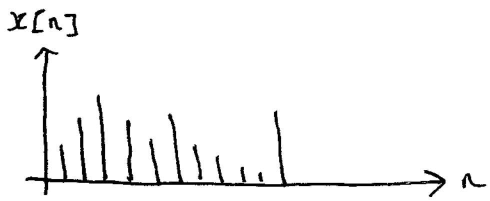
$x[n]$, $n = 0, 1, 2 \ldots, (N-1)$

N

- infinitely long > leakage
- rectangular window



$x'[n] = x[n] .* w[n]$

$x'[n] \neq x[n]$

on the amplitude spectrum, leakage ↓

→ $x(t) = \cos(40\pi t)$

$\omega = 40\pi$ rad/s

$f = \omega / 2\pi = 20$ Hz

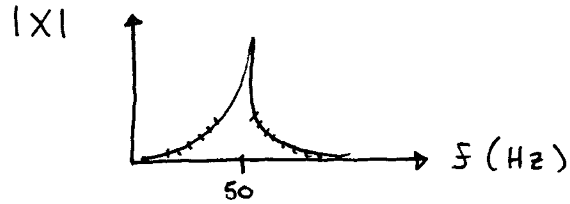
- $f_s = 100$ Hz ;
- $T = 1/f_s$; (sec) - used for demonstration
- $t = 0 : T : 1$;
- $x = x'$;
- $X_r = \text{abs}(\text{fft}(x))$;
- $w = \text{hanning}(\text{length}(x))$;
- $xw = x .* w$;
- $X_{amp} = \text{abs}(\text{fft}(xw))$;
- $L_2 = \text{fix}(\text{length}(X_r)/2)$; (look @ half the signal
- Freq $= (0 : (L2-1) / L2 * (fs/2))$ → will send code

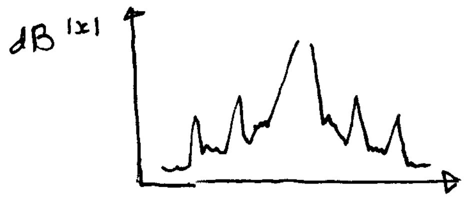
MATLAB code

$X[K] = \sum_n x[n]e^{-j2\pi kn/N}$

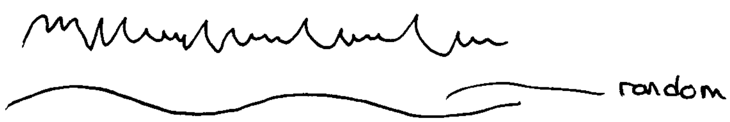
$X(\Omega)$ = DTFT

dB // 1 dB = $20\log_{10}|A/B|$





Kurtosis,



$KU = \mu_4/\sigma^4$

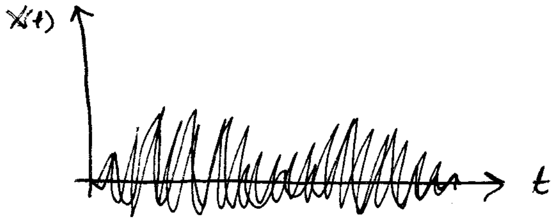
Crest Factor, $CF = X_{max}/\sigma$

For a heacthg System, signal → gaussian pdf

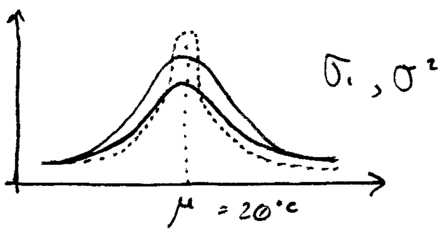
$KU = 3$

Gaussian Signal

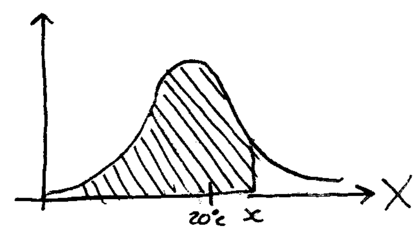
Peak amplitude probability (%)	CF	KU
4.6	2	3
0.1	3.3	3
0.01	3.9	3



Gaussian p.d.f.



