Example two-pole butterworth w/ transfer Fxn:

$$H(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\,\omega_c s + \omega_c^2} \quad ; \quad \text{with } \omega_c = 2 \text{ rad/s}, \; T = 0.2$$

→ If $\omega_c = 2$ rad/s

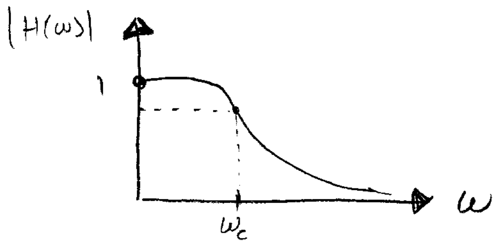
↳ $\omega_p = 2.027$ rad/s

$$H_d(z) = \frac{0.0309\,(z^2 + 2z + 1)}{z^2 - 1.444z + 0.5682}$$

Example

$$x = 1 + \underbrace{\cos(t)}_{\omega = 1 \text{ (rad/s)},\; f = 1/2\pi \text{ (Hz)}} + \underbrace{\cos(5t)}_{\omega = 5 \text{ (rad/s)},\; f = 5/2\pi \text{ (Hz)}}$$

Remove $\cos(5t)$ using 2-pole Butterworth



Choose $\omega_c = 2$ rad/s

$f_{max} = 5/2\pi$

$f_s = 2 f_{max} = 5/\pi \approx 1.59$ Hz

$f_s = 5$ Hz , $T = 1/f_s = 0.2$

In MATLAB ; Filter
butter

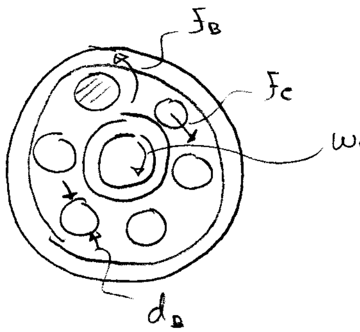
5.4 Fault Detection in Rolling Element Bearing

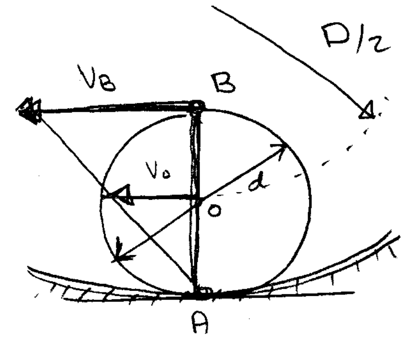
Bearings → main cause of defects in rotating machinery
Small / medium size machines (75 %)

Types of defects :

- distributed defects (wear, ...)
- localized

bearing materials are subjected to dynamic loading





$$V_B = \omega_r\left(\tfrac{D}{2} - d/2\right)$$

$$V_o = \frac{V_B}{2} = \frac{\omega_r}{2}\left(\frac{D}{2} - \frac{d}{2}\right)$$

cage speed

$$\text{Cage Frequency} = \frac{V_o}{D/2} = \frac{\omega_r}{D}\left(\frac{D}{d} - \frac{d}{2}\right)$$

Cage Freq.

$$f_c = \frac{f_r}{D}\left(\frac{D}{2} - \frac{d}{2}\right) = \frac{f_r}{D}\cdot\frac{D}{2}\left(1 - \frac{d}{D}\right)$$

Ball rotating freq.

$$\omega_B = \frac{V_B}{d} = \frac{\omega_r}{\cancel{2}d}\left(\frac{D}{2} - \frac{d}{2}\right)$$

$$f_b = \frac{f_r}{\cancel{2}d}\left(\frac{D}{2} - \frac{d}{2}\right)$$

Cage $\quad f_c = \dfrac{f_r}{2}\left(1 - d/D \cos\alpha\right)$

Ball (rotating) $\quad f_b = \dfrac{f_r D}{2d}\left(1 - \dfrac{d^2}{D^2}\cos^2\alpha\right)$

Outer race: $\quad f_{od} = \dfrac{Z f_r D}{2d}\left(1 - \dfrac{d}{D}\cos\alpha\right)$

Inner race : $\quad f_{id} = \dfrac{Z f}{2}\left(1 + \dfrac{d}{D}\cos\alpha\right)$

Inner race defect :

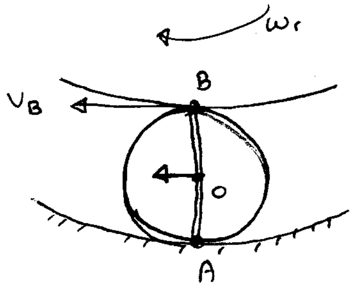
$$\omega_{id} = Z(\omega_r - \omega_c)$$

$$f_{id} = Z(f_r - f_c)$$

Rolling element damage

$$\omega_{rd} = 2\omega_b$$

$$f_{rd} = 2f_b = \frac{f_r D}{d}\left(1 - \frac{d^2}{D^2}\cos^2\alpha\right)$$



$$V_B = \omega_r\left(\frac{D}{2} - \frac{d}{2}\right)$$

$$V_o = \frac{V_B}{2} = \frac{\omega_r}{2}\left(\frac{D}{d} - \frac{d}{2}\right)$$

$$\omega_c = \frac{V_o}{D/2} = \frac{\omega_r}{2}\left(1 - \frac{d}{D}\right)$$

Cage: $$f_c = \frac{f_r}{2}\left(1 - \frac{d}{D}\cos\alpha\right)$$

Rolling Element: $$f_b = \frac{D f_r}{2d}\left(1 - \frac{d^2}{D^2}\cos^2\alpha\right)$$

Outer race: $$\omega_{od} = Z\omega_c$$

$$f_{od} = \frac{Z f_r}{2}\left(1 - \frac{d}{D}\cos\alpha\right)$$

Inner race defect: $$\omega_{id} = Z(\omega_r - \omega_c)$$

$$= Z\left[\omega_r - \frac{\omega_r}{2}\left(1 - \frac{d}{D}\cos\alpha\right)\right]$$

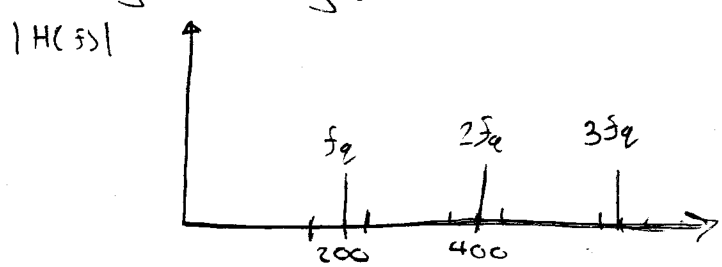
$$= \frac{Z\omega_r}{2}\left[2 - 1 + \frac{d}{D}\cos\alpha\right]$$

$$f_{id} = \frac{Z f_r}{2}\left(1 + \frac{d}{D}\cos\alpha\right)$$

Rolling element $\omega_{od} = 2\omega_b$

$$f_{od} = 2 f_b = \frac{D f_r}{d}\left(1 - \frac{d^2}{D^2}\cos^2\alpha\right)$$

Healthy Bearing : f_r



$f_{od} = 30$ Hz

5.3 Gear System Monitoring

1) Damage :

dynamic loading : Fatigue

contact Force : pitting

tensile : breakage

dynamics , stiffness

Gear signal is periodic

2) Time syncronous Filtering