No class on Thursday.
↳ Midterm covers material until end of today.

Example 3.6:

Consider the signal with the rectangular form of the DFT given by:

$$X[k] = \begin{cases} 6 & ; \ k = 0 \\ -1-j & ; \ k = 1 \\ 0 & ; \ k = 2 \\ -1+j & ; \ k = 3 \end{cases}$$

Compute the inverse DFT

DFT:

$x[n]$ = $n = 0, 1, 2 \ldots N-1$

Mod N

$$X[k] = \sum_{0}^{N-1} x[n] e^{-j(2\pi kn/N)}$$

$k = 0, 1, 2 \ldots N-1$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$
$$e^{j\theta} = \cos\theta + j\sin\theta$$

IDFT:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}$$

$n = 0, 1, 2, \ldots, N-1$

Solution:

$N = 4$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \left(\cos\left(\frac{2\pi kn}{4}\right) + j\sin\left(\frac{2\pi kn}{4}\right) \right)$$

$$= \frac{1}{4} \sum_{0}^{3} X[k] \left(\cos\left(\frac{\pi kn}{2}\right) + j\sin\left(\frac{\pi kn}{2}\right) \right)$$

when $n = 0$:

$$x[0] = \frac{1}{4}\left[\left(6 \times \cancelto{1}{\cos(0)} + j\cancelto{0}{\sin(0)}\right) + (-1-j) \times \left(\cancelto{1}{\cos(0)} + j\cancelto{0}{\sin(0)}\right) \ldots \right.$$

$$\left. \ldots + (0) + (-1+j) \times \left(\cancelto{1}{\cos(0)} + j\cancelto{0}{\sin(0)}\right) \right] = 1$$

$$x[1] = \left(\frac{1}{4}\right)\left[6 \times \left(\cancelto{1}{\cos(0)} + j\cancelto{0}{\sin(0)}\right) + (-1-j) \times \left(\cancelto{0}{\cos(\pi/2)} + j\cancelto{1}{\sin(\pi/2)}\right) \ldots \right.$$

$$\left. \ldots + 0 + (-1+j) \times \left(\cancelto{0}{\cos\left(\frac{3\pi}{2}\right)} + j\cancelto{-1}{\sin\left(\frac{3\pi}{2}\right)}\right) \right] =$$

$$= \left(\frac{1}{4}\right)\left[6 + (-1-j)j + 0 + [-1+j](-j) \right]$$

$$= 2$$

$x[2] = 2$

$x[3] = 1$

4) Properties of DFT

$x[n] \longleftrightarrow X[k]$, $v[n] \longleftrightarrow V[k]$

- Linearity

$a\,x[n] + b\,v[n] \longleftrightarrow a\,X[k] + b\,V[k]$

- Circular time shift

$x[n-q,\ \text{mod } N] \longleftrightarrow X[k]e^{-j2\pi kq/N}$

Proof :

$$X[k] = \sum_{n=0}^{N-1} x[n-q,\ \text{mod } N]e^{-j2\pi kq/N}$$

$u = n-q \quad ; \quad n = u+q$

limits : $-q$; $N-1-q$

$$X[k] = \sum_{-q}^{N-1-q} x[u,\ \text{mod } N]e^{-j2\pi k(u+q)/N}$$

$$= \sum_{-q}^{N-1-q} x[u,\ \text{mod } N]e^{-j2\pi ku/N} \cdot e^{-j2\pi kq/N}$$

$$= X[k]e^{-j2\pi kq/N}$$

- Time Reversal

$x[-n,\ \text{mod } N] \longleftrightarrow X[-k,\ \text{mod } N]$

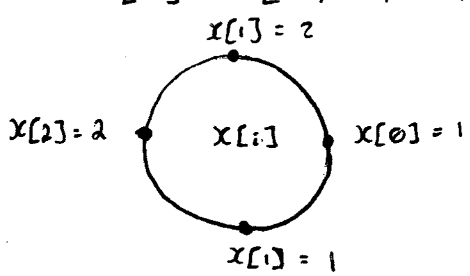
- Circular convolution

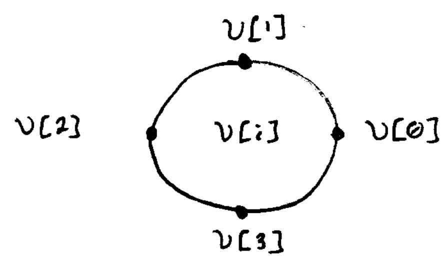
$$y[n] = \underset{(i)}{x[n] \circledast v[n]} = \sum_{i=-\infty}^{\infty} x[i]v[n-i]$$

