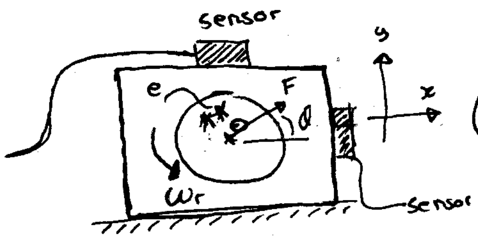
Nov. 25/19



For a rotor :

(center of mass
(center of rotation
(should be the same)

but in reality, that's not how it works.

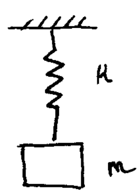
mass = m

centrifugal force → $F = m\omega_r^2$

$\theta = \omega_r t$

$y = e\sin\theta = e\sin(\omega_r t)$

mass - spring

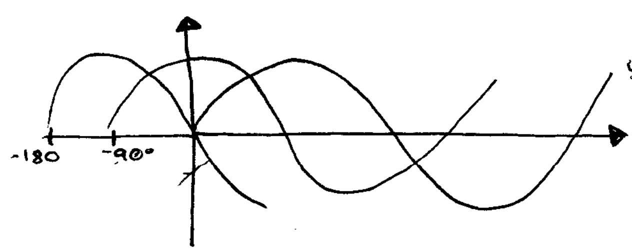


$\xi = 1/\sqrt{2} = 0.707$

Natural Freq: $\omega_n = \sqrt{K/m}$

(1) displacement : $y = e\sin(\omega_r t)$

(2) velocity : $V = \dot{y} = \frac{dy}{dt} = e\omega_r\cos(\omega_r t)$
$= e\omega_r\sin(\omega_r t + \pi/2)$



$y'(t) = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}$

non-causal

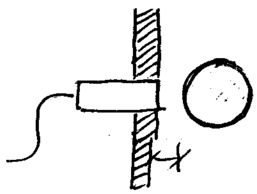
(3) acceleration : $a = \frac{dv}{dt} = e\omega_r^2\cos(\omega_r t + \pi/2)$
$= e\omega_r^2\sin(\omega_r t + \pi)$

- Impedence
  $\omega_n = \sqrt{K/m}$ harmonics

- Sensors measure response to the vibrating Forces.
- Oil Film

Displacement transducers (sensors)

magnetic disp. sensor



optical sensor

output ~ distance

- Velocity transducers

output ~ velocity of the vibrations

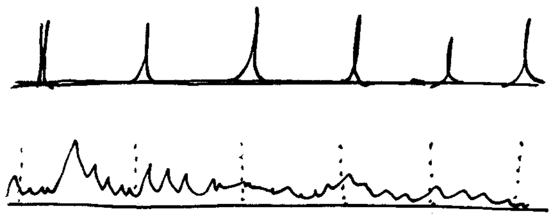
- accelerometer

output ~ acceleration

Gear signal vibration is periodic

T.S.A.

Signal average



Gear ratio = 1.2

Interpolation

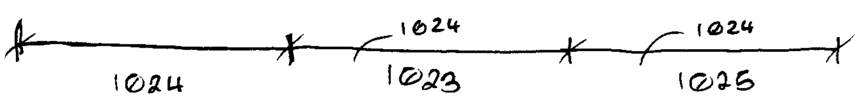
GR = 2

$Z_1 = 17$ 19, 21, 23 ...

- → The number of revolutions, R, can be determined according to noise reduction requirements.
- → due to shaft speed variations, the number of samples may not be equal from one revolution to another. Interpolation should be carried out to resample the data revolution by revolution.

$f_s = 20000$ Hz

20000 samples/sec



Interpolation

→ resample, m

5.3 Amplitude and Phase Demodulation

Z_1 = # of teeth

f_r = rotation speed

Mesh Freq.

$f_m = M Z_1 f_1$; where $M = 1, 2, 3, \ldots$

$$x[n] = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + \dots + A_K\cos(\omega_K t + \phi_K)$$
$$= \sum_{k=1}^{K} A_k \cos(\omega_k t + \phi_k)$$

A_k = amplitude corresponding to the k^{th} mesh freq.

ϕ_k = phase

$\omega_k = 2\pi f_k = 2\pi k \times z_1 f_1$

Signal average :

$$x[n] = \sum_{k=1}^{K} A_k \cos(2\pi k z_1 f_1 t + \phi_k) \qquad n = 1, 2, 3, \dots, N-1$$

$$x[n] = \sum_{k=1}^{K} (A_k + a_k)\cos(2\pi k z_1 f_1 t + \underbrace{a_k + \phi_k}_{\alpha_k})$$

Amplitude demodulation :

Phase ———

- Analytical signal

  Hilbert transform

  Add an imaginary part

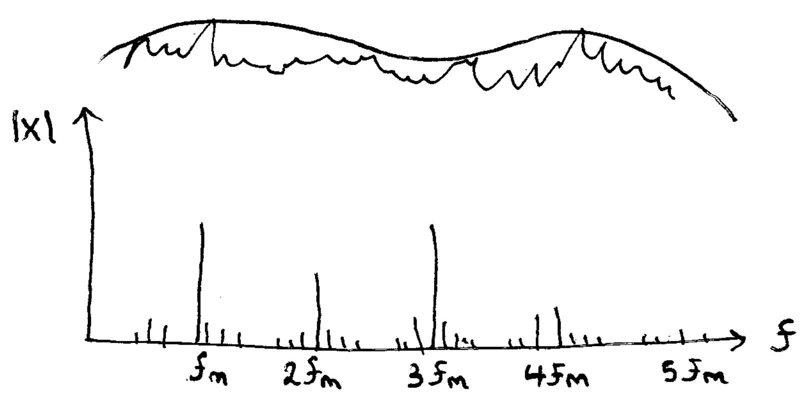
$$\underbrace{y[n]}_{\text{complex valued signal}} = \underbrace{x[n]}_{\text{real}} - \underbrace{jH(x[n])}_{\text{imag. part}}$$

