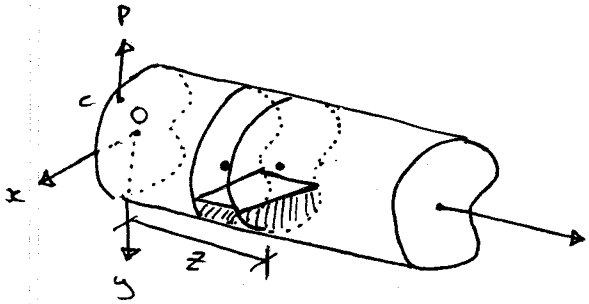
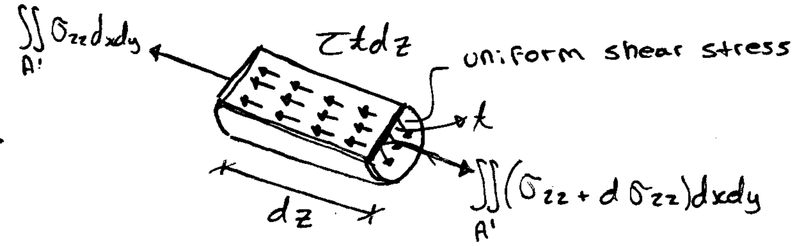
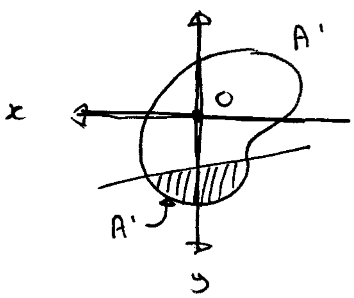


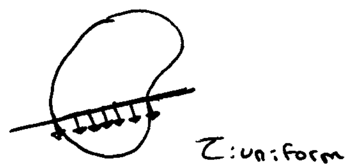




$\Sigma F_z = 0:$

$$\iint_{A'} (\sigma_{zz} + d\sigma_{zz})\,dx\,dy - \cancel{\iint_{A'} \sigma_{zz}\,dx\,dy} - \tau t\,dz = 0$$

$$= q = \tau t = \iint_{A'} \frac{d\sigma_{zz}}{dz}\,dx\,dy$$




Since $\sigma_{zz} = \dfrac{M_x I_y + M_y I_{xy}}{\Delta}\, y - \dfrac{M_y I_x + M_x I_{xy}}{\Delta}\, x$

$$\Rightarrow \frac{d\sigma_{zz}}{dz} = \frac{\frac{dM_x}{dz} I_y + \frac{dM_y}{dz} I_{xy}}{\Delta}\, y - \frac{\frac{dM_y}{dz} I_x + \frac{dM_x}{dz} I_{xy}}{\Delta}\, x$$

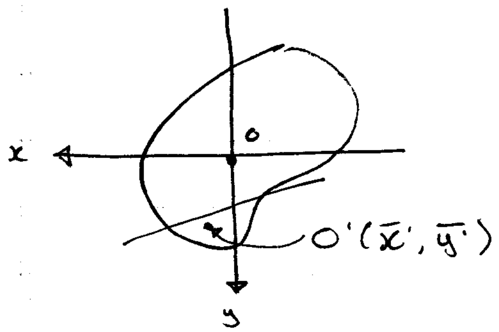
because of: $\dfrac{dM_x}{dz} = V_y \qquad \dfrac{dM_y}{dz} = -V_x$

$$\Rightarrow \frac{d\sigma_{zz}}{dz} = \frac{V_y I_y - V_x I_{xy}}{\Delta}\, y - \frac{-V_x I_x + V_y I_{xy}}{\Delta}\, x$$

$$= \frac{V_y I_y - V_x I_{xy}}{\Delta}\, y + \frac{V_x I_x + V_y I_{xy}}{\Delta}\, x$$

$$\Rightarrow q = \tau t = \iint_{A'} \frac{d\sigma_{zz}}{dz}\,dx\,dy$$

$$= \frac{V_y I_y - V_x I_{xy}}{\Delta} \iint_{A'} y\,dx\,dy + \frac{V_x I_x - V_y I_{xy}}{\Delta} \iint_{A'} x\,dx\,dy$$




