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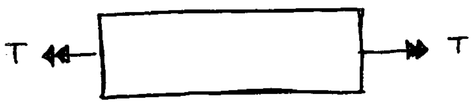
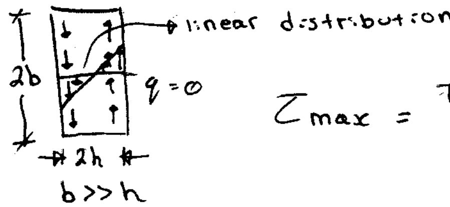
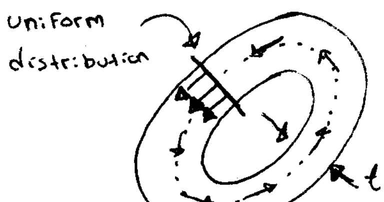




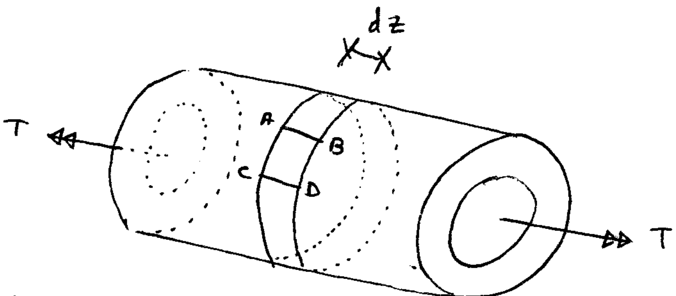
$$\tau_{max} = T/J\,(2H)_{max}$$

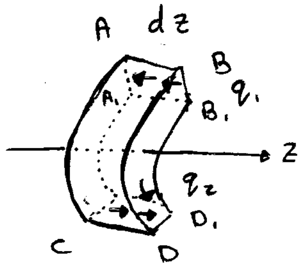


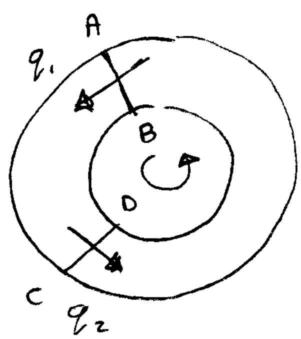
thin-wall cylinder
(closed)

$q = \tau t$ (shear flow)

q = const. (along the cross section)







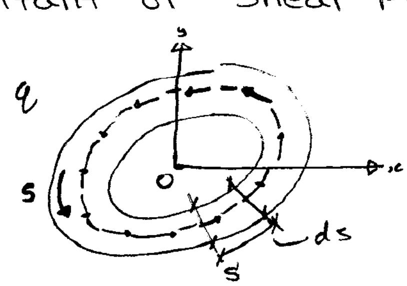
$\Sigma F_z = 0$:

$$-q_1 dz + q_2 dz \Rightarrow q_1 = q_2$$

Resultant of Shear Flow over the cross-section

Internal Force :

T

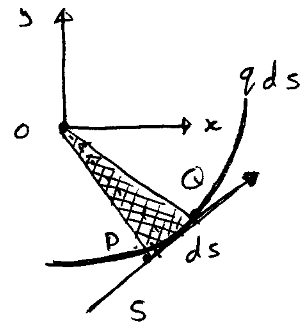


Internal stress

τ

$$\Sigma F_x = 0 , \quad \Sigma F_y = 0$$

$$\Sigma M_O = T$$

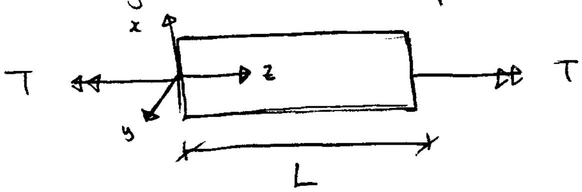


$$d\,q\,ds = q \cdot d \cdot ds$$
$$= q \cdot 2 S_{\Delta OPQ}$$
$$\Sigma M_o = q \cdot 2A = T$$

A: the area enclosed by the mean perimeter of the cross-section

$$\boxed{q = \tau t = \frac{T}{2A}}$$

Angle of twist per unit length θ



rotation between end sections

$$\beta = L\theta$$

work done is: $(\frac{1}{2}) T\beta = (1/2) TL\theta$

Stress components : $\sigma_{zx} \neq 0$, $\sigma_{zy} \neq 0$

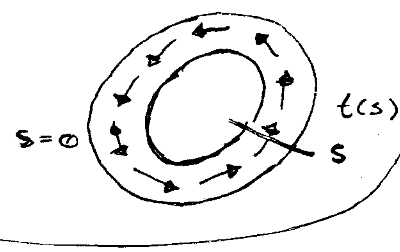
Strain energy density :

$$U_o = \frac{1}{\nu E}(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2) - \nu/E(\sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz}) \cdots$$
$$\cdots + \frac{1}{2G}(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2)$$
$$\Rightarrow \frac{1}{2G}(\sigma_{xz}^2 + \sigma_{yz}^2)$$
$$\Rightarrow \frac{1}{2G}\tau^2$$

