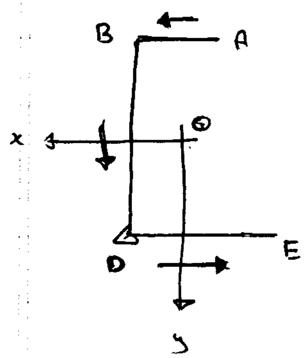
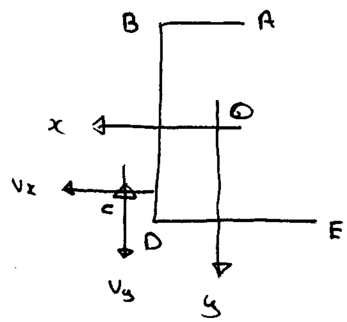
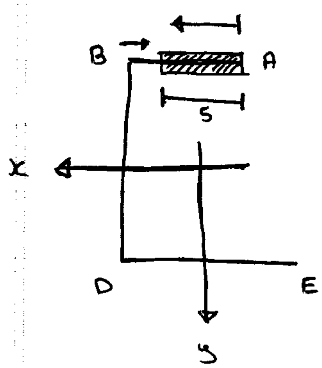
Nov. 20/18





Find shear flow on $\underline{\underline{AB}}$

D is the moment center



Find shear flow on $\underline{\underline{AB}}$

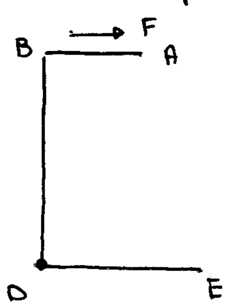
The resultant on AB

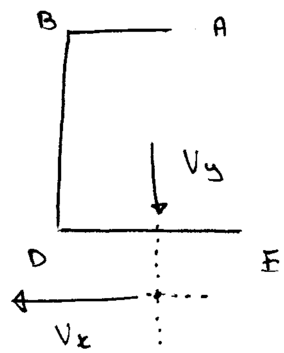
$$F = \int_A^B q(s)\,ds$$

$$= C_1 V_x + C_2 V_y$$

C_1 and C_2 are constants

If F is positive,





Method 2 :

Step 1 : Consider shear force V_x only

↳ identify the line of action of $\underline{\underline{V_x}}$

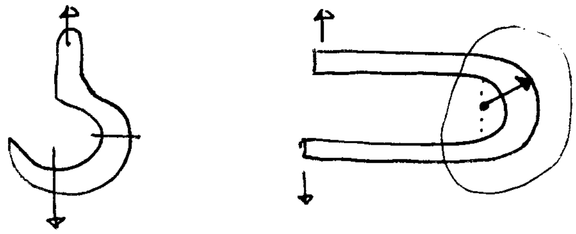
Step 2 : Consider V_y only

↳ identify the line of action of $\underline{\underline{V_y}}$

Chapter 9 - Curved Beams

9.1 - Introduction

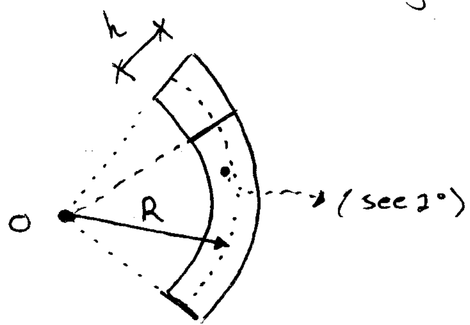
Crane hook:



9.2 Circumfrential stresses in a curved beam

Geometry: (1) The cross-section has a symmetric axis, and the beam has a symmetric plane

(2) The area of cross-section is constant through the axis of the beam.

