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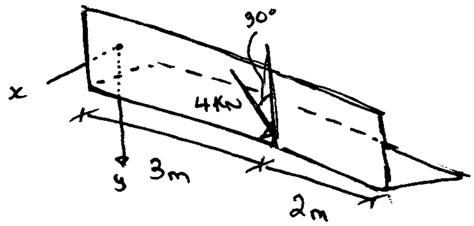
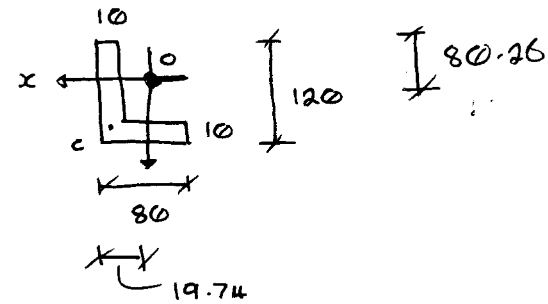
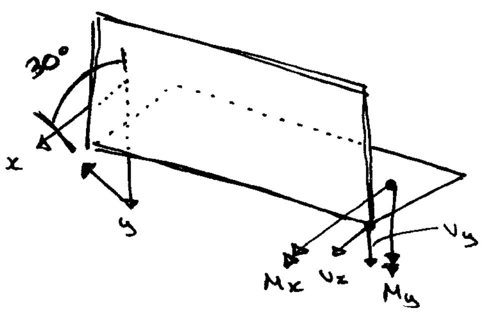



$I_x = 2.783 \times 10^6 \ mm^4$

$I_y = 1.003 \times 10^6 \ mm^4$

$I_{xy} = 0.9726 \times 10^6 \ mm^4$




Normal Stress

$$\sigma_{zz} = \frac{M_x I_y + M_y I_{xy}}{\Delta} y - \frac{M_y I_x + M_x I_{xy}}{\Delta} x$$

Here, $\Delta = I_x I_y - I_{xy}^2$

$\Delta = 1.8454 \times 10^{12} \ (mm^8)$

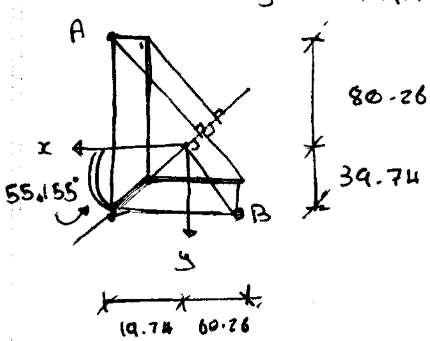
$M_x I_y + M_y I_{xy} = 1.8352 \times 10^{12}$

$M_y I_x + M_x I_{xy} = 2.6361 \times 10^{12}$

$$\Rightarrow \sigma_{zz} = \left(\frac{1.8352 \times 10^{12}}{1.8454 \times 10^{12}}\right) y - \left(\frac{2.6361 \times 10^{12}}{1.8454 \times 10^{12}}\right) x$$

$\sigma_{zz} = 0.99449 (y - 1.4364 x)$

$x, y$ : mm $\quad \sigma_{zz}$ : MPa




$y - 1.4364 x = 0$

Let: $\tan\alpha = (1.4364)$

$\alpha = 55.155°$

A (19.74, −80.26)
B (−60.26, 39.74)

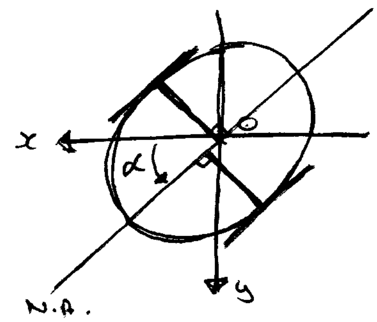
$\sigma_A = 0.99449(-80.26 - 1.4364 \times 19.74)$

$= -108.0$ MPa

$\sigma_B = 0.99449(39.74 - 1.4364 \times (-60.26))$

$\sigma_B = 125.6$ MPa

$$\left[\sigma_{zz} = \frac{M_x I_y + M_y I_{xy}}{\Delta}\, y \cdots \right.$$
$$\left. \cdots - \frac{M_y I_x + M_x I_{xy}}{\Delta}\, x \right] \quad (*)$$




