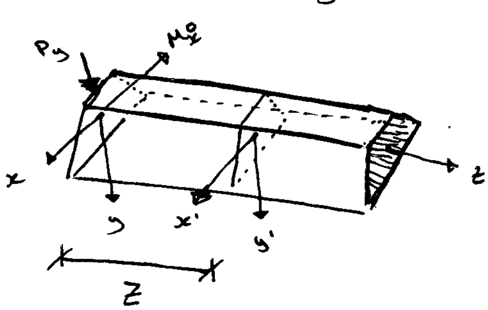
$L/D \geq 5$

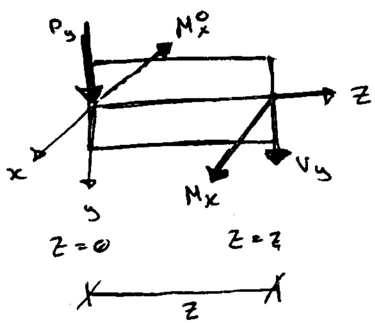
Homogeneous and isotropic (material assumption)



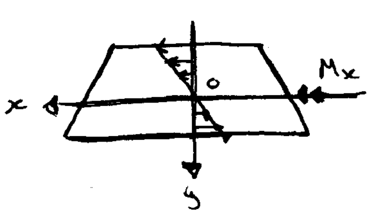
no twisting (only bending deformation)

Beam has a symmetrical plane

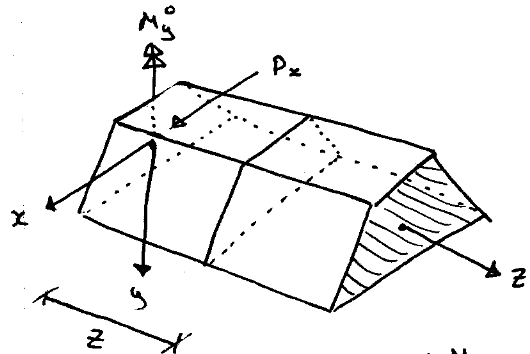


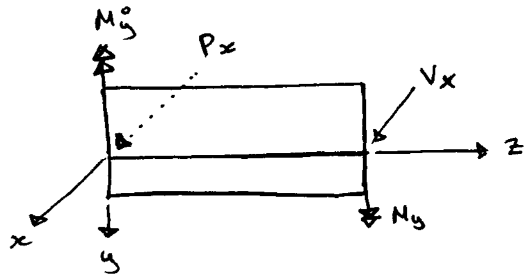


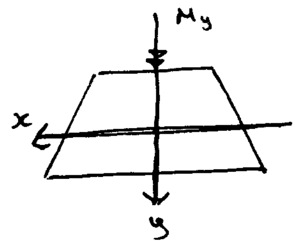
$$\sigma_{zz} = \frac{M_x y}{I_x}$$



$$I_x = \iint_A y^2 \, dx \, dy$$







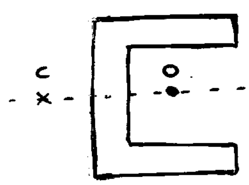
$$\sigma_{zz} = -\frac{M_y x}{I_y}$$

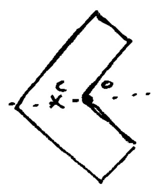
$$\Rightarrow \boxed{\sigma_{zz} = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}}$$

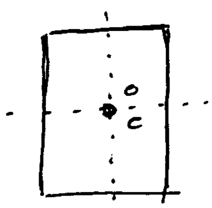
(no twisting)

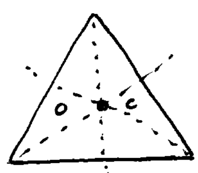
Forces P_x and P_y are passing through the shear center of the beam.

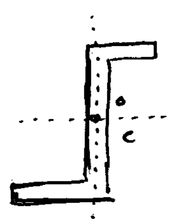
* Shear centre



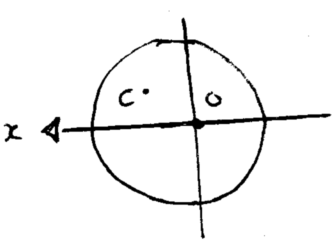




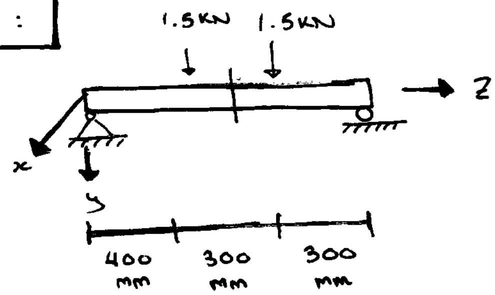


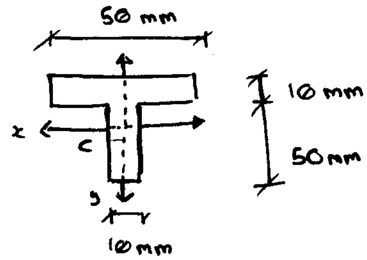


(anti-symmetric)



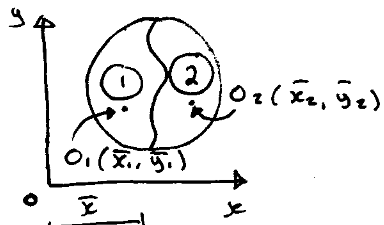
Example :





Find the max tensile and compressive normal stress at the middle span of the beam.