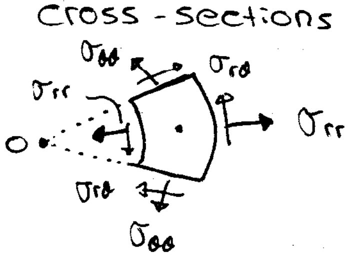


$\frac{R}{h} > 5 \Rightarrow$ straight beam theory

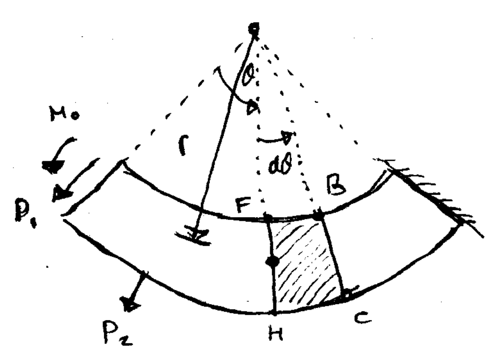
Deformation:

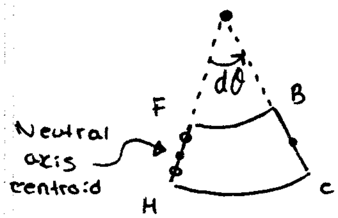
1° Plane cross-sections remain plane after loading

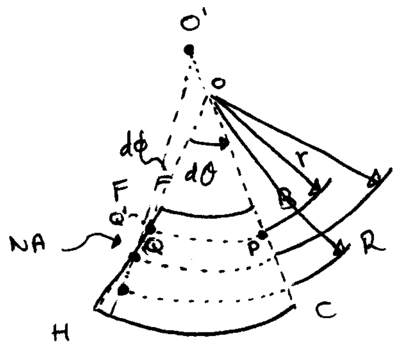
2°

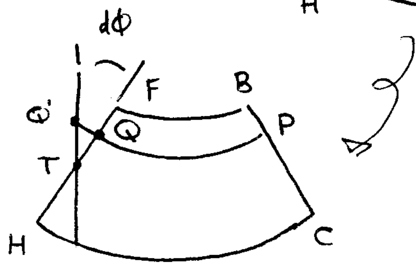


σ_{yy} and $\sigma_{r\theta}$ are sufficiently small









Due to the rotation

$PQ \to PQ'$

$PQ = r\,d\theta$

$QQ' = TQ \cdot d\phi$

$TQ = R_n - r$

$QQ' = (R_n - r)\,d\phi$

The normal strain of the line segment PQ

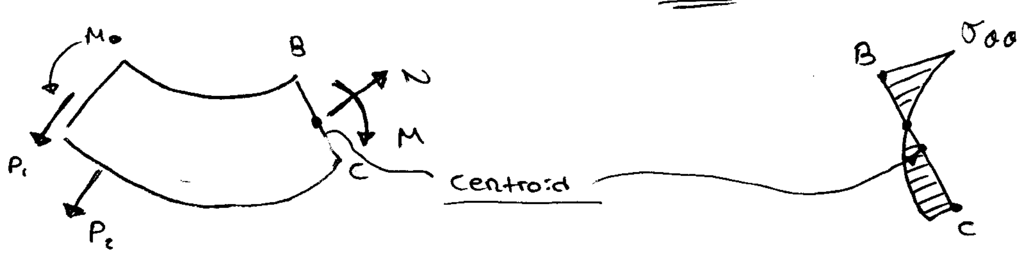
$$\epsilon_{\theta\theta} = \frac{QQ'}{PQ} = \frac{(R_n - r)\,d\phi}{r\,d\theta}$$

Define

$\omega = d\phi / d\theta \quad \Rightarrow \quad \epsilon_{\theta\theta} = \frac{R_n - r}{r}\,\omega$

Stress : $\sigma_{\theta\theta} = E\,\epsilon_{\theta\theta} = E\omega\,\frac{R_n - r}{r}$

Statics : Cross-section BC



$$\begin{cases} N = \iint_A \sigma_{\theta\theta}\,dA \\ M = \iint_A \sigma_{\theta\theta}(R - r)\,dA \end{cases}$$

Sub $\sigma_{\theta\theta}$ into the above eqns

$$\begin{cases} \iint_A E\omega\,\frac{R_n - r}{r}\,dA = N \\ \iint_A E\omega\,\frac{R_n - r}{r}(R - r)\,dA = M \end{cases} \Rightarrow E\omega = \frac{A_m}{A(RA_m - A)}\cdot M - \frac{N}{A}$$