$$\iint_{A'} y\,dx\,dy = A'\bar{y}'$$

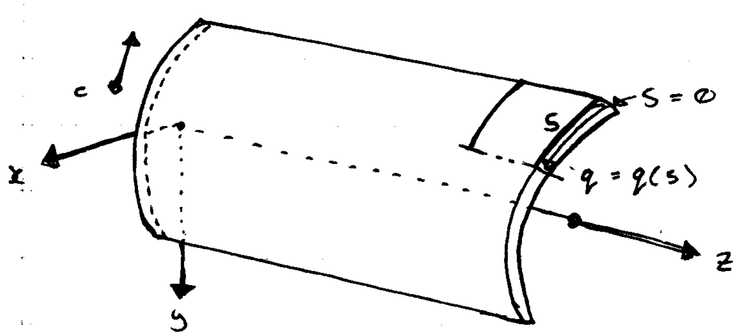
$$\iint_{A'} x\,dx\,dy = A'\bar{x}'$$

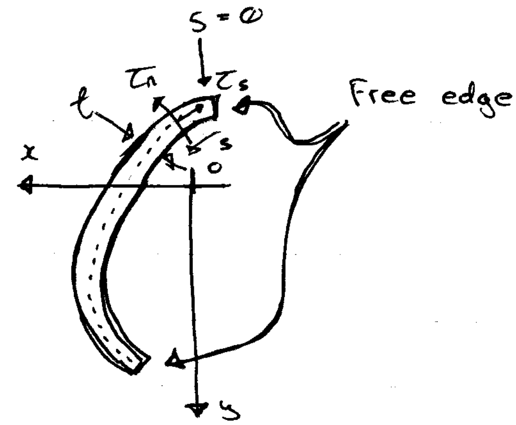
$$\boxed{q = \tau t = \frac{V_y I_y + V_x I_{xy}}{\Delta} A'\bar{y}' + \frac{V_x I_x + V_y I_{xy}}{\Delta} A'\bar{x}'}$$

If $I_{xy} = 0$, $q = (V_y / I_x) A'\bar{y}' + (V_x / I_y) A'\bar{x}'$

$\boxed{\text{If } V_x = 0, \quad q = (V_y / I_x) A'\bar{y}'}$

2. Thin-wall open section



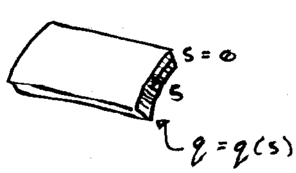




1° $\tau_n = 0$ (thickness very small)

2° $\tau_s$ is uniform through thickness of the wall

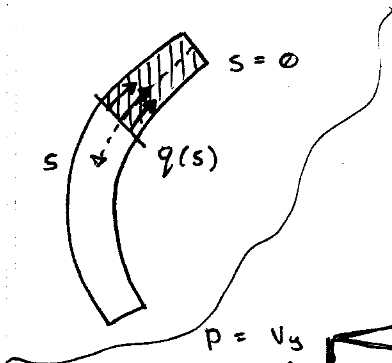
3° $q = \tau t$ : shear flow

4° Shear flow is zero at the free edge

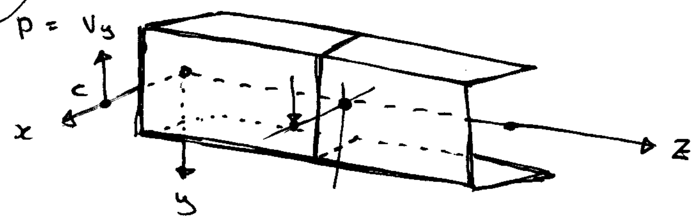


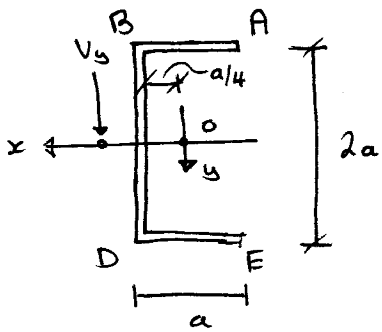

Positive shear flow:

the shear flow points into the area

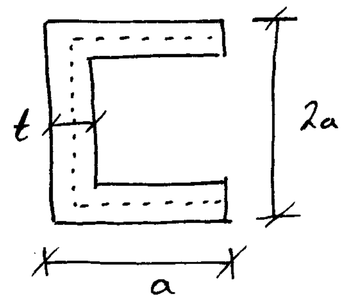



Example: Determine the shear Flow in a C channel section due to a shear Force $V_y$ through it's shear center.