Solution:

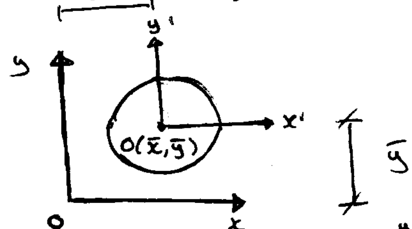


$$A = A_1 + A_2$$

$$\begin{cases} \bar{x} = \dfrac{A_1\bar{x}_1 + A_2\bar{x}_2}{A} \\ \bar{y} = \dfrac{A_1\bar{y}_1 + A_2\bar{y}_2}{A} \end{cases}$$

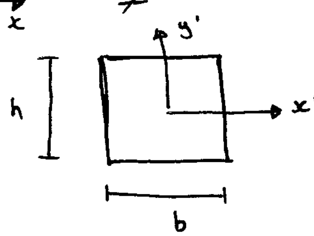
$I_{x'}, I_{y'}, I_{xy'}$

$\Rightarrow I_x \quad I_y \quad I_{xy}$



Parallel axis theorem:

$$\begin{cases} I_x = I_{x'} + A\bar{y}^2 \\ I_y = I_{y'} + A\bar{x}^2 \\ I_{xy} = I_{x'y'} + A\bar{x}\bar{y} \end{cases}$$



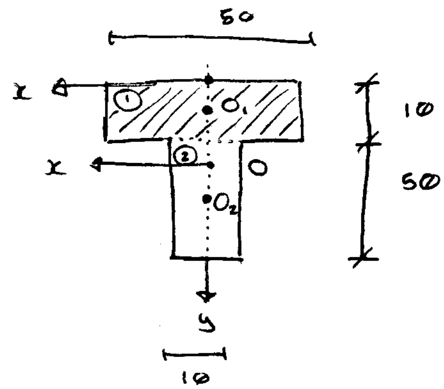
$$I_{x'} = \tfrac{1}{12} bh^3$$
$$I_{y'} = \tfrac{1}{12} hb^3$$
$$I_{x'y'} = 0$$

$O_1(\bar{x}_1, \bar{y}_1)$

$O_2(\bar{x}_2, \bar{y}_2)$

$\bar{y}_1 = (10/2) = 5$

$\bar{y}_2 = \left(\frac{50}{2}\right) + 10 = 35.$



$A_1 = (50)\times(10) = 500$

$A_2 = (50)(10) = 500$

$$\therefore \bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{500(5) + (500)(35)}{(500+500)} = 20$$

