$$\int | \underbrace{x(t) e^{-j\omega t}}_{\text{not integrable}} | dt < \infty$$

$$= \infty \quad \text{DNE}$$

Example $X(\omega) = \frac{1}{j\omega + 1}$

(3) $v(t) = t^2 x(t)$

Solution: $\frac{d}{d\omega} X(\omega) = -\frac{j}{(j\omega+1)^2}$

$$\frac{d^2}{d\omega^2} X(\omega) = -\frac{j}{(j\omega+1)^3}(-2)j$$

$$= \frac{2(-1)}{(j\omega+1)^3} = -\frac{2}{(1+j\omega)^3}$$

$$V(\omega) = j^2 \frac{-2}{(1+j\omega)^3} = \frac{2}{(1+j\omega)^3}$$

Note: rectangular, polar

$$V(\omega) = \frac{2(1-j\omega)^3}{[(1+j\omega)^3(1-j\omega)^3]} = \frac{2(1-j)(1-j\omega)^2}{(1+\omega^2)^3}$$

$$= Re + j Im$$

(4) $v(t) = x(t)\cos(t)$

$$v(t) \longleftrightarrow V(\omega) = (\tfrac{1}{2})[X(\omega+4) + X(\omega-4)]$$

$$V(\omega) = \left(\frac{1}{2}\right)\left[\frac{1}{j(\omega+4)+1} + \frac{1}{j(\omega-4)+1}\right]$$

$$= \left(\frac{1}{2}\right)\left[\frac{1-j(\omega+4)}{[1+j(\omega+4)][1-j(\omega-4)]} + \frac{1-j(\omega-4)}{[1+(j(\omega-4)][1-j(\omega+4)]}\right]$$

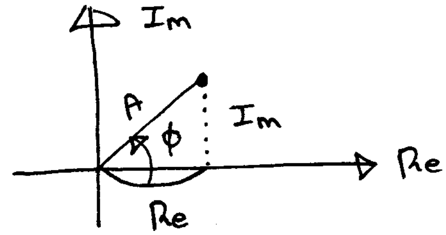
$$= \left(\frac{1}{2}\right)\left[\frac{1-j(\omega+4)}{1+(\omega+4)^2} + \frac{1-j(\omega-4)}{1+(\omega-4)^2}\right]$$

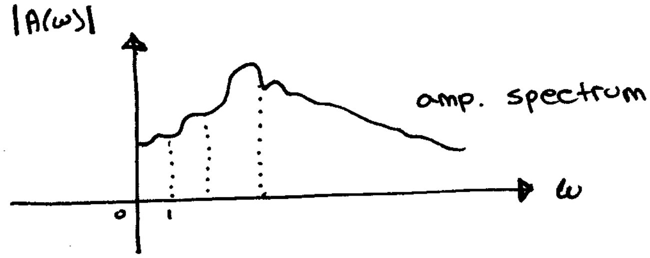
$$= \left(\frac{1}{2}\right)\underbrace{\left[\frac{1}{1+(\omega+4)^2} + \frac{1}{1+(\omega-4)^2}\right]}_{Re} + \left(\frac{j}{2}\right)\underbrace{\left[\frac{-(\omega+4)}{1+(\omega+4)^2} + \frac{-(\omega-4)}{1(\omega-4)^2}\right]}_{Im}$$

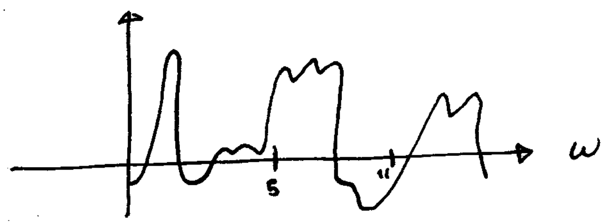
$$= \sqrt{Re^2 + Im^2} = e^{j\phi}$$

$$\phi = \arctan\left[\frac{Im}{Re}\right]$$

$$V(\omega) = A(\omega)e^{j\phi(\omega)}$$
$$A(\omega) = \sqrt{Re^2(\omega) + Im^2(\omega)}$$
$$\phi(\omega) = \arctan\left(\frac{Im(\omega)}{Re(\omega)}\right)$$







