(3.5) LT & TF

$x(t)$

LT: $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$

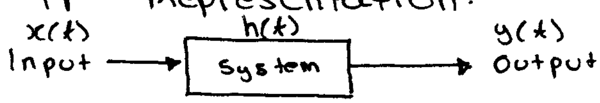
$-\infty < s < \infty$

$s = j\omega$

ILT: $x(t) = \int_{-\infty}^{\infty} X(s) e^{st} \, ds$

$-\infty < s < \infty$

TF Representation:



$y(t) = h(t) \otimes x(t)$

$Y(s) = H(s) * X(s)$

$\longrightarrow$ TF : $H(s) = \dfrac{Y(s)}{X(s)}$

$$H(s) = \frac{b_m (s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}$$

Roots:

$z_1, z_2, \ldots, z_m$ ~ zeros

$p_1, p_2, \ldots, p_N$ ~ poles

order $= N$

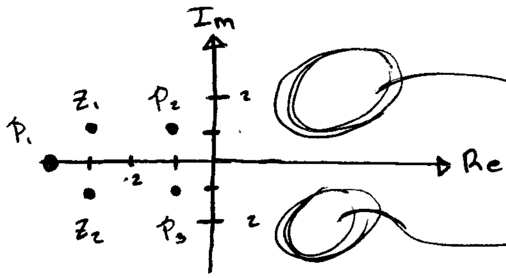
$N \uparrow\uparrow$
- complexity increases
- costs increase
- heat generated increase

↳ as temperature ↑, $\mu \downarrow$ (viscosity decreases)

$\boxed{\text{Example 3.8}}$ Determine roots + order of system:

$$H(s) = \frac{2s^2 + 12s + 20}{s^3 + 6s^2 + 10s + 8}$$

$\rightarrow$ $z_1 = -3 + j$, $z_2 = -3 - j$ $\rightarrow$ $p_1 = -4$, $p_2 = -1 + j$, $p_3 = -1 - j$



If poles were here, system would be unstable.

$\boxed{\text{Example 3.10}}$ Given frequency fxn, determine time signal

$$H(s) = \frac{s+2}{s^3 + 4s^2 + 3s}$$

$\rightarrow$ $z_1 = -2$ $p_1 = 0$, $p_2 = -1$, $p_3 = -3$

$$H(s) = \frac{a}{s-0} + \frac{b}{s+1} + \frac{c}{s+3}$$

Method I: direct comparison (3 unknowns)

$$\frac{s+2}{s(s+1)(s+3)} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+3}$$

$$= \frac{a(s+1)(s+3) + b(s)(s+3) + c(s)(s+1)}{s(s+1)(s+3)}$$

$$a(s^2+4s+3) + b(s^2+3s) + c(s^2+s) = s+2$$

$\rightarrow$ $as^2 + a4s + a3 + bs^2 + b3s + cs^2 + cs = s + 2$

$$\begin{cases} a + b + c = 0 \\ 4a + 3b + c = 1 \\ 3a = 2 \end{cases}$$

Method II

$$\frac{s+2}{s(s+1)(s+3)} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+3}$$

Multiple s. Let $s = 0$

$$\left.\frac{s+2}{\cancel{s}(s+1)(s+3)} * \cancel{s}\right|_{s=0} = \left.\frac{a}{\cancel{s}} * \cancel{s} + \frac{b}{\cancel{s+1}} * s + \frac{c}{\cancel{s+3}} * s\right|_{s=0}$$

$$(2/3) = a$$

$* s+1$, let $s = -1$

$$\rightarrow \left.\frac{s+2}{s(\cancel{s+1})(s+3)} \times (\cancel{s+1})\right|_{s=-1} = \left.\frac{a}{\cancel{s}}(s+1) + \frac{b}{\cancel{s+1}} * (\cancel{s+1}) + \frac{c}{\cancel{s+3}}(s+1)\right|_{s=-1}$$

$$\rightarrow \frac{-1+2}{(-1)(2)} \Rightarrow \frac{-1}{2} = 0 + b + 0$$

then $b = (-1/2)$

$* s+3$, let $s = -3$

$$\rightarrow \left.\frac{s+2}{s(s+1)(\cancel{s+3})} (\cancel{s+3})\right|_{s=-3} = \left.\frac{a}{\cancel{s}}(s+3) + \frac{b}{\cancel{s+1}}(s+3) + \frac{c}{\cancel{s+3}}(\cancel{s+3})\right|_{s=-3}$$

$$\rightarrow \frac{(-3)+2}{(-3)(-2)} \Rightarrow \frac{-1}{6} = 0 + 0 + c$$

then $c = (-1/6)$

$$H(s) = \frac{(2/3)}{s} + \frac{(-1/2)}{s+1} + \frac{(-1/6)}{s+3}$$

$$h(t) = (2/3)e^{-0t} - (1/2)e^{-t} - (1/6)e^{-3t}$$

3.6 Z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$x[n]$, $-\infty < n < \infty$

DTFT:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

The Z-transform :