Strain energy of the torsional member

$$U = \iiint_V U_o \, dV = \iint_A L U_o \, dA$$

$$= L \int \int U_o \, dt$$
$$= L \oint \left(\int_0^{t(s)} U_o \, dt \right) ds$$
$$= L \oint \left(\int_0^t \frac{\tau^2}{2G} dt \right) ds$$



$$= L \oint \frac{\tau^2}{2G} t \, ds$$
$$= L \oint \frac{\tau q}{2G} ds$$
$$= \frac{Lq}{2G} \oint \tau \, ds$$

$$\frac{1}{2} T \cancel{L'} \theta = \frac{\cancel{L'} q}{2G} \oint \tau \, ds$$

Since $q = \frac{T}{2A}$ $\Rightarrow$ $\theta = \frac{q}{GT} \oint \tau \, ds$

$$\boxed{\theta = \frac{1}{2GA} \oint \tau \, ds}$$ (Angle of twist)

Since $\tau = \frac{q}{t}$

$$\Rightarrow \theta = \frac{1}{2GA} \oint q/t \, ds = \frac{q}{2GA} \oint 1/t \, ds$$

$$\theta = \frac{T}{4GA} \oint \frac{1}{t} \, ds$$

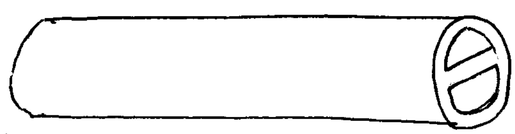
Define $J = \frac{4A^2}{\oint 1/t \, ds}$

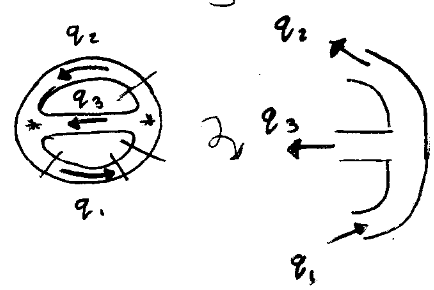
$$\Rightarrow \theta = \frac{T}{GJ}$$

Special Case: t = const.

$\oint 1/t \, ds = s/t$ $\quad$ s: the wall mean perimeter length

6.7.1 Hollow thin-wall member having several compartments

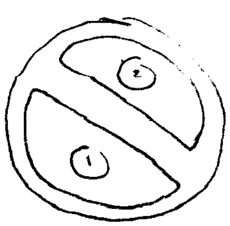




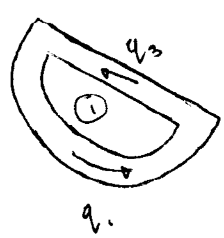
$q_1 = q_2 + q_3$

$$\boxed{\therefore q_3 = q_1 - q_2}$$

$$\theta = \frac{1}{2GA} \oint \tau \, ds$$

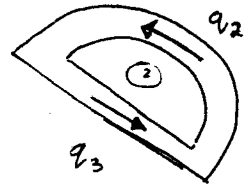


Compartment 1



Compartment 2 :

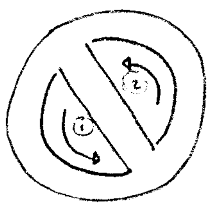
$$\theta = \frac{1}{2GA} \oint \tau \, ds$$

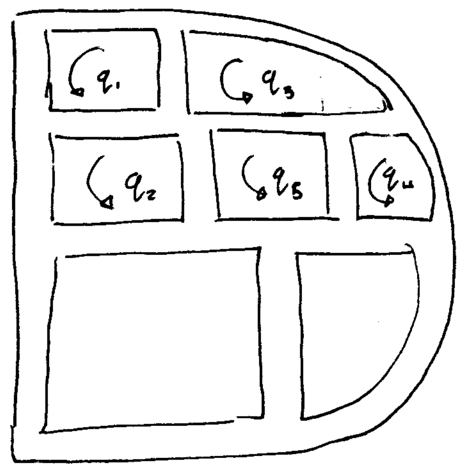


θ must be the same for either compartment.

