$x(t) \longrightarrow \boxed{\text{System}} \xrightarrow{y(t)}$ $\qquad y(t) = 3x(t)e^{t} + \cancel{\text{I.E.}}$

I.E. (initial energy)

- Causal

  non-causal : offline processing

- linear

  Superposition

Input	Output
$x_1 = u$	y_1
$x_2 = 3u$	y_2
$x_3 = x_1 + x_2 = 4u$	$y_3 = y_1 + y_2$

- Time-invariance

Input	Output
$x_1 = x(t)$	$y_1 = 3x(t)e^{x}$
$x_2 = x(t-2)$	$y_2 = 3x(t-2)e^{x}$

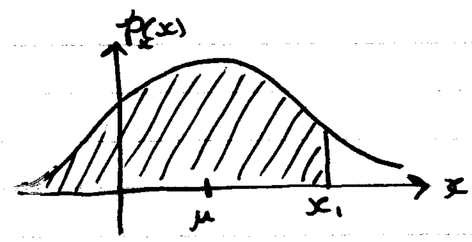
$$\not\approx$$

$$y_3(t-2) = 3x(t-2)e^{t-2}$$

(2.3) Review of Probability Concepts

pdf - prob. distribution function

$p_x(x)$



prob.

$P_x(x \le x_1) = \int_{-\infty}^{x_1} p_x(x)\,dx$, $\qquad$ Gaussian pdf

2) Statistical moments

- 1st order moment

$$\mu = \int_{-\infty}^{\infty} x\,p_x(x)\,dx$$
$$= E\{x\}$$

expectation

- 2nd-order moment

$Var(x) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p_x(x)dx$

Variance

Standard dev. σ $= E\{(x-\mu)^2\}$

$\sigma^2 = E\{(x^2 - 2x\mu + \mu^2)\}$

$= E\{x^2\} - 2\mu E\{x\} + \mu^2$

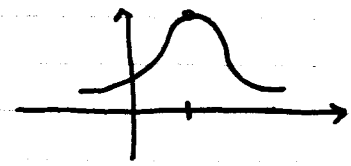
$= E\{x^2\} - 2\mu^2 + \mu^2$

- 3^{rd} moment

$\mu_3 = E\{(x-\mu)^3\}$

$SK = \mu^3 / \sigma^3$; skewness

(unitless)

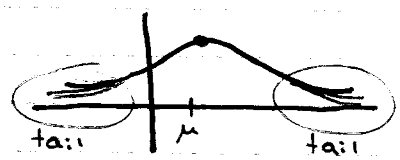


- 4^{th} moment

$\mu_4 = E\{(x-\mu)^4\}$

Kurtosis

$KU = \frac{\mu^4}{\sigma^4}$



Two Variables

$cov(x_1, x_2)$

Coefficient

$\rho_{12}^2 = \frac{cov^2(x_1, x_2)}{Var(x_1)Var(x_2)}$

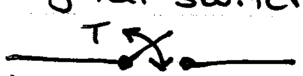
(2.4) Sampling + Aliasing

Collect data, analog signal

Computer ⟶ digital signal

- Sampling

digital switch

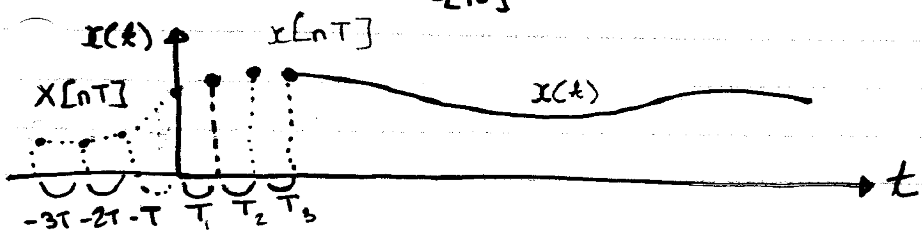


analog signal $x(t)$

digital signal $x[n]$

round brackets for analog signal

square brackets for digital signal



T = time interval (sec)

Analog to Digital converter (ADC)

↳ (DAC) inverse

- control motor, etc.

$x[nT] = x(t)\big|_{t=nT}$

For $n = 0, 1, 2 \ldots$

(in theory, $n = -2, -1, 0, 1 \ldots$)

Sampling Frequency,

$f_s = \frac{1}{T}$ (Hz)