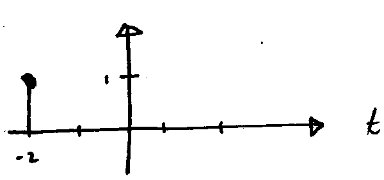
$\boxed{\text{Example}}$ ICFT

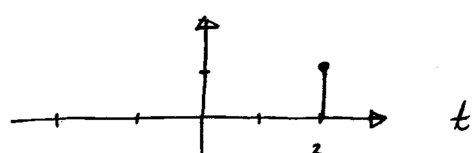
(1) $X(\omega) = \sin(2\omega)$
$x(t) = ?$

$$\begin{cases} \cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j} \end{cases}$$ (modified Euler formula)

$$X(\omega) = (-j/2)[\underbrace{e^{j2\omega}}_{c=-2} - \underbrace{e^{-j2\omega}}_{\& \ c=2}]$$

$$\longleftrightarrow (-j/2)[\delta(t+2) - \delta(t-2)]$$



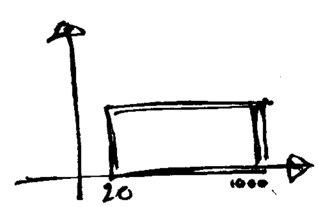


(2) $X(\omega) = \cos^2(2\omega)$

$$= \left[\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right]^2$$

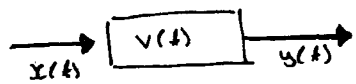
$$= \left(\frac{1}{4}\right)\left[e^{j4\omega} + e^{-j4\omega} + \overset{2}{\cancel{2}}e^{j2\omega - j2\omega}\right]$$

$$x(t) = \left(\frac{1}{4}\right)\left[\delta(t+4) + \delta(t-4) + 2\delta(t)\right]$$



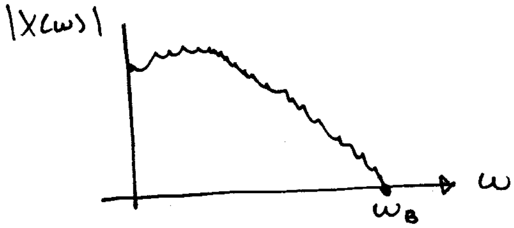
* Passive Filter : 1, 2, 3 order (cheat)

* active filter : op-amp



$$X(\omega) * V(\omega) = Y(\omega) \xrightarrow{ICFT} y(t)$$

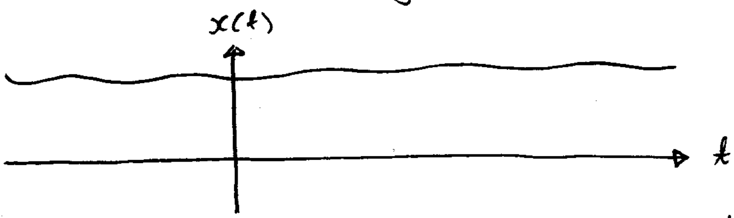
(3.3) Band Limited Signals



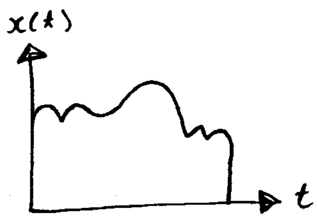
$|X_\omega| \approx 0, \ \omega \geq \omega_B$

$\longrightarrow$ Band limited signal

* Band limited signal cannot be time limited signal



* Time limited signal x(t) cannot be band limited.