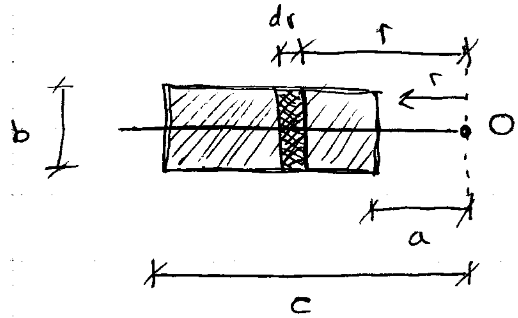
$$\left\{\begin{array}{l} \boxed{E\omega = \dfrac{A_m}{A(RA_m - A)} \cdot M - \dfrac{N}{A}} \\[2ex] \boxed{R_n = \dfrac{MA}{MA_m + N(A - RA_m)}} \end{array}\right.$$

Here, $A_m = \iint_A \frac{1}{r}\, dA$

$A = \iint_A dA$ area of cross section

$$\Rightarrow \quad \boxed{\sigma_{\theta\theta} = \frac{N}{A} + \frac{M}{A(RA_m - A)}\left(\frac{A}{r} - A_m\right)}$$

Calculate A_m:



$$A_m = \iint_A \frac{1}{r}\, dA$$

$$= \int_a^b \frac{1}{r}\, b \cdot dr = b\ln(c/a)$$

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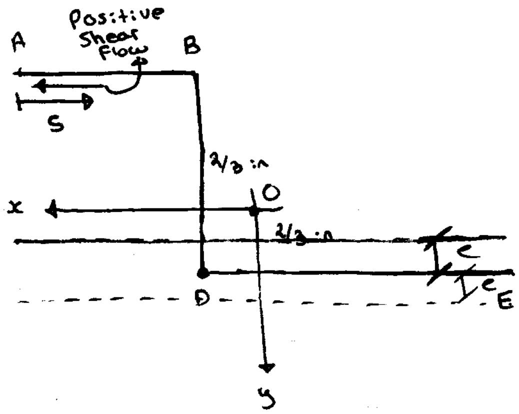


D : moment center

Internal Shear Forces are positive

Step 1: positive shear force V_x

if Sheer Flow is positive, the line of action of V_x is above D

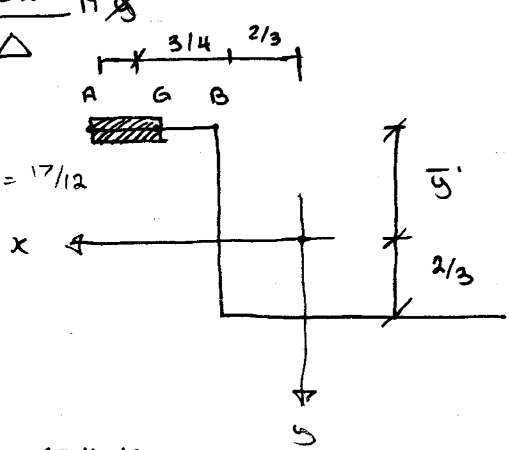
$$e = \frac{\frac{AB}{6}(q_A + 4q_G + q_B) \times BD}{V_x}$$

should be $\bar{x}'$

$$q = \frac{V_y I_y - V_x I_{xy}}{\Delta} A'\bar{y}' + \frac{V_x I_x - V_y I_{xy}}{\Delta} A'\cancel{\bar{y}}'$$

$(V_y = 0)$

$$q = -\frac{V_x I_{xy}}{\Delta} A'\bar{y}' + \frac{V_x I_x}{\Delta} A'\cancel{\bar{y}}' \rightarrow \bar{x}'$$