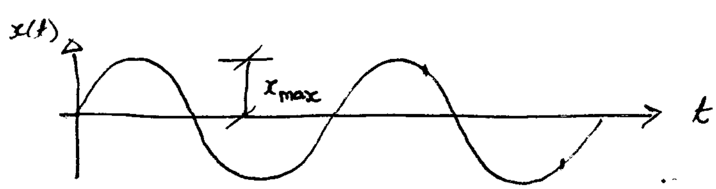
Z-table



$P(x)$ ↯ $CF = x_{max}/\sigma$

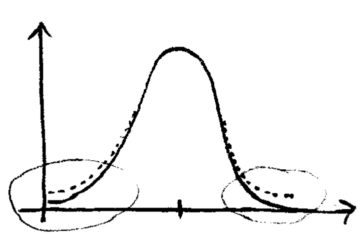
Table 5.2 : The CF and KU for various functions

Signal details	CF	KU
Sine wave pulse train	2	1.5

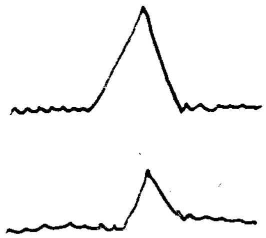


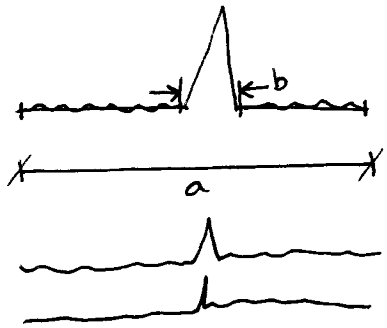
$CF = x_{max}/\sigma$

$KU = \mu_4/\sigma^4$ ~ p.d.f. tail properties



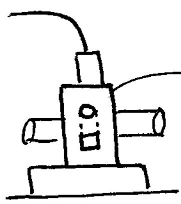
Duty Factor





$\frac{b}{a}$

(*) → can do Q1-4 on A3.



→ bearing housing

time signal

≈ Gaussian

$x[n]$ ADC

→ Anti-aliasing Filtering first

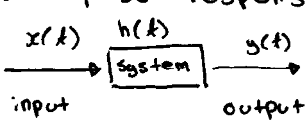
→ ADC second

→ need to read material for bearing fault detection (Ch.12?)

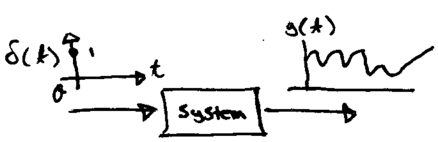
Chapter 4 : Design of Digital Filters

4.1 Analysis of Ideal Filter

1) Impulse response

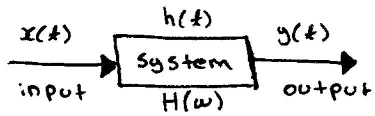


$h(t)$ = impulse (unit pulse) response fn



$$y(t) = h(t) \otimes x(t)$$
$$y[n] = h[n] \otimes x[n], \quad n = 0, 1, 2, \ldots, N-1$$