Neutral axis:

$\sigma_{zz} = 0$

$$\Rightarrow \frac{M_x I_y + M_y I_{xy}}{\Delta}\, y - \frac{M_y I_x + M_x I_{xy}}{\Delta}\, x = 0$$

$$\Rightarrow \quad y = x \tan\alpha = 0$$

and $$\tan\alpha = \frac{M_y I_x + M_x I_{xy}}{M_x I_y + M_y I_{xy}}$$

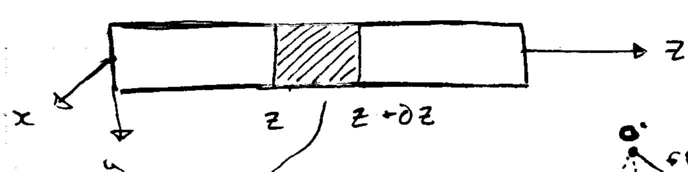
The normal stress

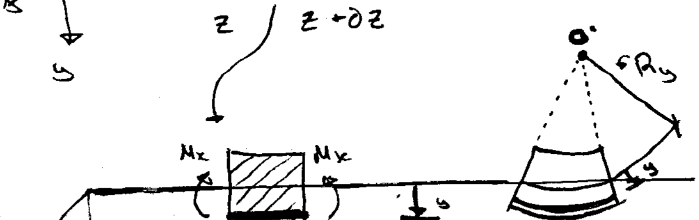
$$\sigma_{zz} = \frac{M_x I_y + M_y I_{xy}}{\Delta}\,(y - x\tan\alpha)$$

IF $M_x \neq 0$

$$\sigma_{zz} = \frac{M_x}{I_x - I_{xy}\tan\alpha}\,(y - x\tan\alpha)$$

## 7.3 Deflections of Straight Beams Subjected to nonsymmetrical bending.



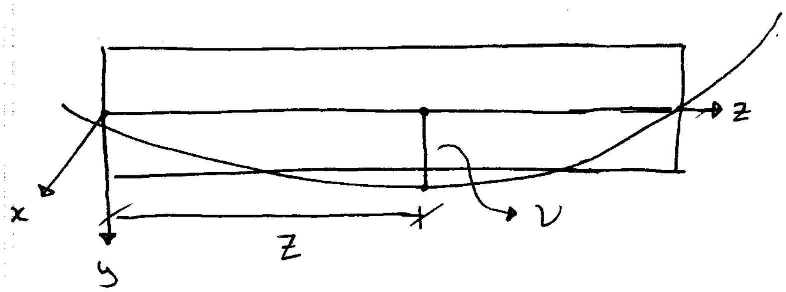




$$R_y\, d\theta_x = dz$$

$$dz \longrightarrow (R_y + y)\, d\theta_x$$

Normal Strain

$$\epsilon_{zz} = \frac{(R_y + y)\, d\theta_x - dz}{dz} = \frac{y}{R_y}$$




$$\frac{1}{R_y} = \frac{\left|\frac{\partial^2 v}{\partial z^2}\right|}{\left(\sqrt{1 + \left(\frac{\partial v}{\partial z}\right)^2}\right)^3}$$

Small deformation

$$\left|\frac{\partial v}{\partial z}\right| \ll 1$$

$$\Rightarrow \frac{1}{R_y} = \left|\frac{\partial^2 v}{\partial z^2}\right| = -\frac{\partial^2 v}{\partial z^2}$$

$$\Rightarrow -\frac{\partial^2 v}{\partial z^2} = \frac{\epsilon_{zz}}{y} = \frac{\sigma_{zz}}{E y}$$

Since $\sigma_{zz} = \dfrac{M_x I_y + M_y I_{xy}}{\Delta} y - \cancelto{0}{\dfrac{M_y I_x + M_x I_{xy}}{\Delta}} x$

Let $x = 0$:

Then : $\boxed{-\dfrac{\partial v^2}{\partial z^2} = \dfrac{M_x I_y + M_y I_{xy}}{E\Delta}}$