At G, $A' = 0.5t$

$\bar{x}' = 3/4 + 2/3 = 17/12$

$\bar{y}' = -4/3$

$q_G = 0.02944\,V_x$

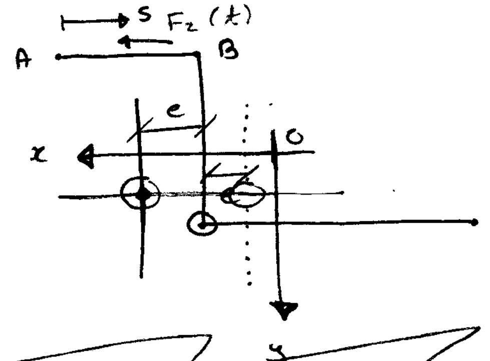
At B, $q_B = -0.01674\,V_x$

the resultant $F = \frac{AB}{6}(q_A + 4q_G + q_B) = 0.01684\,V_x$

red line, distance

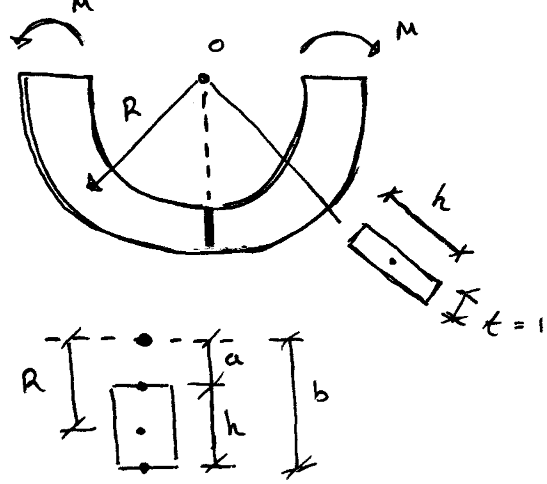
$$e = \frac{F \times BD}{V_x} = 0.03368 \text{ in}$$

Step 2 : Positive internal shear force V_y



The resultant shear flow on AB F_z,

If F_z is positive,



1° curved beam theory

$$\sigma_{\theta\theta} = \frac{M}{A(RA_m - A)}\left(\frac{A}{r} - A_m\right)$$

M : internal bending moment

A : area of cross-section $[A = th]$

R : centroidal radius

$$A_m = \iint_A \frac{1}{r}\, dA$$

$$\rightarrow = t \ln\left(\frac{R+h/2}{R-h/2}\right)$$

Case : $\frac{R}{h} = 1$

Inner radius : $a = R - \frac{h}{2} = \frac{h}{2}$

Outer radius : $b = R + \frac{h}{2} = \frac{3h}{2}$

$\Rightarrow b = 3a$

$$\sigma_{\theta\theta} = \frac{M}{th(h\,t\ln(3) - th)}\left(\frac{th}{r} - t\ln(3)\right)$$

$$\sigma_{\theta\theta} = \frac{M}{th^2(\ln 3 - 1)}\left(\frac{h}{r} - \ln(3)\right)$$

At inner surface, $r = a = h/2$

2

$$\sigma_{\theta\theta,max} = \frac{M}{th^2} \cdot \frac{1}{\ln(3)-1}(2-\ln 3)$$

$$= 2.28518 \frac{M}{th^2} \Leftarrow \text{approximate solution}$$

<u>Elasticity</u> :

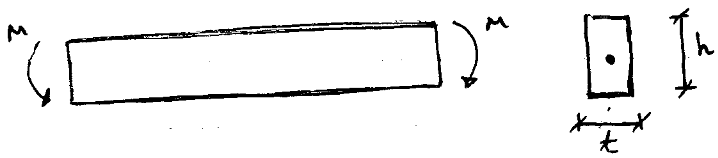
$$\sigma_{\theta\theta} = \frac{4M}{Q}\left[\frac{-a^2b^2}{r^2}\ln\left(\frac{b}{a}\right) + b^2\ln\left(\frac{r}{b}\right) + a^2\ln\left(\frac{a}{r}\right) + b^2 - a^2\right]$$

$$Q = 4a^2b^2\left(\ln\left(\frac{b}{a}\right)\right)^2 - (b^2-a^2)$$

at $r = a$

$$\sigma_{\theta\theta} = 2.29199 \frac{M}{th^2} \Leftarrow \text{exact solution}$$

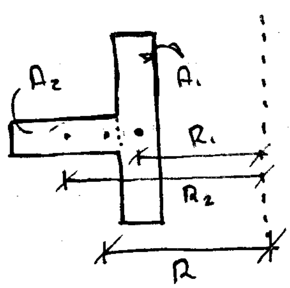
<u>Straight Beam Theory</u> :