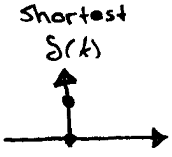


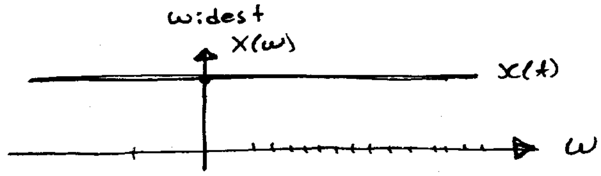
* If $x(t)$ is longer $\longrightarrow$ band $\downarrow$

.... shorter $\longrightarrow$ band $\uparrow$

$\delta(t)$ Band

Shortest widest





$$x(at) \longleftrightarrow \frac{1}{a} X(\omega/a)$$

(3.4) Continuous Time FT (CTFT)

Given $x[n]$, $n = 0, 1, 2, \cdots ; N-1$

DTFT : $X(\Omega) = \sum_{n=0}^{N-1} x[n] e^{-j\Omega n}$

$-\infty < \Omega < \infty$

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \longrightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

$\sum_{n=0}^{N-1} |x[n]| < \infty$

- $X(\Omega)$ is a periodic Fxn with 2π

$X(\Omega + 2\pi) = \sum_{n=0}^{N-1} x[n] e^{-j(\Omega + 2\pi)n}$

$= \sum_{n=0}^{N-1} x[n] e^{-j\Omega n} * e^{-j2\pi n}$

$e^{-j2\pi n} = \cos(-2\pi n) + j\sin(-2\pi n)$

$= \underbrace{\cos(2\pi n)}_{\text{always } = 1} - j\underbrace{\sin(2\pi n)}_{\text{always } = 0} = 1$

Then $X(\Omega + 2\pi) = X(\Omega)$

$0 \le \Omega \le 2\pi$, $-\pi < \Omega < \pi$

IDTFT :

$x[n] = \int_{0}^{2\pi} X(\omega) e^{j\Omega n} d\Omega$

$n = 0, 1, 2, \cdots ; N-1$

$e^{j\Omega n} = e^{j(\Omega + 2\pi)n}$

$= e^{j\Omega n} * \underbrace{e^{j2\pi n}}_{1}$

3.4 DTFT

$x[n]$: $n = 0, 1, 2, 3, \dots, N-1$

$x[n]$: if $n < 0$, $n \geq N$

$$X(\Omega) = \sum_{n=0}^{N-1} x[n] e^{-j\Omega n}$$

$X(\Omega) \sim$ periodic fxn. $\quad -\infty < \Omega < \infty$ (continuous) $\quad 2\pi$ (discrete)

IDTFT

$$x[n] = \frac{1}{2\pi} \int_{-\infty \; (0/-\pi)}^{\infty \; (2\pi/\pi)} \underbrace{X(\Omega) e^{j\Omega n}}_{\text{Periodic fxn w/ } 2\pi} d\Omega$$

$$x[n] = \frac{1}{2\pi} \int_0^{2\pi} X(\Omega) e^{j\Omega n} d\Omega, \quad n = 0, 1, \dots, N-1$$

Example 3.4 : Compute the DTFT of a discrete-time signal defined by:

$$x[n] = \begin{cases} 0 & ; \quad n < 0 \\ a^n & ; \quad 0 \leq n \leq q \\ 0 & ; \quad n > q \end{cases}$$

Where a is a nonzero real constant and q is a positive integer.