**Special case :** $I_{xy} = 0$

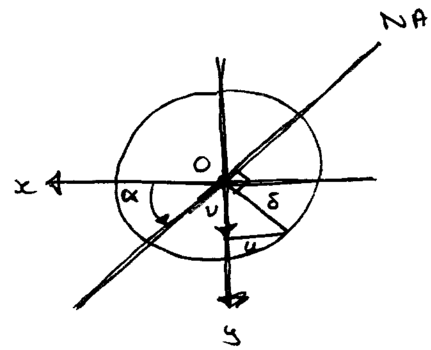
then $\Delta = I_x I_y$

$\Rightarrow \boxed{\dfrac{-\partial^2 v}{\partial z^2} = \dfrac{M_x}{E I_x}}$ $\qquad -EIv'' = M$

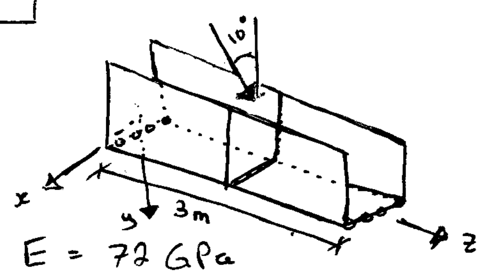
The total deflection:

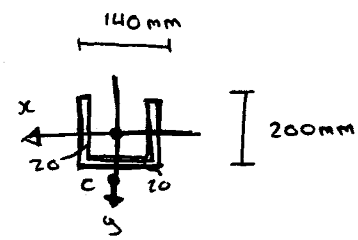
$$u = -v \tan\alpha$$

$$\delta = \sqrt{u^2 + v^2}$$




Example




Given $E = 72$ GPa

Find the max deflection of the beam

Solution

$$y_o = 82.0 \text{ mm}$$

$$I_x = 39.69 \times 10^6 \text{ mm}^4$$

$$I_y = 30.73 \times 10^6 \text{ mm}^4$$

$$I_{xy} = 0$$

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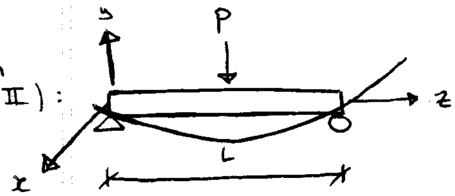
From (MM II):




$EI$ = const.

$$\boxed{\frac{-\partial^2 v}{\partial z^2} = \frac{M_x}{EI_x}}$$

For unsymmetrical beam bending

$$\frac{-\partial^2 v}{\partial z^2} = \frac{M_x I_y + M_y I_x}{E\Delta}$$

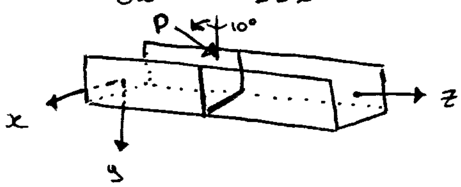
$$\boxed{\frac{-\partial^2 v}{\partial x^2} = \frac{M_x}{E(I_x - I_y \tan\alpha)}}$$

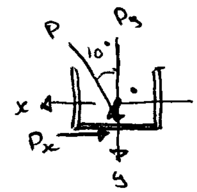
$M_x$ is a function of $Z$

(And $P = 35$ kN)

Since $I_{xy} = \varnothing$ :

$$\frac{-\partial^2 v}{\partial x^2} = \frac{M_x}{EI_x}$$



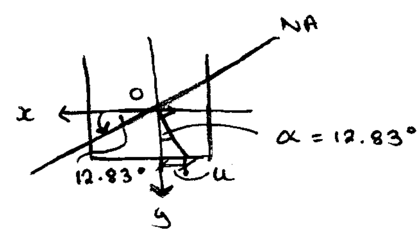

Since $P_y = P\cos(10°)$

$$v_{max} = \frac{P_y L^3}{48 EI_x} \quad \text{From table}$$

$$v_{max} = \frac{PL^3 \cos(10°)}{48 EI_x}$$

$$v_{max} = \frac{(35 \times 10^3)(3 \times 10^4)(\cos 10°)}{(48)(72 \times 10^3)(39.69 \times 10^6)}$$