$f_s = 20$ Hz, $T = 1/f_s = 1/20 = 0.05$ sec

$f_s = 15000$ Hz, $T = 1/f_s \ldots$

$x[n]$

n = discrete time value

$f_s \uparrow\uparrow$, $T = 1/f_s \downarrow\downarrow$

data size $\uparrow\uparrow$, Processing speed $\downarrow\downarrow$

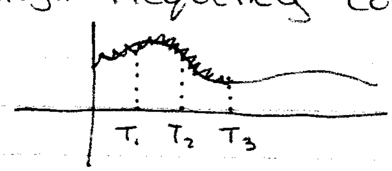
$T \geq 0.003$ sec

online control, real time (diagnosis prognosis)

machine learning

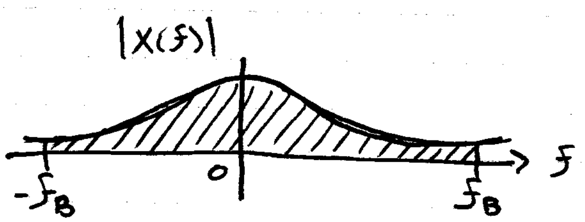
If $f_s \downarrow\downarrow$, $T = 1/f_s \uparrow\uparrow$

high frequency components would be lost

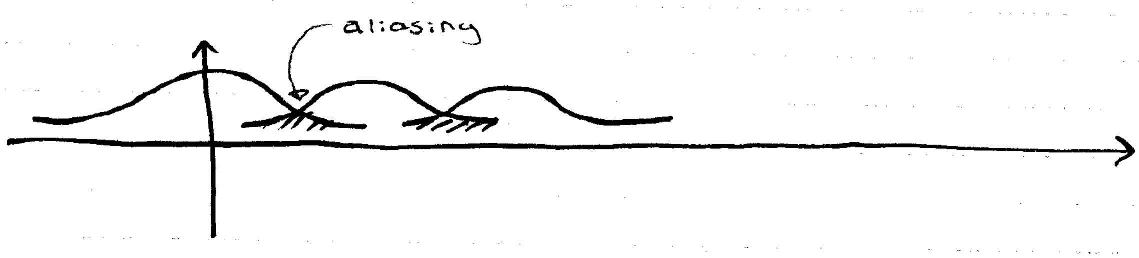


- Actual signals are time-limited

  $-f_B \leq f \leq f_B$

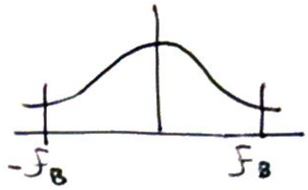


sampling ($2f_B$)



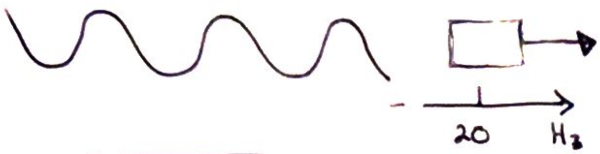
2) Aliasing

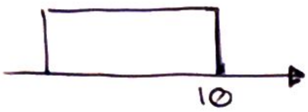
All the signals are time limited



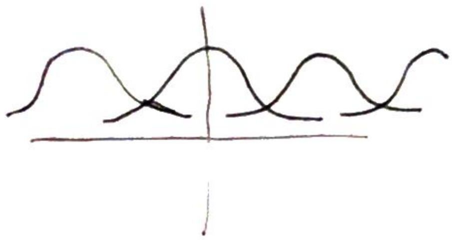
$f_s = 15{,}000$ Hz

Frequency components would be non-limited





Sampled continuously:

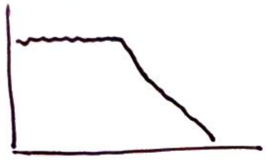


$x[n] = x[nT] = x(t)\big|_{t=nT}$

$n = \dots -2, -1, 0, 1, 2 \dots$

$f_{max} = f_B$ Hz,

FFT



Most important Freq. components $(-f_B, f_B)$

$f_s = 2f_B$

$[0, f_B]$ or $[-f_B, f_B]$

Contains extra frequency components overlapped from high freq. region to the low freq. region.

$(f_B, +\infty)$

~ aliasing

$f_s = 40$ shots/sec

$= 20 \sim 25$ pictures/sec

Before doing ADC

anti-aliasing filter to remove freq. (Low pass filter) analog filter

components higher than f_B

Nyquist Freq

$f_n = 2f_B$

Sampling Freq

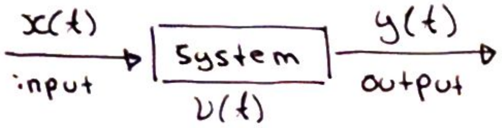
$f_s \geq f_n = 2f_B$

CD

20 KHZ

$f_s = 44.1$ KHZ

(2.4) - Convolution representation



$$y(t) = x(t) \otimes v(t)$$

1) Convolution for continuous signals

$$x(t) \otimes v(t) = \int_{-\infty}^{\infty} x(\lambda)\, v(t-\lambda)\, d\lambda$$

$$\begin{cases} x(t) = 0, \text{ if } t < 0 \\ v(t) = 0, \text{ if } t < 0 \end{cases}$$