$(t \ll a)$

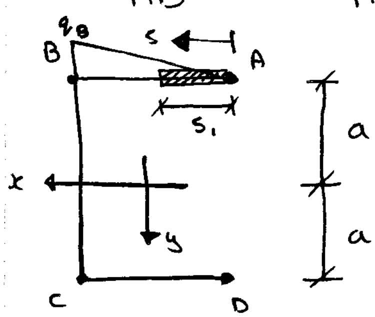



$$q = \tau t = \frac{V_y}{I_x} A' \bar{y}'$$

$$I_x = \left(\tfrac{1}{12}\right) t (2a)^3 + \left[\left(\tfrac{1}{12}\right)(a)(t)^3 + at \cdot a^2\right] \times 2$$

$$= (8/3)\, a^3 t \qquad (a t^3 \ll a^3 t)$$

On AB: $A' = s_1 t$ $\qquad \bar{y}' = -a$




$$q(s_1) = \frac{V_y}{I_x} A' \bar{y}'$$

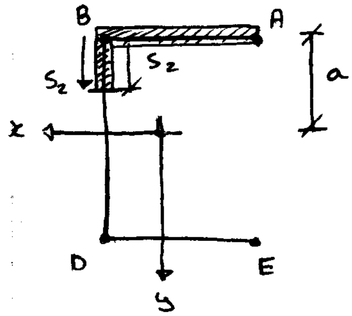
$$= \frac{V_y}{8/3\, a^3 t} (s_1 t)(-a) = -\frac{3 V_y (s_1)}{8a^2}$$

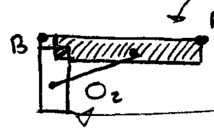
(when $0 \le s_1 \le a$)

at B, $s_1 = a$

$$q_B = -\frac{3V_y}{8a^2}(a) = \frac{-3V_y}{8a}$$

On BD:




$$A'\bar{y}' = A_1\bar{y}_1' + A_2\bar{y}_2'$$

$$q(s_2) = q_B + \frac{V_y}{I_x} A_2 \bar{y}_2'$$

$$\Rightarrow \frac{-3V_y}{8a} - \frac{V_y}{8/3\, a^3 t} \cdot (s_2 t)(a - s_2/2)$$

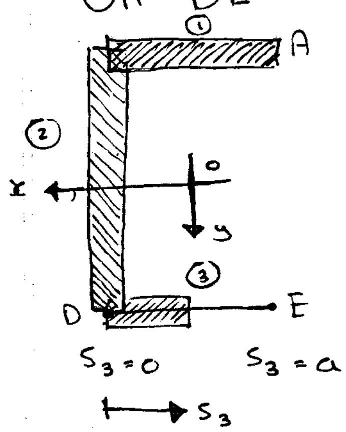
$$= \frac{3V_y}{8a}\left(\frac{1}{2}\left(\frac{s_2}{a}\right)^2 - \frac{s_2}{a} - 1\right)$$

$$\boxed{\begin{aligned} A_2 &= s_2 t \\ \bar{y}_2' &= -(a - s/2) \end{aligned}}$$

At $D_1$, $s_2 = 2a$

$$q_D = q(2a) = -\frac{3V_y}{8a} = q_B$$

On DE




$$q(s_3) = (V_y/I_x)\,A'\bar{y}'$$

$$A'\bar{y}' = A_1'\bar{y}_1' + A_2'\bar{y}_2' + A_3'\bar{y}_3'$$

$$q(s_3) = \frac{V_y}{I_x}\left[A_1'\bar{y}_1' + A_2'\bar{y}_2' + A_3'\bar{y}_3'\right]$$

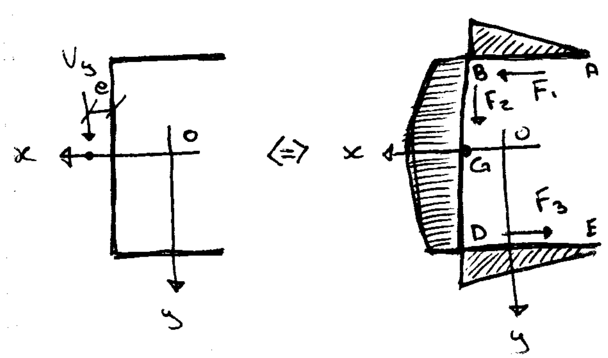
$$= q_D + \frac{V_y}{I_x} A_3'\bar{y}_3'$$