$$y_{amp}[n] = \sqrt{Re^2[y[n]] + Im^2[y[n]]}$$

Phase : $$y_{phase}[n] = \arctan\left(\frac{Im(y[n])}{Re(y[n])}\right)$$

amplitude modulation

⟶ envelope



- Overall residual signal

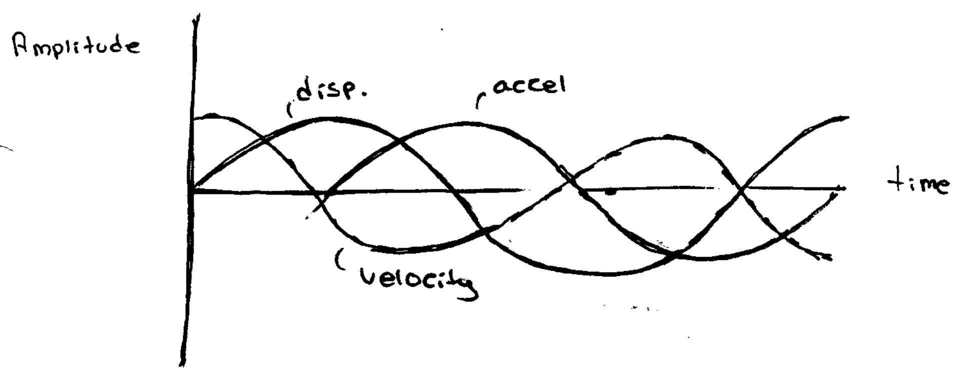
multiple bandstop filter to remove gear mesh Freq. & it's harmonics
$X[n]$ ~ signal average (TSA)

Analytical signal :
$y_o[n] = x_o[n] - jH(x_o[n])$

amplitude modulation :

$$y_{o,amp} = \sqrt{Re^2[y_o] + Im^2[y_o]}$$

$$y_{o,phase} = \arctan\left(\frac{Im[y_o]}{Re[y_o]}\right)$$



$V = 90°(d)$
$a \sim 90°(v)$
$= \sim 180°(d)$

Displacement Transducers

a) Displacement transducers measure relative motion between the shaft and the output

b) Typically, the actual useful frequency range of proximity probes is up to 500 Hz

c) Shaft surface scratches, out-of-roundness, and variation in electrical properties will produce signal errors. Surface treatment and run-out subtraction can be used to solve these problems.

disp. ~ 500 Hz

Velocity (~ 2000 Hz)

accelerometer

$$\omega_n = \sqrt{K/m}$$

Piezoelectric accelerometer

charge → $F = ma$

Selecting the right transducer for an application:

a) The parameter of interest
- disp x
- velocity $v = \dot{x}$
- acceleration $a = \dot{v} = \ddot{x}$

b) Mechanical impedence considerations

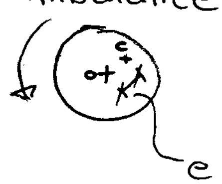
c) Frequency considerations.

If the freq. of the vibration is $> 1000\,Hz$, you must use accelerometer. If $10 \rightarrow 1000\,Hz$, either velocity or acceleration transducers can be used.

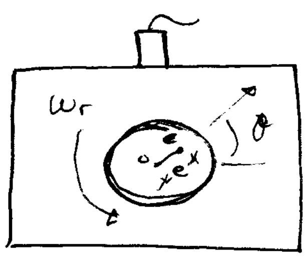
Bearings & Gears
↳ most common source of defects (rolling element)

5.5 Fault Diagnosis in other Machinery Systems

i) Imbalance



o : centre of rotation
c : centre of mass
e : difference between the two.



$F = m\omega_r^2 r$

$F_y = F\sin\theta = m\omega_r^2 \sin(\omega_r t)$

once per revolution

Characteristics of imbalance :

→ it is Sinusoidal at a Freq. of once per revolution

→ it is a rotating vector

→ its amplitude increases with speed because the imbalance force

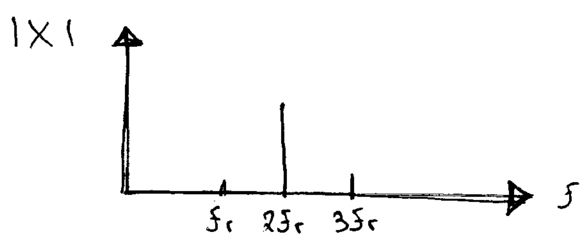
→ phase plays a key role in detecting & analyzing imbalance

2) Misalignment

Characteristics of Misalignment

→ It is usually characterized by a $2 \times f_r$ Frequency component, with large number of harmonics.

→ It has high axial vibration levels



→ the ratio of $1 \times f_r$ to $2 \times f_r$ component levels can be used as an indicator of misalignment severity.

3) Resonance

Force Frequencies or it's harmonics $\approx$ structure natural freq.

Natural Freq. in machinery vibration analysis

1) resonance of the structure can cause changes in vibration level

2) the dynamics of rotating shafts change significantly near natural Freq.

3) resonance of transducers will limit the operating frequency range of measurement.