2) Filters and Filtering



Input : $x(t) = A\cos(\omega_0 t)$

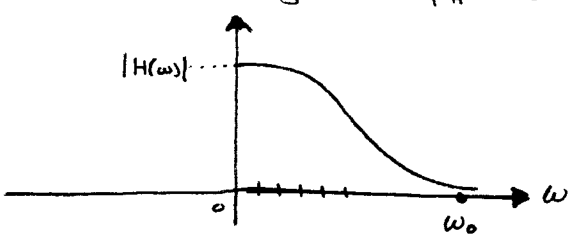
$x(t) \longleftrightarrow X(\omega)$

$Y(\omega) = X(\omega) * H(\omega)$

$Y(\omega) \longleftrightarrow y(t)$

$y(t) = A\,|H(\omega_0)|\cos(\omega_0 t + \phi_H)$

ϕ_H = phase delay : $\phi_H = \arctan \dfrac{Im(H(\omega_0))}{Re(H(\omega_0))}$



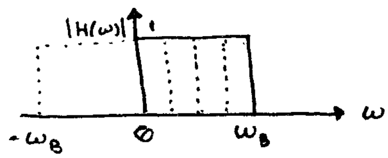
$|H(\omega_0)| = \varnothing$

$y(t) = \varnothing$

Filter, Filtering

3) Ideal Filters

① Ideal low pass Filter



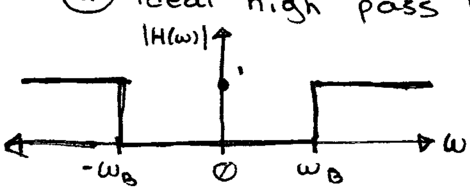
$$|H(\omega)| = \begin{cases} 1, & -\omega_B < \omega < \omega_B \\ \varnothing, & \omega > \omega_B, \quad \omega < -\omega_B \end{cases}$$

$-\omega_B \sim \omega_B$ = pass band

$\omega_B \sim +\infty$, $-\infty \sim -\omega_B$ = stop band

band width $\sim \omega_B$

② Ideal high pass Filter



$$|H(\omega)| = \begin{cases} 1, & \omega \geq \omega_B, \quad \omega \leq -\omega_B \\ \varnothing, & \text{otherwise} \end{cases}$$

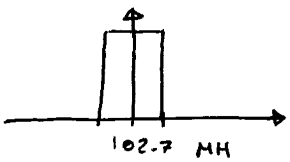
pass bands = $-\infty \sim -\omega_B$

" " = $\omega_B \sim \infty$

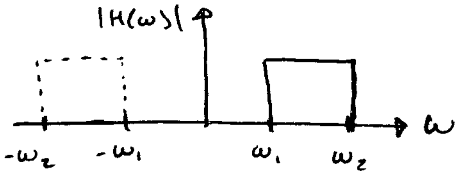
stop band = $-\omega_B \sim \omega_B$

LU Radio Station

102.7 MHz

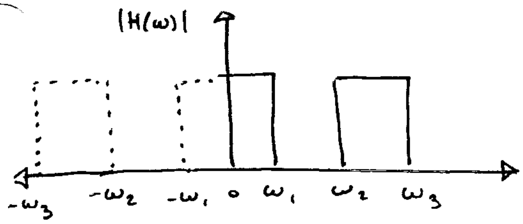


(3) Ideal band pass Filter



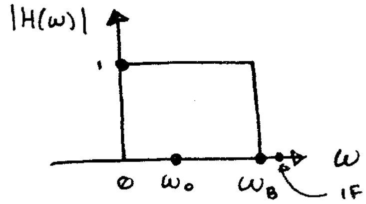
$$|H(\omega)| = \begin{cases} 1 , & \omega_1 \sim \omega_2 ,\ -\omega_2 \sim -\omega_1 \\ 0 , & \omega < -\omega_2 \\ & -\omega_1 \sim \omega_1 ,\ \omega > \omega_2 \end{cases}$$

(4) Ideal band stop Filters



$$|H(\omega)| \begin{cases} 1 ; & -\omega_1 \sim \omega_1 ,\ \omega_2 \sim +\infty , \\ 0 ; & -\omega_2 \sim -\omega_1 ,\ \omega_1 \sim \omega_2 \end{cases}$$

LPF :



ω_B = cutoff freq.

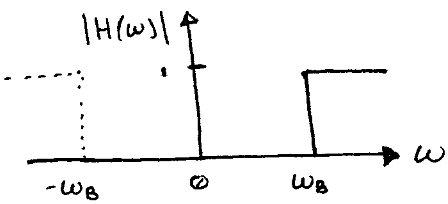
low pass filter

IF this was ω_0 this amplitude becomes zero.