Internal resultant:

$$T = 2A_1 q_1 + 2A_2 q_2$$





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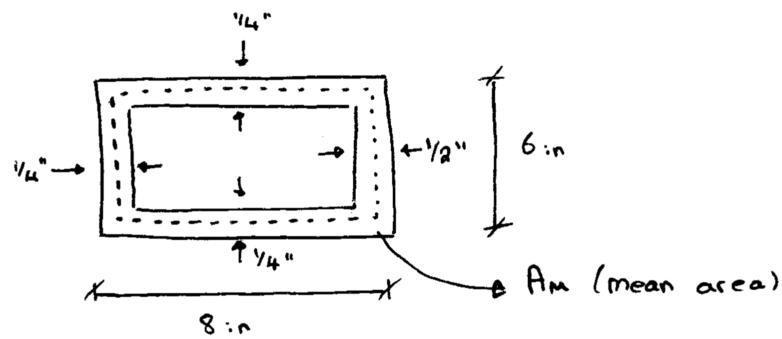
Example:



$T = 50$ kip-in

Find the max shear stress developed in the cross-section

Also find the effective polar moment of inertia.

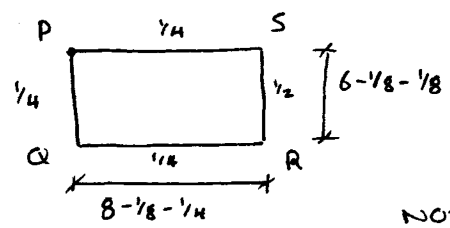
Solution: $q = \dfrac{T}{2A}$ (mean area)

$A = (8 - 1/8 - 1/4) \times (6 - 1/8 - 1/8) = 43 \text{ in}^2$

$\therefore \; q = \dfrac{50 \text{ kip-in}}{(2)(43 \text{ in}^2)} \rightarrow q = \tau \cdot t \quad \therefore \; \tau_{max} = \dfrac{q}{t_{min}}$

$\therefore \; \tau_{max} = \dfrac{50}{(2)(43)}\left(\dfrac{1}{1/4}\right) = 2.28 \text{ ksi}$

$J = \dfrac{4A^2}{\oint \frac{1}{t}\, ds}$



$\oint 1/t \, ds = \int_{SPQR} 1/t \, ds + \int_{RS} 1/t \, ds$

NOTE:

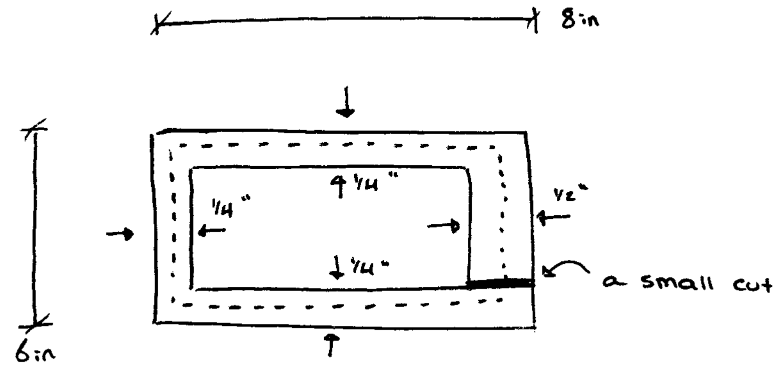
$\int_{SP} 1/t \, ds = 1/t \int_{SP} ds = SP/t$

$= 1/(1/4)\,(SP + PQ + QR) + 1/(1/2)\,(RS)$

t

$\Rightarrow 4((8 - 1/8 - 1/4) + (6 - 1/8 - 1/8) + (8 - 1/8 - 1/4) + 2(6 - 1/8 - 1/8) = 95.5$

and $J = \dfrac{4A^2}{\oint \frac{1}{t}\, ds} = \dfrac{4 \times 43.8^2}{95.5} = 80.354 \text{ in}^2$



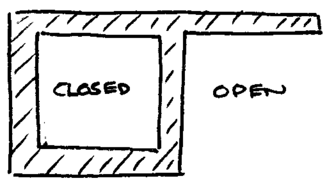
For the open section, find J

$$J = \Sigma \tfrac{1}{3}(2b)(2h)^3$$

$$= (\tfrac{1}{3})\left[(8-\tfrac{1}{8}-\tfrac{1}{4}) + (6-\tfrac{1}{8}-\tfrac{1}{8}) + (8-\tfrac{1}{8}-\tfrac{1}{4})\right] \times (\tfrac{1}{4})^3 \cdots$$