$$q(s_3) = -\frac{3V_y}{8a} + \frac{V_y}{8a^3 t} s_3 t$$

$$= \frac{-3V_y}{8a} + \frac{3V_y}{8a}\cdot\frac{s_3}{a}$$

@E, $s_3 = a$

$$\therefore\ q_E = \frac{-3V_y}{8a} + \frac{3V_y}{8a} = 0$$




Since $q_B = \frac{3V_y}{8a}$

$$\therefore\ F_1 = \frac{1}{2} q_B \cdot AB$$

$$= \frac{1}{2}\left(\frac{3V_y}{8a}\right) a = \frac{3}{16} V_y$$

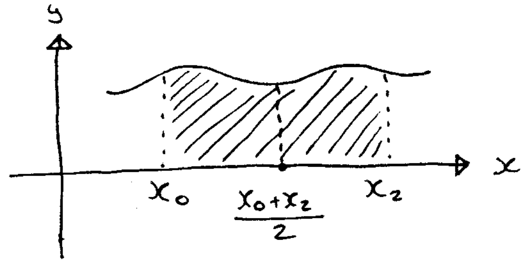
D is the moment center

$$F_1 \cdot 2a = V_y e$$

$$\Rightarrow e = \frac{F_1 \cdot 2a}{V_y} = \frac{3/16\, V_y\, 2a}{V_y} = \frac{3a}{8}$$

negative

$$F_2 = -\int_0^{2a} q(s_2)\, ds_2$$



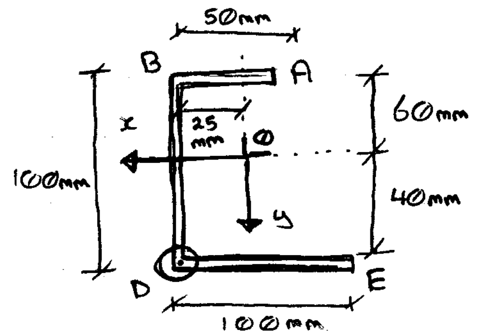

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{x_2 - x_0}{6}\left(f_0 + 4f_1 + f_2\right)$$

$$f_0 = f(x_0),\quad f_1 = f\left(\frac{x_0+x_2}{2}\right)$$

$$f_2 = f(x_2)$$

$$F_2 = \frac{2a}{6}\left(q_B + 4q_G + q_D\right)$$
$$= \frac{a}{3}\left(\frac{-3V_y}{8a} - 4\left(\frac{9V_y}{16a}\right) - \frac{3V_y}{8a}\right)$$
$$F_2 = -V_y$$

Example: Find the shear center of a C-section.




$t = 4mm$

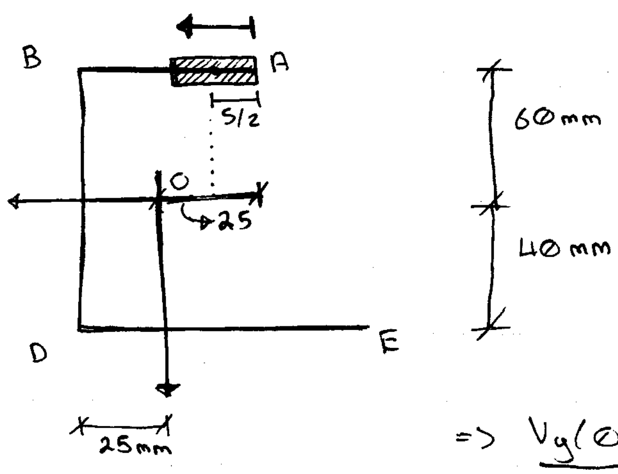
Solution: 1° Find the centroid and moment of area about centroid axes

$I_x = 1.733 \times 10^6 \text{ mm}^4$

$I_y = 0.876 \times 10^6 \text{ mm}^4$

$I_{xy} = -0.500 \times 10^6 \text{ mm}^4$

2° Find internal shear flow due to the internal shear forces $V_x$ and $V_y$




At $0 \le s \le 50$

$A' = st$

Centroid $O_1 (s/2 - 25, -60)$

$$\therefore q(s) = \frac{V_y I_y - V_x I_{xy}}{\Delta} A'\bar{y}' + \ldots$$
$$\ldots + \frac{V_x I_x - V_y I_{xy}}{\Delta} A'\bar{x}'$$