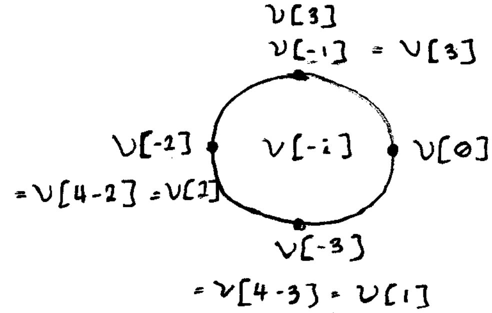
$$x[n] \circledast v[n] = \sum_{i=0}^{N-1} x[i]v[n-i,\ \text{mod } N]$$

$x[n] = [\,1,\ 2,\ 2,\ 1\,]$

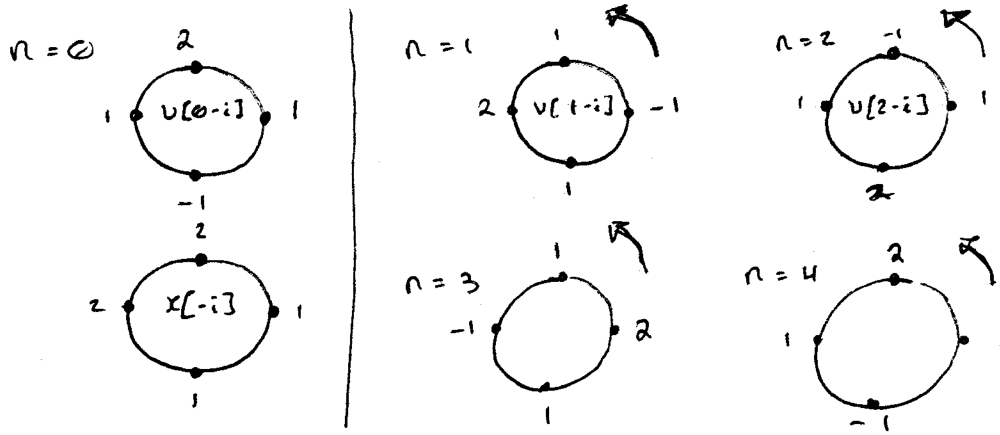
$v[n] = [\,1,\ -1,\ 1,\ 2\,]$







$v[-n] = v[N-n]$



$n = 0$

$1 + 4 + 2 - 1 = 6$

$n = 1$

$-1 + 2 + 4 + 1 = 6$

$n = 2$

$1 + (-2) + 2 + 2 = 3$

$n = 3$

$2 + 2 + (-2) + 1 = 3$

$n = 4$

$1 + 4 + 2 - 1 = 6$

5) Relationship between DTFT & DFT

DTFT:

$$X(\Omega) = \sum_{n=0}^{N-1} x[n] e^{-j\Omega n}$$

DFT:

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kN}{N}}$$

$$\Omega = \frac{2\pi k}{N} \qquad f_s \qquad T = \frac{1}{f_s}$$

$$\Delta f = \frac{f_s}{N} = \frac{1}{NT}$$

$$\Delta\omega = 2\pi \Delta f = 2\pi\, k/N$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

Given k, $k = 0, 1, 2 \ldots N-1$

# of multiplications

$\geq N^2$

If $N = 1024$, $N^2 = 1024^2 = 1{,}048{,}576$

FFT (decimation in time)

$$\frac{N \log_2(N)}{2}$$

$N = 1024$; $\# = 5129$

If N is an even integer, $N/2$ is an integer

$$x[n] \begin{cases} a[n] = x[2n], & n = 0, 1, 2 \ldots (N/2 - 1) \\ b[n] = x[2n+1], & n = 0, 1, 2 \ldots (N/2 - 1) \end{cases}$$

If $N/2$ is an even integer,

$$a[n] \begin{cases} a_1[n] \begin{cases} \text{etc...} \\ \ldots \end{cases} \\ a_2[n] \begin{cases} \ldots \\ \ldots \end{cases} \end{cases}$$

$$b[n] \begin{cases} b_1[n] \\ b_2[n] \end{cases}$$

- N should be an integer

  1025

  $N = 2^q$