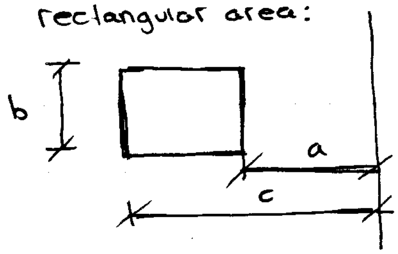
$$\sigma_{max} = \frac{M}{I} \cdot \frac{h}{2} = \frac{M}{\frac{1}{12}th^3} \cdot \frac{h}{2}$$

$$= 6\frac{M}{th^2}$$

composite area :



rectangular area :



$$A_M = \iint \frac{1}{r}\, dA$$

$$= A_m = \iint_{A_1} \frac{1}{r}\, dA + \iint_{A_2} \frac{1}{r}\, dA$$

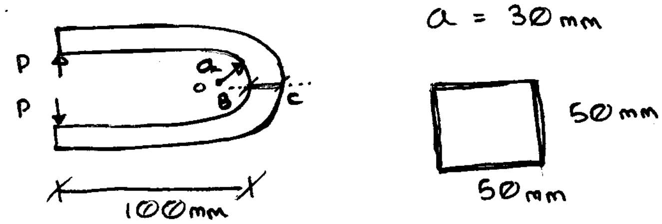
$$\rightarrow A_m = A_{m_1} + A_{m_2}$$

$$R = \frac{A_1R_1 + A_2R_2}{A_1 + A_2}$$

$$A_m = b\ln\left(\frac{c}{a}\right)$$

Example :

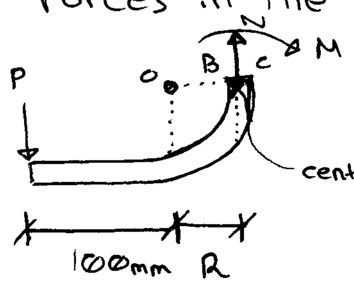
$a = 30\,mm$



Determine the value of the max tensile and max compressive stresses in the frame.

Solution : BC

Internal Forces in the cross-section BC



(M - Flattens part)

✱ (N - positive direction, tensile)

centroid of the cross - section

$\hookrightarrow R = 30 + (50/2) = 55\,mm$

$\Rightarrow N = P = 9.50\,kN = 9500\,N$

$M = P(100 + 55) = 1472500\,N\cdot mm$

Geometry

$R = 55\,mm$

$A = 50 \times 50 = 2500\,mm^2$

$A_m = b\ln(c/a) = 50\ln\left(\frac{80}{30}\right)$

$\Rightarrow A_m = 49.0415\,mm$ (use all decimals)

Normal stress

$$\sigma_{\theta\theta} = \frac{N}{A} + \frac{M}{A(RA_m - A)}\left(\frac{A}{r} - A_m\right)$$

$$= \frac{9500}{2500} + \frac{1472500}{(2500)((55 \times 49.0415) - 2500))} \times \left(\frac{2500}{r} - 49.0415\right)$$

r : mm

$\sigma_{\theta\theta}$: MPa

At $r = a = 30$ mm

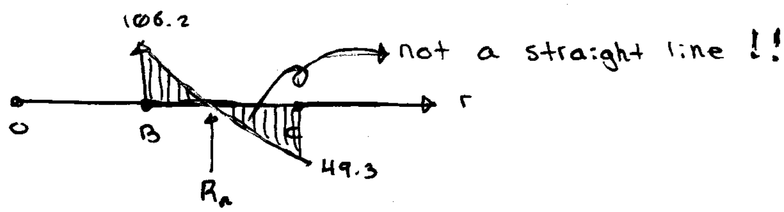
$$\sigma_{\theta\theta} = 3.8 + 2.9856\left(\frac{2500}{30} - 49.0415\right)$$
$$= 106.2 \text{ MPa}$$

At $r = 30 + 50 = 80$ mm

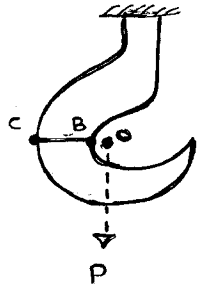
$$\sigma_{\theta\theta} = 3.8 + 2.9856\left(\frac{2500}{80} - 49.0415\right)$$
$$= -49.3 \text{ MPa}$$