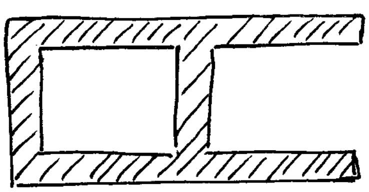
$$\cdots + (\tfrac{1}{3})(6-\tfrac{1}{8}-\tfrac{1}{8})(\tfrac{1}{2})^3$$

$$J = 0.34896 \text{ in}^4$$



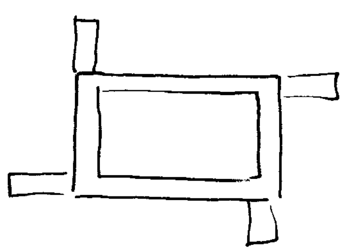
$$J = J_{close} + J_{open}$$

$$= \frac{4A^2}{\oint \frac{1}{t} ds} + \tfrac{1}{3}(2b)(2h)^3$$



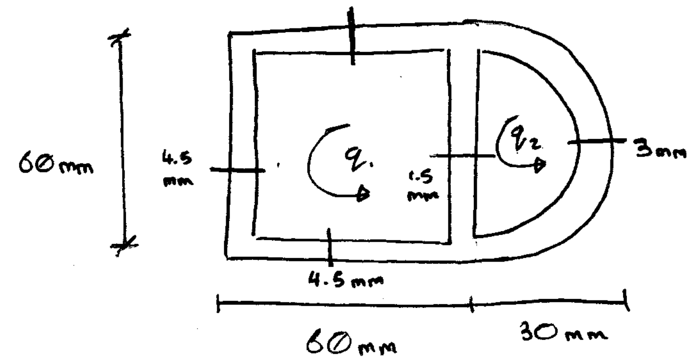
$$J = J_{close} + J_{open_1} + J_{open_2}$$

$$= \frac{4A^2}{\oint \frac{1}{t} ds} + \tfrac{1}{3}(2b)(2h)^3 + \tfrac{1}{3}(2b)(2h)^3$$



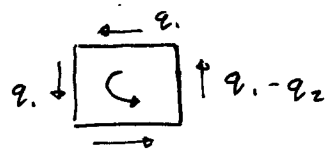
Same thing.

Example :



Given $G = 26.0$ GPa the max shear stress is 40 MPa, Find the max T the member can support.

Cell 1 :

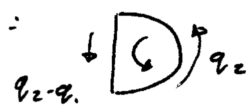


$$\theta_1 = \frac{1}{2GA_1} \oint \tau \, ds = \frac{1}{2GA_1} \oint \frac{q}{t} \, ds$$

Since $A_1 = 60^2 = 3600$

$$\theta = \frac{1}{2G(3600)}\left[\left(\frac{q_1}{4.5} \times 3 \times 60\right) + \left(\frac{q_1 - q_2}{1.5} \times 60\right)\right]$$

Cell 2 :



$$A_2 = \tfrac{1}{2}\pi r^2 = (\tfrac{1}{2})\pi(30)^2$$

$$= 450\pi$$

$$\theta_2 = \frac{1}{2GA_2} \oint \frac{q}{t} \, ds$$

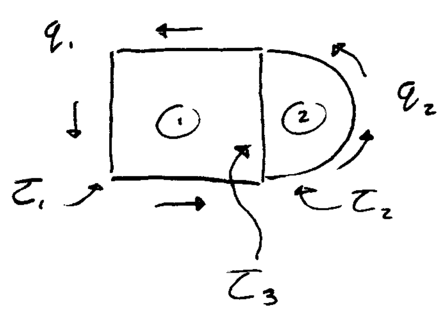
$$= \frac{1}{2G(450\pi)}\left[\frac{q_2}{3}\pi(30) + \frac{q_2 - q_1}{1.5}(60)\right]$$

Since its a rigid body, $\theta_1 = \theta_2$

$$\frac{1}{2G(3600)}\left[\frac{q_1}{4.5} \times 180 + \frac{q_1 - q_2}{1.5} \times 60\right]$$

$$\Rightarrow \frac{1}{2G(450\pi)}\left[\frac{q_2}{3} \times 30\pi + \frac{q_1 - q_2}{1.5}(60)\right]$$

$$\Rightarrow q_1 = 1.220\, q_2$$



$$\tau_1 = \frac{q_1}{4.5} \qquad \tau_2 = \frac{q_2}{3.0}$$

$$\tau_3 = \frac{q_1 - q_2}{1.5}$$

$$\tau_{max} = \max(\tau_1, \tau_2, \tau_3) = 40$$

$$T = 2A_1 q_1 + 2A_2 q_2$$