Solution :

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=0}^{q} a^n e^{-j\Omega n} = \sum_{n=0}^{q} (a e^{-j\Omega})^n$$

$$\sum_{n=P_1}^{P_2} r^n = \frac{r^{P_1} - r^{P_2+1}}{1-r}$$

$P_1 = 0$, $P_2 = q$, $r = ae^{-j\Omega}$

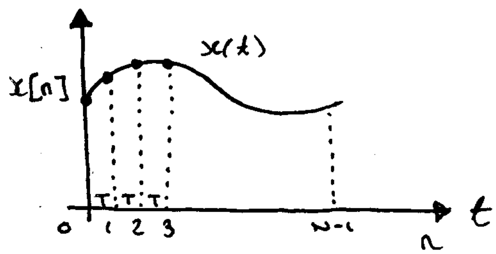
$$X(\Omega) = \frac{(ae^{-j\Omega})^0 - (ae^{-j\Omega})^{q+1}}{1 - ae^{-j\Omega}}$$

$$= \frac{1 - (ae^{-j\Omega})^{q+1}}{(1 - ae^{-j\Omega})}$$

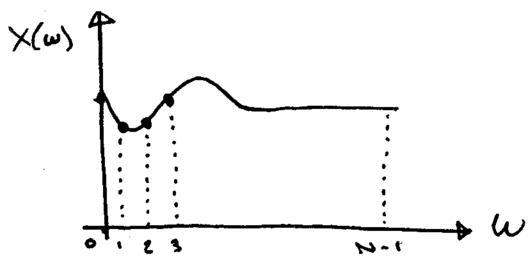
3.5 DFT

$x(t)$, ADC, f_s Hz

$T = 1/f_s$ (sec)



$$x[n] = x(t)\big|_{t=nT} \quad ; \quad n = 0, 1, 2, 3, \ldots, N-1$$



$f_s = 1/T$

$\Delta f = f_s/N$

$\omega = 2\pi f$

$\Delta\omega = 2\pi\Delta f = 2\pi f_s/N$

$\Delta f = f_s/N$ (Hz)

$\Delta\omega = 2\pi\Delta f = 2\pi(f_s/N)$ rad/s

DTF

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

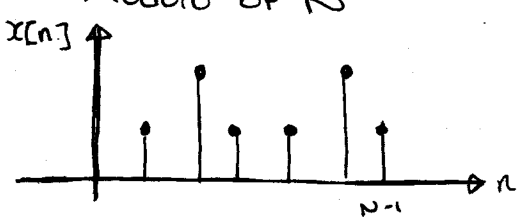
$k\Delta\omega$ $\qquad$ $k = 0, 1, \ldots, N-1$

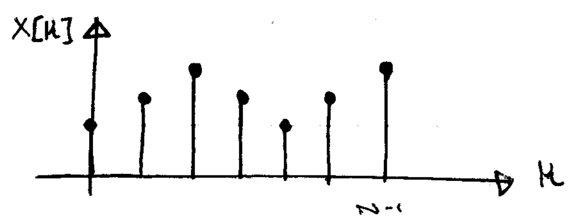
$k\Delta\Omega$

$x[n] = 0$, if $n < 0$, $n \geq N$ $\quad$ ($n = 0, 1, 2, \ldots, N-1$)

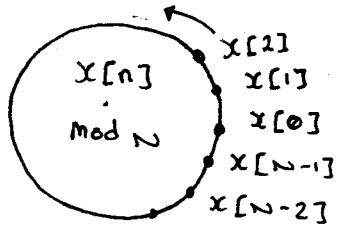
$X[k] = 0$, if $k < 0$, $k \geq N$ $\quad$ ($k = 0, 1, 2, \ldots, N-1$)

Modulo of N





Circular representation



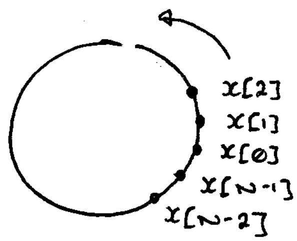
$N = 1024$

$1024 - 1$

$x[-2] = x[N-2]$

$x[-n] = x[N-n]$

$x[n+N] \quad x[n]$



$N = 1024$

$x[-k] = X[N-k]$

$x[k+N] = X[k]$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

$$X[k] = Re[k] + jIm[k]$$

$$x[n] = 0, \quad \text{if} \quad n < 0, \quad n \geq N \qquad ; \quad n = 0, 1, 2, \ldots, N-1$$