$x(t) = A\cos(\omega_0 t)$

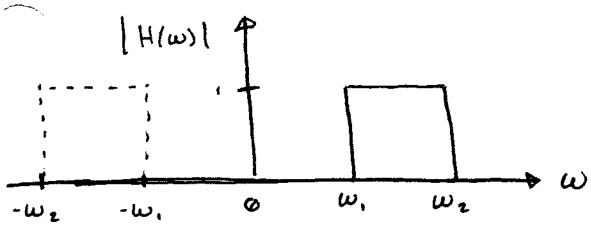
$y(t) = A\,|H(\omega_0)|\cos(\omega_0 t + \phi_n)$

HPL :



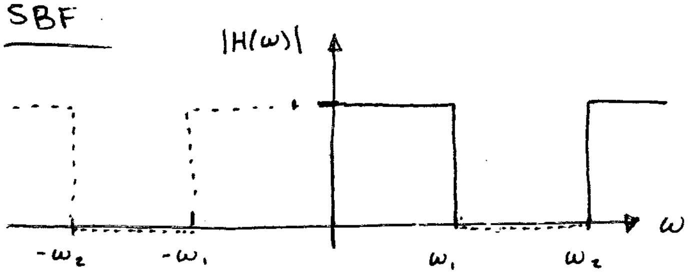
halt pass Filter

BPF



band pass Filter

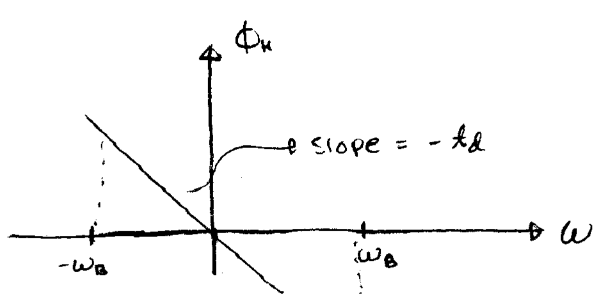
SBF



Stop-band Filtel

4.2 Phase Function of Ideal Filters

A linear phase over the passband



$\phi_H = -\omega t_d$

$x(t) = A\cos(\omega_0 t)$

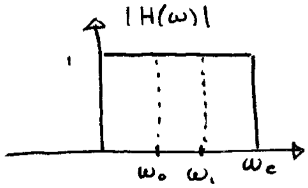
Output :

$y(t) = A|H(\omega_0)| \cos(\omega_0 t - \omega_0 t_d)$

$= A \times 1 \times \cos(\omega_0(t - t_d))$

delay

$x = A_0 \cos(\omega_0 t) + A_1 \cos(\omega_1 t)$

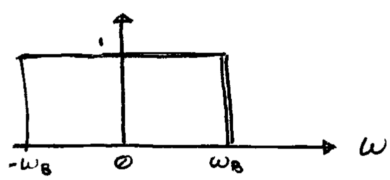


$y(t) = A_0|H(\omega)| \cos(\omega_0 t - \omega_0 t_d) + A_1|H(\omega)| \cos(\omega_1 t - \omega_1 t_d)$

delay

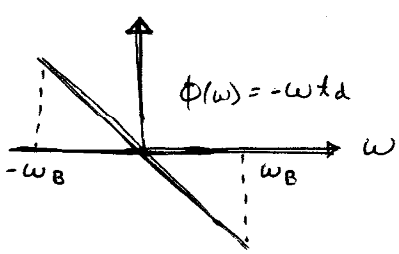
If ϕ_H is not a linear function of ω over the PB

$\phi_H = C$



CFT

$h(t)$



$$H(\omega) = \begin{cases} 1 \; ; & -\omega_B \le \omega \le \omega_B \\ \emptyset \; ; & \text{otherwise} \end{cases}$$

$$\phi_H = \begin{cases} -\omega t_d \; ; & -\omega_B \le \omega \le \omega_B \\ \emptyset & \end{cases}$$

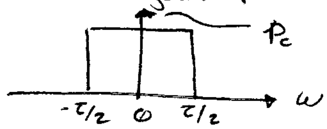
Polar ex.