$$= \mathbf{6.78} \text{ mm}$$




$$\tan\alpha = \frac{M_y I_x + M_x I_{xy}}{M_x I_y + M_y I_{xy}}$$

$$M_y = \left(\tfrac{1}{2}\right)P_x \cdot \left(\tfrac{L}{2}\right) = \tfrac{1}{4}P(\sin 10°)L$$

$$M_x = \left(\tfrac{1}{2}\right)P_y \cdot \left(\tfrac{L}{2}\right) = \tfrac{1}{4}P(\cos 10°)L$$

$$\tan\alpha = \frac{M_y I_x}{M_x I_y} = \frac{\cancel{(\tfrac{1}{4})L}\sin(10°) \times 39.69 \cancel{\times 10^6}}{\cancel{(\tfrac{1}{4})L}\cos(10°) \times 30.73 \cancel{\times 10^6}} = 0.227$$

$\alpha = 12.83°$

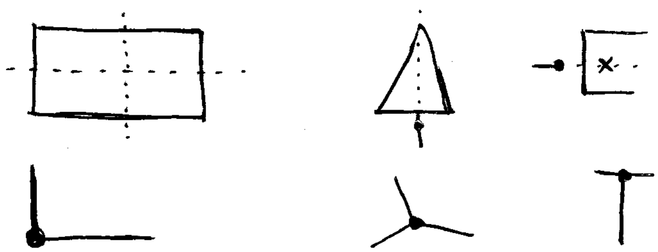
∴ max deflection

$$\delta_{max} = \frac{v_{max}}{\cos\alpha} = \frac{6.78}{\cos(12.83°)} = 6.95 \text{ mm}$$

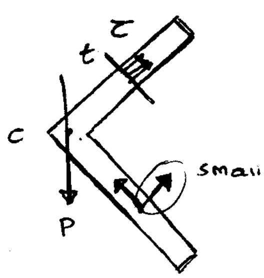
# Ch. 8 - Shear Center for Thin-wall beam Cross-section

## 8.1 Approximation for shear in thin-wall cross section.

Sheer center : A point in the cross-section of a beam through which the loads must pass for the beam to be subjected to only bending deformation. No torsion is caused by the transverse loads that act through the shear center.



Shear stress in the thin wall :



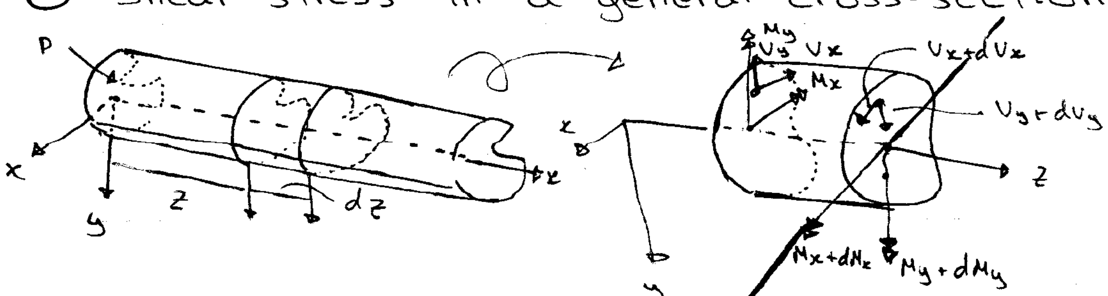

1° Shear stress is parallel to the boundary

2° Shear stress is uniform through the wall thickness

The resultant of the shear stress through the wall thickness : $q = \tau t$ (shear flow)