$x[n]$

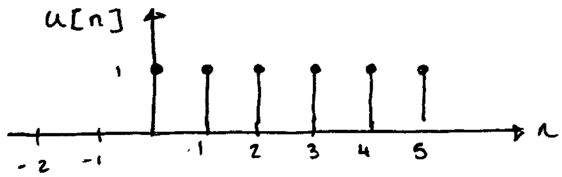
$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$z = e^{j\Omega}$$

$$\Omega = \omega T \qquad \& \qquad s = j\omega$$

$\boxed{\text{Example } 4.8}$

$$u[n] = \begin{cases} 1 & ; \; n = 0, 1, 2, \ldots, \infty \\ 0 & ; \; n = -\infty, \ldots, -2, -1, 0 \end{cases}$$



$$U(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$
$$= \sum_{n=0} 1 \times z^{-n}$$
$$= 1 + z^{-1} + z^{-2} + \ldots$$

$$\sum_{n=q_1}^{q_2} r^n = \frac{r^{q_1} - r^{q_2}}{1-r} = \frac{(z^{-1})^0 - (z^{-1})^\infty}{(z^{-1})}$$
$$= \frac{1 - 0}{1 - 1/q} = \frac{z}{z-1}$$

Analog Filter < passive Filter / active Filter

R.C.   op. amp

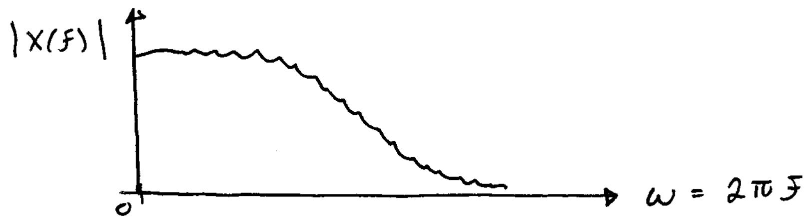
Inductor

3.7 Window Fxn

Physical quantity → $\boxed{\text{sensor}}$ → output $x(t)$

$\downarrow$ ADC

$x[n]$



$\frac{30000}{60}$ rev/min $\qquad$ $\frac{1800}{60}$ rpm

500 Hz $\qquad$ = 30 Hz

$x[n]$ $\qquad$ length(x) = ∞

Infinite length, $n = 0, 1, 2, 3, \ldots$

$x[n] = \cos(40\pi n)$, $n = 0, 1, \cdots, \infty$

$\omega = 40\pi$; $f = 20$ Hz

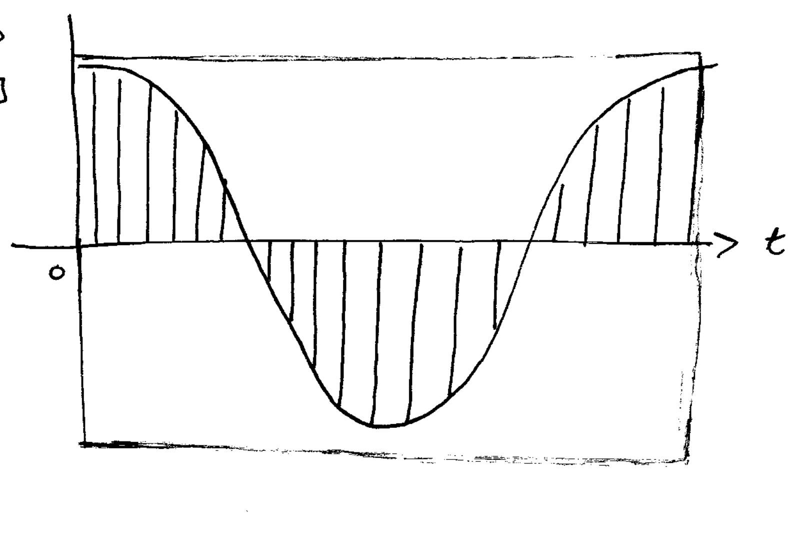
$N = 10\,000$

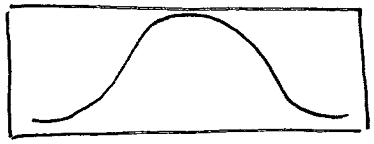
$N = 1000$

$N = 100 \longrightarrow$ leakage

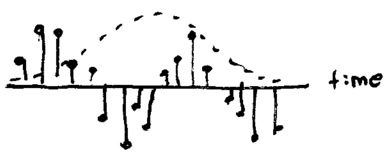
- Signal length
- $x(t)$

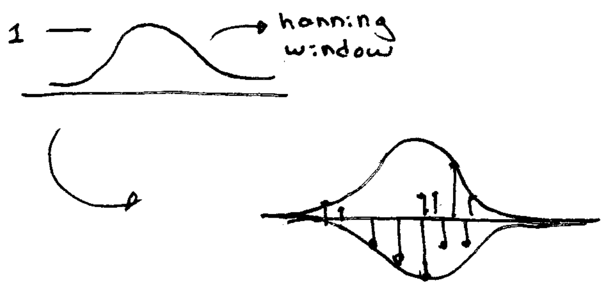
  $x[n]$



- rectangular 
- Hanning window $W[n] = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$; $n = 0, 1, \cdots, N-1$
- Hamming window $W[n] = 0.5 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$; $n = 0, 1, \cdots, N-1$

$x[n] * w[n]$





3.8 Kurtosis Analysis

$$KU = \frac{\mu^4}{\sigma^4} = \frac{E\{(x-\mu)^4\}}{\sigma^4}$$

Pulses will change pdf properties @ tail . . .