$I_x = I_x^{①} + I_x^{②}$

$I_x^{①} = I_{x_1}^{①} + Ad^2$

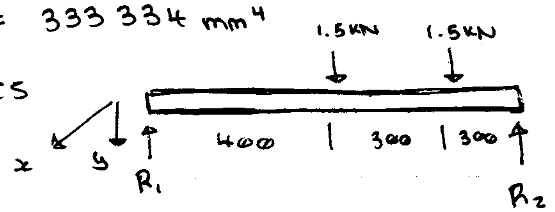
$= (1/12)(50)(10)^3 + (500)(20-5)^2$

$I_x^{②} = I_{x_2}^{②} + Ad^2$

$= (1/12)(10)(50)^3 + (500)(20-35)^2$

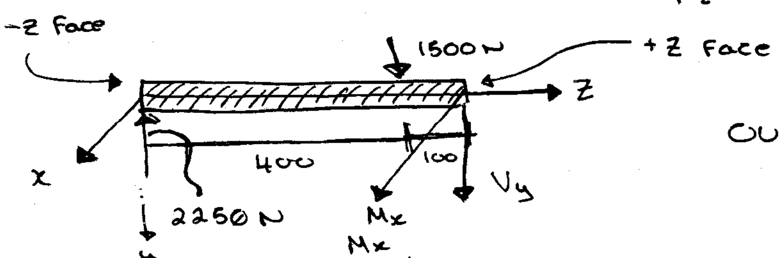
$I_x = 333\,334 \text{ mm}^4$

Statics

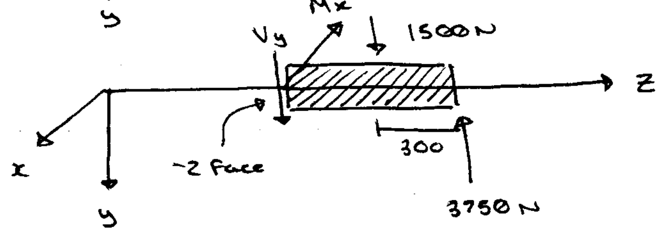


→ $R_1 = 2250$ N

$R_2 = 3750$ N



Outer normal



$\Sigma M_{x_1} = 0$: $M_x - (2250)(500) + (1500)(100)$

$M_x = 975000$ N-mm

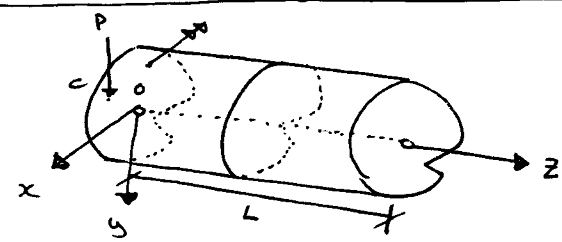
Normal Stress

$$\boxed{\sigma_{zz} = \frac{M_x}{I_x} y}$$

At the top, $y = -20$

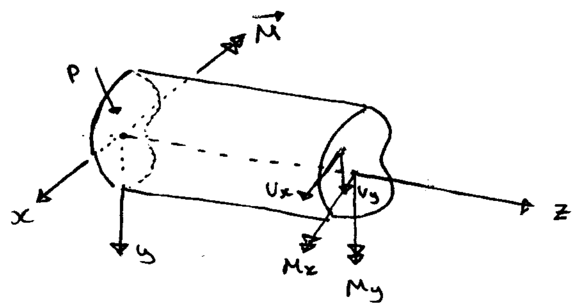
$$\sigma_{zz} = \frac{975000}{333334} \times (-20) = -58.8 \text{ MPa}$$

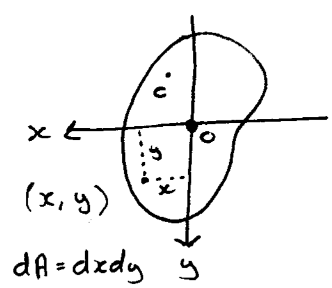
7.2 Bending Stress in Beams Subjected to Non-symmetric bending



P: through shear center C
(no twisting)
(Plane cross-section remain Plane)

Method of Section





$dF_z = \sigma_{zz}\, dx\, dy$

Resutant

$\iint_A \sigma_{zz}\, dx\, dy = 0$ (no axial force)

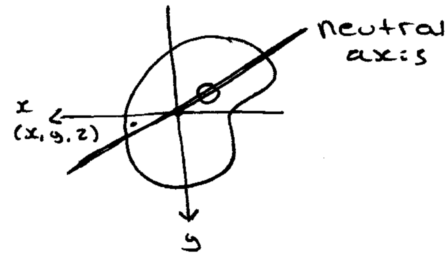
$\rightarrow \iint_A y\, \sigma_{zz}\, dx\, dy = M_x$

$\iint_A x\, \sigma_{zz}\, dx\, dy = -M_y$

Cross-section has a rigid body rotation

The disp. at point (x, y, z)

$$\begin{cases} u = 0 \\ v = 0 \\ w = a''_{(z)} + x\, b''_{(z)} + y\, c''_{(z)} \end{cases}$$



Strains : $\epsilon_{xx} = 0\,, \quad \epsilon_{yy} = 0$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} = a'(z) + x\, b'(z) + y\, c'(z)$$

Stress : $\sigma_{zz} = E\,\epsilon_{zz} = a(z) + x\, b(z) + y\, c(z)$

$$\iint_A \sigma_{zz}\, dx dy = \iint_A \left(a(z) + b(z)\, x + c(z)\, y\right) dx\, dy$$

$$= a(z) \iint_A dx dy + b(z) \iint_A x dx dy + c(z) \int y dx dy = 0$$

$\Rightarrow \quad a(z) \cdot A = 0 \quad \Rightarrow \quad a(z) = 0$

$\Rightarrow \quad \sigma_{zz} = b(z)\, x + c(z)\, y$

$$\Rightarrow \quad \iint_A y\, \sigma_{zz}\, dx dy = \iint_A \left(b(z) xy + c(z) y^2\right) dx dy$$

$$= b(z) \iint_A xy dx dy + (c(z) \iint_A y^2 dx dy \quad \dots$$

$$= b(z) \iint_A x^2 dx dy + (c(z) \iint_A xy\, dx dy = -M_x$$

Define $I_x = \iint_A y^2\, dx dy$

$I_y = \iint_A x^2\, dx dy$

$I_{xy} = \iint_A xy\, dx dy$

$$\rightarrow \begin{cases} b(z)\, I_{xy} + c(z)\, I_x = M_y \\ b(z)\, I_y + c(z)\, I_{xy} = -M_x \end{cases}$$

$$\rightarrow \quad b(z) = -\frac{M_y I_x + M_x I_{xy}}{\Delta} \qquad \{ \; \Delta = I_x I_y - I_{xy}^2$$

$$\sigma_{zz} = \frac{M_x I_y + M_y I_{xy}}{\Delta} y - \frac{M_y I_x + M_x I_{xy}}{\Delta} x$$

If $I_{xy} = 0$, then $\Delta = I_x I_y$

$$\Rightarrow \boxed{\sigma_{zz} = \frac{M_x}{I_x} y - \frac{M_y}{I_y} x}$$