## 8.2 Shear Flow in Thin-wall beam Cross-section

① Shear stress in a general cross-section

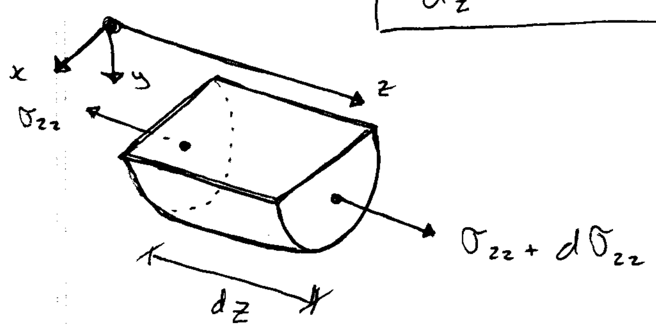


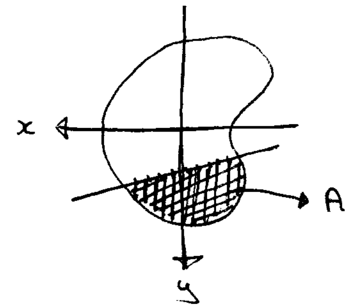

$\Sigma M_x = 0$ : $M_x + dM_x - M_x - V_y dz = 0$

$$\frac{dM_x}{dz} = V_y$$

$$\Sigma M_y = 0 : \quad M_y + dM_y - M_y + V_x dz = 0$$

$$\boxed{\frac{dM_y}{dz} = -V_x}$$

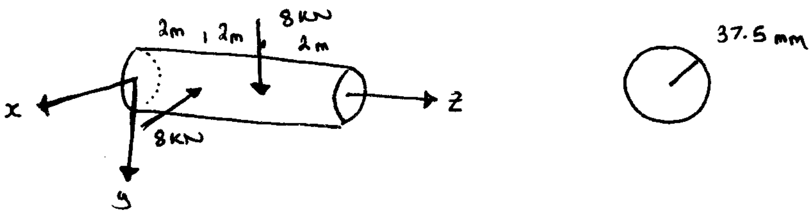



$$\Sigma F_z = 0 : \quad -\iint_{A'} \sigma_{zz} \, dx dy + \iint_{A'} (\sigma_{zz} + d\sigma_{zz}) \, dx dy$$

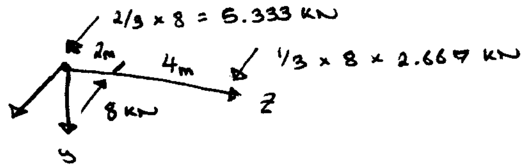
$$- \tau dz \cdot t = 0$$

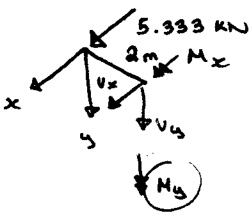
$$\therefore \quad \tau \cdot t = q = \iint_{A'} \frac{d\sigma_{zz}}{dz} \, dx dy$$




Find the location and magnitude of the max normal stress in the beam.

Solution:

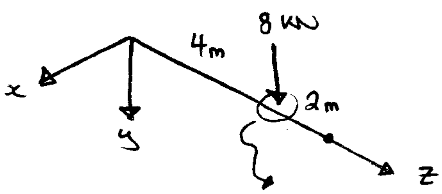


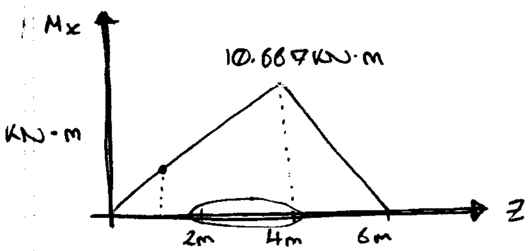

$\Sigma M_y = 0$

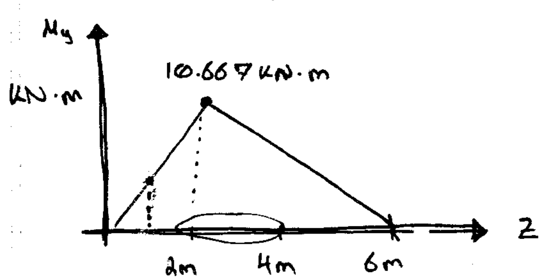
$M_y - 5.333 \times 2 = 0$

$M_y = 10.667$ kN·m




$M_x = 10.667$ kN·m

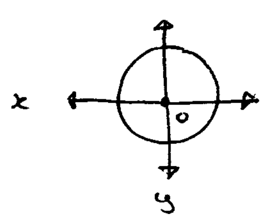



Observation, max normal occurs between $z = 2$, $z = 4$

when $2 \le z \le 4$

$$\begin{cases} M_x(z) = \dfrac{10.667}{4} z \\ M_y(z) = (6 - z)\left(\dfrac{10.667}{4}\right) \end{cases}$$