$$X[k] = 0, \quad \text{if} \quad k < 0, \quad k \geq N \qquad ; \quad k = 0, 1, 2, \ldots, N-1$$

$$X[k] = \sqrt{Re^2 + Im^2}\, e^{j\phi[k]}$$

$$\phi[k] = \arctan\left[\frac{Im(k)}{Re(k)}\right]$$

- IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N} \qquad ; \quad n = 0, 1, 2, \ldots, N-1$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Example 3.5 : Suppose that $x[0]=1$, $x[1]=2$, $x[2]=2$, $x[3]=1$, and $x[n]=0$ for all other integers n. Compute DFT.

Solution : $x[n]=[1,2,2,1]$, $N=4$

$$X[k] = \sum_{n=0}^{3} x[n]e^{-j2\pi kn/4}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} x[n]\left[\cos\left(\frac{-2\pi kn}{N}\right) + j\sin\left(\frac{-2\pi kn}{N}\right)\right]$$

$$= \underbrace{\sum_{n=0}^{N-1} x[n]\cos(2\pi kn/N)}_{Re[k]} - j\underbrace{\sum_{n=0}^{N-1} x[n]\sin(2\pi kn/N)}_{Im}$$

$$Re[k] = \sum_{n=0}^{3} x[n]\cos(\cancel{2}\pi kn/\cancel{4}2)$$

$k=1$ $$Re[1] = x[0]\cos(\pi\times1\times0/2) + x[1]\cos(\pi\times1\times1/2) + x[2]\cos(\pi\times1\times2/2) + \dots$$

$$\dots + x[3]\cos(\pi\times1\times3/2)$$

$$= (1\times1) + (2\times0) + (2\times(-1)) + (1\times0)$$

If $k=0$:

$$Re[0] = x[0]\cos\left(\frac{\pi\times0\times0}{2}\right) + x[1]\cos\left(\frac{\pi\times0\times1}{2}\right) + x[2]\cos\left(\frac{\pi\times0\times2}{2}\right) + x[3]\cos\left(\frac{\pi\times0\times3}{2}\right)$$

$$= (1\times1)+(2\times1)+(2\times1)+(1\times1) = 6$$

$$Re[k] = \begin{cases} 6 & ; \ k=0 \\ -1 & ; \ k=1 \\ 0 & ; \ k=2 \\ -1 & ; \ k=3 \end{cases}$$

If $k=0$:

$$Im[k] = \sum_{n=0}^{3} x[n]\sin\left(\frac{\pi\times k\times n}{2}\right)$$

$$= 1\times\sin\left(\frac{\pi\times0\times0}{2}\right) + 2\times\sin\left(\frac{\pi\times0\times1}{2}\right) + 2\times\sin\left(\frac{\pi\times0\times2}{3}\right) + \dots$$

$$\dots \ 1\times\sin\left(\frac{\pi\times0\times3}{2}\right) = 0$$

If $k=1$:

$$Im[1] = \sum_{n=0}^{3} x[n]\sin\left(\frac{\pi kn}{2}\right) = 1$$

$$Im[k] \begin{cases} 0 & ; \ k=0 \\ -1 & ; \ k=1 \\ 0 & ; \ k=2 \\ 1 & ; \ k=3 \end{cases}$$

$$X[k] = Re[k] + Im[k] \begin{cases} 6 & ; \quad k = 0 \\ -1-j & ; \quad k = 1 \\ 0 & ; \quad k = 2 \\ -1+j & ; \quad k = 3 \end{cases}$$

Polar representation,

$$X[k] = \begin{cases} 6e^{j0} & ; \quad k = 0 \\ \sqrt{2}e^{-j(3\pi/4)} & ; \quad k = 1 \\ 0 & ; \quad k = 2 \\ \sqrt{2}e^{j(3\pi/4)} & ; \end{cases}$$