, if $\lambda < 0$
$x(\lambda) = 0$

$v(t-\lambda) = 0$, if $t - \lambda < 0$
$t < \lambda$

$$x(t) \otimes v(t) = \int_0^t x(\lambda)\, v(t-\lambda)\, d\lambda$$

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

$$\int_{-\infty}^{\infty} |v(t)|\, dt < \infty$$

2) Conv. for discrete signals

$x[n]$, $v[n]$

$\dots, -2, -1, 0 < n < 1, 2, \dots$

$$x[n] \otimes v[n] = \sum_{i=-\infty}^{+\infty} x[i]\, v[n-i]$$

if $x[n] = 0$; $n < 0$

if $v[n] = 0$; $n < 0$

$x[i] = 0$ if $i < 0$

$v[n-i] = 0$ if $n - i < 0$, $i > n$

$$x[n] \otimes v[n] = \begin{cases} \sum_{i=0}^{n} x[i]\, v[n-i] & ; \; i = 0, 1, 2, \dots, n \\ 0 & ; \text{ if } i < 0, i > n \end{cases}$$

Convolution computation procedures :

1) Changing the discrete time index n to i in signals $x[n]$ and $v[n]$. The resulting signals $x[i]$ and $v[i]$ are then functions of the discrete-time index i.

2) Determining $v[n-i]$. The signal $v[n-i]$ is a folded and shifted version of the signal $v[i]$. More precisely, $v[-i]$ is $v[i]$ folded about the vertical axis, and $v[n-i]$ is $v[-i]$ shifted by n steps. If $n > 0$, $v[n-i]$ is an n-step right shift of $v[-i]$. In contrast, if $n < 0$, $v[n-i]$ is an n-step left shift of $v[-i]$

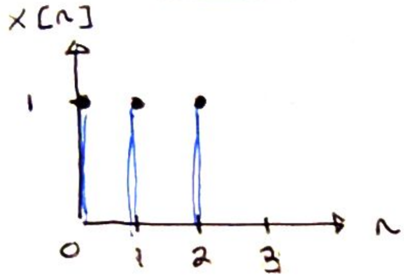
3) Computing the convolution

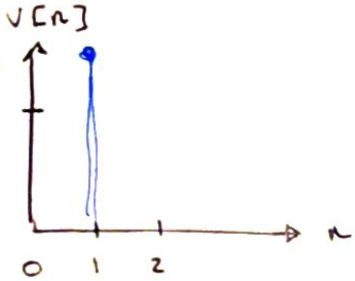
$v[n] \longrightarrow v[i]$, changed index

$\longrightarrow v[-i]$ Folding

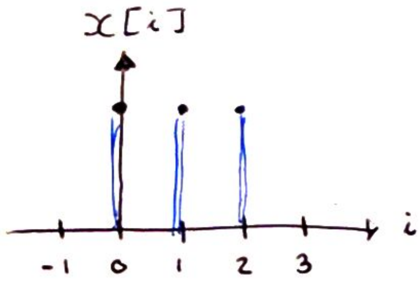
$\longrightarrow v[n-i]$ Shifting

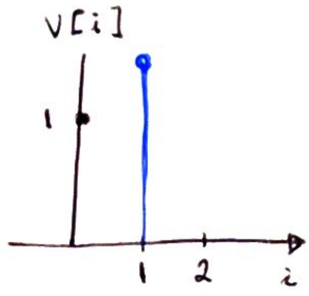
$\boxed{\text{Example 1}}$

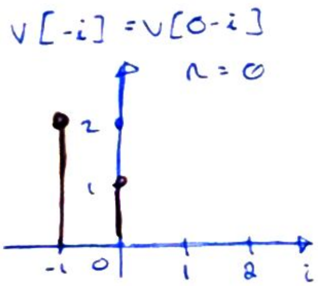


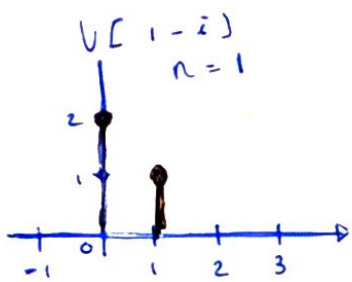


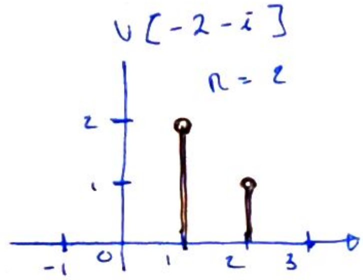
Solution











. . . etc.

3) Conv. Properties

- Associativity,

$x[n]$, $v[n]$, $w[n]$

$$x[n] \otimes (v[n] \otimes w[n]) = (x[n] \otimes v[n]) \otimes w[n]$$

- Communitivity,

$$x[n] \otimes v[n] = v[n] \otimes x[n]$$

$$\sum_{i=-\infty}^{+\infty} x[i]v[n-i] = \sum_{i=-\infty}^{\infty} v[i]x[n-i]$$

- Distributivity with Addition

$$x[n] \otimes v[n] + w[n]) = x[n] \otimes v[n] + x[n] \otimes w[n]$$