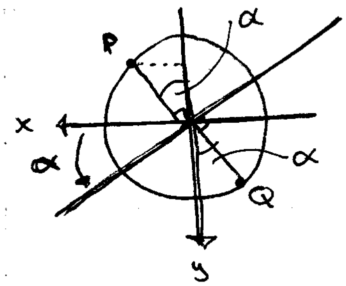

Since $I_x = I_y = (1/4)\pi r^4$

$= (1/4)\pi(37.5)^4$

$= 1.553 \times 10^6 \text{ mm}^4$

$2 \le z \le 4$

$$\therefore \tan\alpha = \frac{M_y I_x + M_x I_{xy}}{M_x I_y + M_y I_{xy}} = \frac{M_y}{M_x} \Rightarrow \boxed{\tan\alpha = \frac{6 - z}{z}}$$




$P(r\sin\alpha,\ -r\cos\alpha)$

$Q(-r\sin\alpha,\ r\cos\alpha)$

Normal Stress

$$\sigma_{zz} = \frac{M_x I_y + \cancel{M_y I_{xy}}\, y}{\Delta} - \frac{M_y I_x + \cancel{M_x I_{xy}}\, x}{\Delta}$$

(where $\Delta = I_x I_y$)

(where $I_{xy} = 0$ for circular cross-section)

$$= \frac{M_x}{I_x} y - \frac{M_y}{I_x} x$$

can use either way

OR

$$\sigma_{zz} = \frac{M_x (y - x\tan\alpha)}{I_x - \cancel{I_{xy}}\tan\alpha}$$

$$\sigma_{zz} = \frac{M_x}{I_x}(y - x\tan\alpha)$$

$\to z$ ... $\to z$

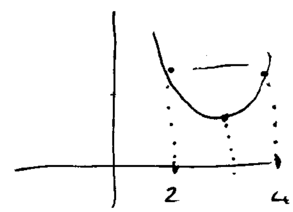
const.

$$\sigma_{zz,P} = \frac{M_x}{I_x}(-r\cos\alpha - r\sin\alpha \cdot \tan\alpha)$$

$$= -\frac{M_x r}{I_x} \cdot \frac{1}{\cos\alpha}$$

$$\sigma_{zz,Q} = \frac{M_y}{I_x}(r\cos\alpha - (-r\sin\alpha)\cdot \tan\alpha)$$

$$= \frac{M_x \cdot r}{I_x} \cdot \frac{1}{\cos\alpha}$$




$$\therefore\ \sigma^{(z)}_{zz,max} = \frac{M_x \cdot r}{I_x} \cdot \frac{1}{\cos\alpha}$$

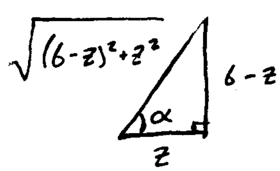
(N·m) (N·mm)

Since $M_x = \dfrac{10.667}{4} z$ kN·m $= \dfrac{10.667}{4} z \times 10^3$

$I_x = 1.553 \times 10^6\ mm^4$

$r = 37.5$ mm

$\tan\alpha = \dfrac{6-z}{z}$




min occurs @ 3000 mm

max occurs @ 2000 mm, 4000 mm

$\sigma_{zz}(2000) = \sigma_{zz}(4000)$

$\Rightarrow 288.0$ MPa

AT $z = 2000$ $\tan\alpha = \dfrac{6000-z}{z} = 2$

$z = 4000$ $\tan\alpha = \dfrac{6000-z}{z} = 1/2$

$$\cos\alpha = \frac{z}{\sqrt{(6-z)^2 + z^2}} \longrightarrow \frac{z}{\sqrt{(6000-z)^2 + z^2}}$$

$$\therefore\ \sigma_{zz_{max}}(z) = \frac{(10.667) z \times 10^3}{4} \times \frac{(37.5)}{(1.553 \times 10^6)} \Bigg/ \frac{z}{\sqrt{(6000-z)^2 + z^2}}$$

$$\therefore\ \sigma_{zz_{max}}(z) = 64.39 \times 10^{-3} \times \sqrt{(6000-z)^2 + z^2}$$

when $2000\text{ mm} \le z \le 4000\text{ mm}$