(3.4) FFT

$x[n]$, $n = 0, 1, 2, \dots, N-1$

$N = 1024$

IF N is an even number

$x[n] \to a[n] = x[2n]$, $n = 0, 1, \dots, \frac{N}{2} - 1$

$ \to b[n] = x[2n+1]$, $n = 0, 1, \dots, \frac{N}{2} - 1$

$N/2 \sim$ even #

$a[n] \to a_1[n] \to a_3[n]$

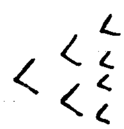
$ \to a_2[n] \to a_4[n]$

$N = 1026,\ 1025$

$N = 1024 = 2^{10}$

- Bit reversing

  $8 = 2^3$



Real-time online

Time point n	Binary	Reverse Bit word	Order
0	000	000	$x[0]$
1	001	100	$x[1]$
2	010	010	$x[2]$
3	011	110	$x[3]$
4	100	001	$x[4]$
5	101	101	$x[5]$
6	110	011	$x[6]$
7	111	111	$x[7]$

(3.5) The Laplace Transform & Transfer Function Representation

1) LT computation

Given $x(t)$, two-sided LT

$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$

s = Freq variable (complex)

CFT : $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$

$s = j\omega$

For one sided LT

$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt$

ILT : Given $X(s)$

$x(t) = \int_{-\infty}^{\infty} X(s) e^{st}\, dt$

$x(t) \longleftrightarrow X(s)$

$v(t) \longleftrightarrow V(s)$

$\boxed{\text{Example 3.7}}$

(a) $20t \longleftrightarrow 20\ 1/s^2$

(b) $2e^{3t} \longleftrightarrow 2\left(\frac{1}{s-3}\right)$

(c) $5\cos(2t) \longleftrightarrow 5\left(\frac{s^2}{s^2+4}\right)$

(d) $2e^{-t}\sin(3t) \longleftrightarrow 2\left(\frac{3}{(s+1)^2+3^2}\right)$

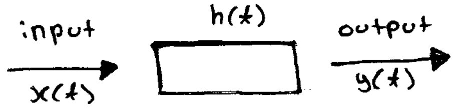
(e) $1 + 2t + e^{-t} \longleftrightarrow \frac{1}{s} + \frac{2}{s^2} + \frac{1}{s+1}$

(f) $2t + 3\frac{dx(t)}{dt} \longleftrightarrow \frac{2}{s^2} + 3[sX(s) - x(0)]$

(g) $2\int x(t)dt \longleftrightarrow 2(1/s)X(s)$

2) Transfer Function Representation

- Impulse response of a system is $h(t)$



$y(t) = x(t) \otimes h(t)$

$Y(s) = X(s)H(s)$

TF : $H(s) = \frac{Y(s)}{X(s)}$

$$H(s) = \frac{b_M s^M + b_{M-1} s^{M-1} + \dots + b_1 s + b_0}{1 * s^N + a_{N-1} s^{N-1} + \dots + a_1 s + a_0}$$

$N \geq M$

Factorial Form :

$$H(s) = \frac{b_m (s - z_1)(s - z_2) * \dots * (s - z_m)}{(s - p_1)(s - p_2) * \dots * (s - p_N)}$$

- $p_1, p_2, \dots, p_n$ ~ roots of denominator polynomial (poles)
- $z_1, z_2, \dots, z_m$ ~ roots of the numerator (zeros)

→ MATLAB : roots m

→ H(s) properties ~ poles, zeros

- N = order of the H(s)

$\boxed{\text{Example 3.8}}$ Given the following frequency function, determine the roots + order of the system

$$H(s) = \frac{2s^2 + 12s + 20}{s^3 + 6s^2 + 10s + 8} = \frac{\overset{-3 \pm j}{2(s^2 + 6s + 10)}}{s^3 + 6s^2 + 10s + 8}$$

Solution :

$$H(s) = \frac{2(s + 3 - j)(s + 3 + j)}{(s + 4)(s + 1 - j)(s + 1 + j)}$$

- 3rd order system

  roots : $-4, \ -1 \pm j$

- zeros : $-3 \pm j$
- poles : $-4 \ ; -1 \pm j$

$$\text{input} \ \xrightarrow[x(t)]{X(s)} \ \underset{h(t)}{\overset{H(s)}{\boxed{\text{system}}}} \ \xrightarrow[y(t)]{Y(s)} \ \text{output}$$

Solution to get $y(t)$

- $h(t) \longrightarrow H(s)$
- $x(t) \longrightarrow X(s)$
- $Y(s) = H(s) * X(s)$
- $y(t) \xleftrightarrow{\text{ILT}} Y(s)$

Example 3.10

Given the following frequency function, determine it's corresponding time signal.

$$H(s) = \frac{s+2}{s^3 + 4s^2 + 3s}$$

Poles :

$$H(s) = \frac{s+2}{s(s^2+4s+3)}$$

$$= \frac{s+2}{s(s+3)(s+1)}$$

zeros : -2

poles : $0, -3, -1$

$$H(s) = \frac{a}{s} + \frac{b}{s+3} + \frac{c}{s+1}$$