1° neutral axis ? $R_n = 52.3355$ mm

2° distribution of $\sigma_{\theta\theta}$ in the radial direction

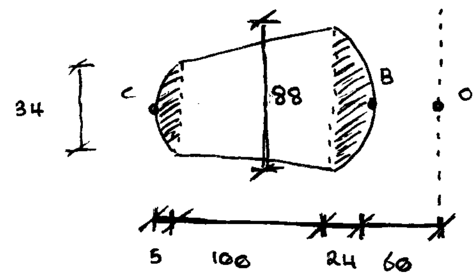


Example :



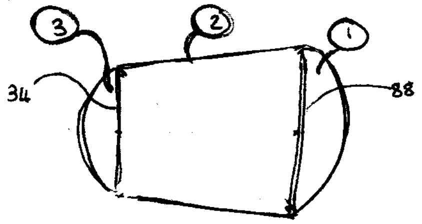
$Y = 500$ MPa

$SF = 2.00$



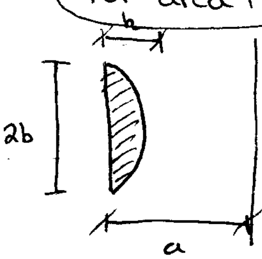
Find the max load the crane hook can support

Solution : A, R, Am (Pg 324 - 325)



elliptical

For area 1 :



$a = 60 + 24 = 84$

$2b = 88 \rightsquigarrow b = 44$

$h = 24$

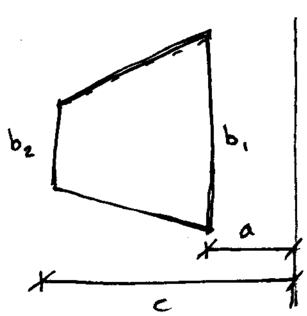
$$A_1 = \frac{\pi b h}{2} = \frac{\pi(44)(24)}{2}$$

$A_1 = 1658.76$

$$R_1 = a - \frac{4h}{3\pi} = 84 - \frac{(4)(24)}{(3\pi)} = 73.81$$

$$A_m' = 2b + \frac{\pi b}{h}\left(a - \sqrt{a^2-h^2}\right) - \frac{2b}{h}\sqrt{a^2-h^2}\,\arcsin\left(\frac{h}{a}\right) = 22.64$$

For area 2 :



$a = 24 + 60 = 84$

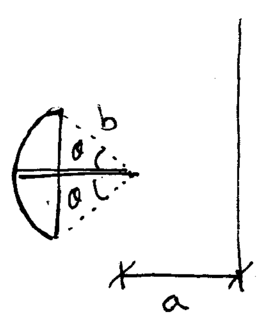
$c = 100 + a = 184$

$b_1 = 88$, $b_2 = 34$

$A_2 = 6100$

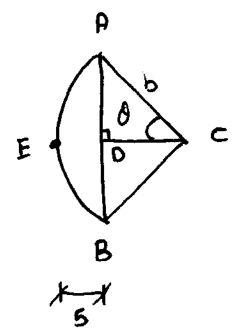
$R_2 = 196.62$

$A_{m_2} = 50.57$



Area 3

circular arc



$AC = b$

$CD = b - 5$

$AD = \frac{1}{2}(34) = 17$

$\triangle ACD \; : \; AC^2 = AD^2 + CD^2$

$b^2 = 17^2 + (b-5)^2$

$\rightarrow b = 31.4$

$\sin\theta = \frac{AD}{AC} = \frac{17}{31.4} \Rightarrow \theta = 32.78°$

and $\quad a = 100 + 24 + 60 - (31.4 - 5)$

$= 157.6 > b = 31.4$

$A_3 = 115.27$

$R_3 = 186.01$

$A_{m_3} = 0.62$

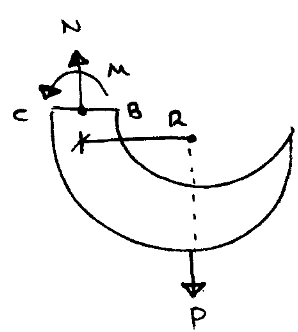
For the cross section:

$A = A_1 + A_2 + A_3 = 7874.03 \text{ mm}^2$

$A_m = A_{m1} + A_{m2} + A_{m3} = 73.83 \text{ mm}$

$R_1 = \dfrac{A_1R_1 + A_2R_2 + A_3R_3}{A} = 116.37 \text{ mm}$

Statics:



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