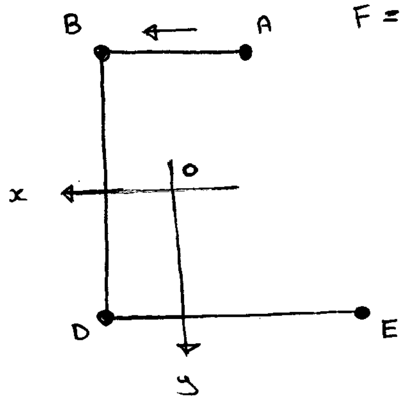
$$\Rightarrow \frac{V_y(0.876)(10^6) - V_x(-0.500)(10^6)}{(1.268 \times 10^{12})} \cdot s(4)(-60) \ldots$$

$$\ldots \frac{V_x(1.733)(10^6) - V_y(-0.500)(10^6)}{(1.268 \times 10^{12})} s(4)(s/2 - 25)$$

$$\Rightarrow V_x\left[2.7335(s-50) - 94.635\right](10^6) + V_y\left[0.7886(s-50) - 165.795\right](10^6)$$

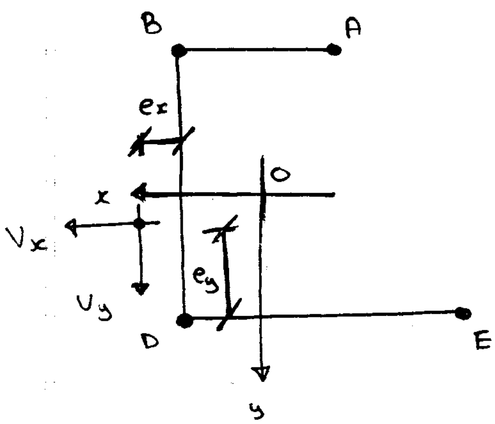
Resultant on AB :

$$F = \int_0^{50} q(s)\,ds$$
$$= -(0.1752\,V_x + 0.2237\,V_y)$$




$$F = 0.1752V_x + 0.2237\,V_y$$

$$\Sigma M_D = (0.1752V_x + 0.2237V_y)(100)$$
$$= 17.52V_x + 22.37V_y$$




Shear Force $V_x$ and $V_y$

$$\Sigma M_D = V_x e_y + V_y e_x$$

$$\Rightarrow \begin{cases} e_y = 17.52 \text{ mm} \\ e_x = 22.37 \text{ mm} \end{cases}$$

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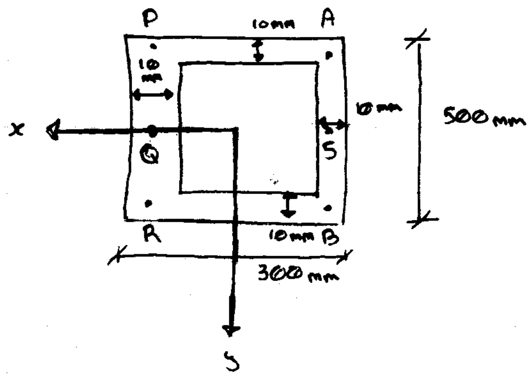
$\boxed{\text{Example}}$ Box Beam




1° thin wall

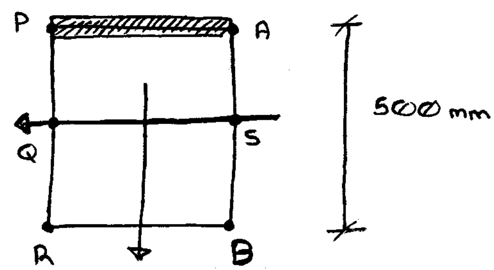
2° x-axis is symmetric axis

3° only internal shear force

$V_y$ is needed

Take $V_y = I_x$

4° cut at point A



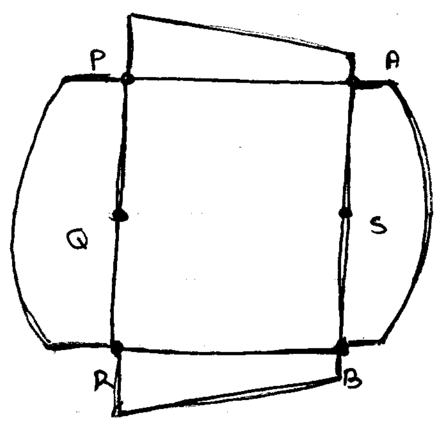

4° cont'd

$$q_P = q_A + \frac{V_y}{I_x} A'\bar{y}'$$
$$= q_A + (300)(10)(-250)$$
$$= q_A - 750000$$

$$q_Q = q_P + \frac{V_y}{I_x} A'\bar{y}'$$
$$= (q_A - 750000) + (250)(20)(-125)$$
$$= q_A - 1375000$$