$H(\omega) = 1 \times e^{-j\omega t_d} = \cos(-\omega t_d) + j\sin(-\omega t_d)$ $\quad |e^{j\theta}| = \cos\theta + j\sin\theta$

$H(\omega) = P_{2B}\, e^{-j\omega t_d}$

P_{2B} = rectangular Function $\quad -\omega_B \sim \omega_B$

P_τ = rectangular pulse with τ



$\rightarrow \quad \tau \,\text{sinc}\left(\dfrac{\tau t}{2\pi}\right) \longleftrightarrow 2\pi P_\tau$

$\rightarrow \quad \dfrac{\tau}{2\pi} \,\text{sinc}\left(\dfrac{\tau t}{2\pi}\right) \longleftrightarrow P_\tau$

$\rightarrow \quad \tau = 2\omega_B$

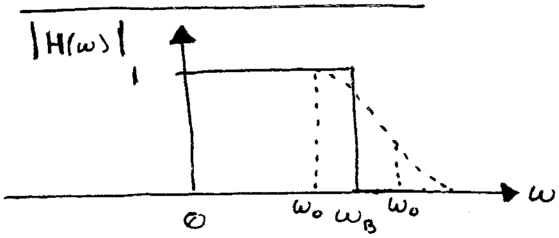
$\rightarrow \quad \dfrac{\cancel{2}\omega_B}{\cancel{2}\pi} \,\text{sinc}\left(\dfrac{\cancel{2}\omega_B t}{\cancel{2}\pi}\right) \longleftrightarrow P_{2B}$

$\delta(t-c) \longleftrightarrow e^{-j\omega c}$

$c = t_d$

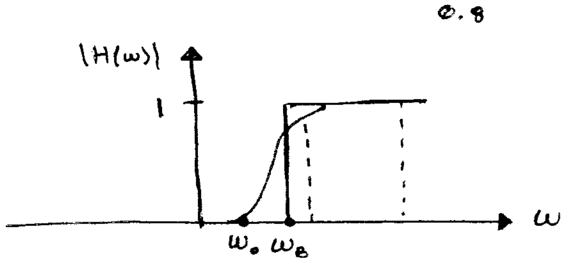
$\underbrace{\left(\dfrac{\omega_B}{\pi}\right)\text{sinc}\left(\dfrac{\omega_B(t-t_d)}{\pi}\right)}_{h(t)\ \rightarrow\ \text{non-causal}} \longleftrightarrow P_{2B}\, e^{-j\omega t_d}$

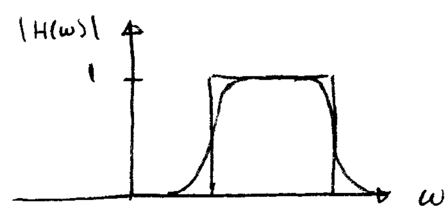
4.2 Causal Filters



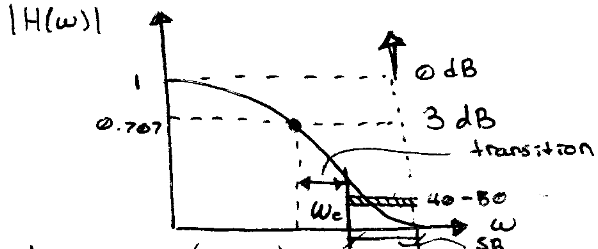
Input : $\quad x(t) = A\cos(\omega_o t)$

Output: $\quad y(t) = A\underbrace{|H(\omega)|}_{0.8}\cos(\omega_o t + \phi_H)$



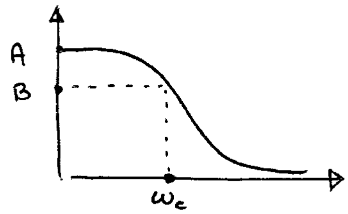


Passband of LPF



$|H(\omega)| \geq (1/\sqrt{2}) \approx 0.707 \sim 3\text{dB}$ (magic number)

$\text{dB} = 20 \log_{10} |H(\omega)|$



$3\text{dB} = 20 \log (A/B) = 20 \log \frac{A}{A/\sqrt{2}}$

$B = A/\sqrt{2}$

ω_c = cut-off freq. of the LPF

Pass-band : $0 \sim \omega_c$

- Stop band : drop $40 \sim 50$ dB
- transition region should be as narrow as possible
- Butterworth Filters

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\zeta > 0$ damping ratio

$s = j\omega$

$$H(\omega) = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2}$$