$q_R = q_P$ ; $q_B = q_A$

$$\rightarrow q_S = q_B + \frac{V_y}{I_x} A'\bar{y}'$$
$$= q_A + (250)(10)(125)$$
$$= q_A + 313500$$




Angle of twist (per unit length):

$$\theta = \frac{1}{2GA} \oint q/t \, dl = 0$$

$$\Rightarrow \int_{AP} q/t \, dl + \int_{PR} q/t \, dl + \int_{RB} q/t \, dl + \int_{BA} q/t \, dl$$

$$\Rightarrow \frac{1}{10}\int_{AP} q \, dl + \frac{1}{20}\int_{PR} q \, dl + \frac{1}{10}\int_{RB} q \, dl + \frac{1}{10}\int_{BA} q \, dl$$

$$\int_{AP} q \, dl = \frac{1}{2}(q_A + q_P)(AP) = \frac{1}{2}(q_A + q_A - 750000)(300)$$
$$= 300 q_A - 112500000$$

$$\int_{PR} q\,dl = \frac{PR}{6}\left(q_P + 4q_Q + q_R\right)$$
$$= \frac{500}{6}\left(q_A - 750000 + 4(q_A - 1375000) + q_A - 750000\right)$$
$$= 500\,q_A - 20(29166667)$$

$$\int_{RB} q\,dl = \frac{1}{2}(q_R + q_B)(RB)$$
$$= 300(q_A) - 112500000$$

$$\int_{BA} q\,dl = \frac{AB}{6}\left(q_B + 4q_S + q_A\right)$$
$$= 500(q_A) + 10416667 \times 10$$

$$\Rightarrow \left(\tfrac{1}{10}\right)(300q_A - 112500000)$$
$$+ \left(\tfrac{1}{20}\right)(500q_A - (20)(29166667))$$
$$+ \left(\tfrac{1}{10}\right)(300\,q_A - 112500000)$$
$$+ \left(\tfrac{1}{10}\right)(500q_A + 10416670) = 0$$

$$\Rightarrow q_A = 305556 \text{ N/mm}$$

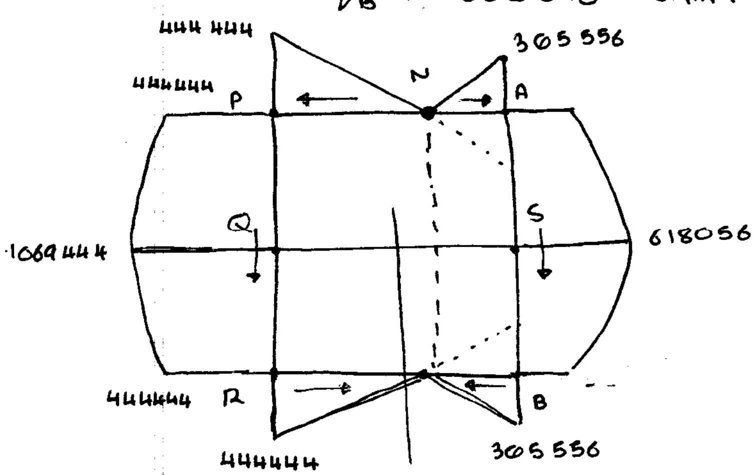
$\longrightarrow$ $q_P = -444\,444$ N/mm

$q_R = -444444$ N/mm

$q_B = 305556$ N/mm

$q_Q = -1069444$ N/mm

$q_S = 618056$ N/mm




1° Symmetrical?

2° Edge (Parallel) Shear Force
$\Rightarrow$ quadratic

3° Edge (perpendicular) Shear Force
$\Rightarrow$ linear

Finding shear center (using Point A)

A : moment center

$$\boxed{e = 203.0 \text{ mm}}$$

Here $I_x = 687500000 \text{ mm}^4$