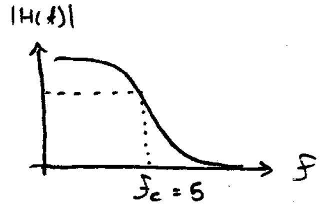
- Corresponding LPF (BW, CV-1 order) - H(s)
  - $\omega_c = 1$ rad/s
  - Get H(s)
  - Transform H(s) $\longrightarrow$ desired Filter

    Frequency transform

$\boxed{\text{Example 4.2}}$ Design a three-pole Butterworth low-pass Filter with a bandwidth of 5 Hz.

<u>Solution</u> :

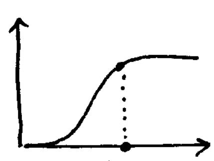


Given: $$H(s) = \frac{(s-z_1)}{(s-p_1)(s-p_2)(s-p_3)}$$

$$H(s) = \frac{2s+1}{s^3+2s^2+s+4}$$ ∘ ∘ ∘ MATLAB

$\boxed{\text{Example 4.3}}$ Design a 3-pole high-pass Filter with cutoff Frequency $\omega = 4$ Hz

<u>Solution</u> :



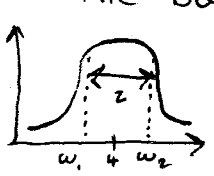
$\omega_c =$

$f_c = 4$ Hz

$\omega_c = 2\pi \times 4$ (rad/s)

$\boxed{\text{Example 4.4}}$ In example 4.2, a three-pole Butterworth lowpass Filter was transformed to a bandpass Filter with the passband centered at $\omega = 4$ Hz. The bandwidth is equal to 2 Hz.

<u>Solution</u> :



4.3 - Design of Digital Filters

1) Digital Filter

$$x(t) \longrightarrow \boxed{H(s)} \longrightarrow y(t)$$

$$x[n] \longrightarrow \underbrace{\boxed{H_d(z)}}_{\text{digital}} \longrightarrow y[n]$$

- DTFT. (discrete time Fourier transform)

$$X(\Omega) = \sum_{n=0}^{N-1} x[n] e^{-j\Omega n} \quad ; \quad \text{where} \quad -\pi \le \Omega \le \pi$$

- DFT (discrete Fourier transform)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi k n}{N}\right)} \quad ; \quad \text{where} \quad \Omega = \frac{2\pi k}{N} \sim 2\pi f\left(\frac{1}{N}\right) \sim \omega$$

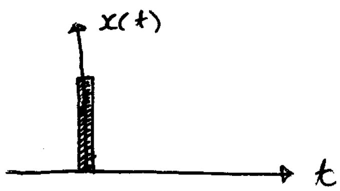
$$\Omega = \omega T$$

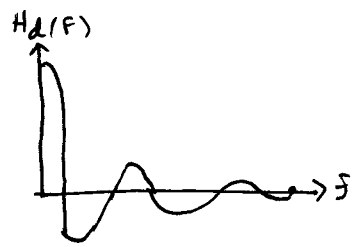
- ZT (z-transform)

$$X(z) = \sum_{n=0}^{N-1} x[n] z^{-n} \quad ; \quad \text{where} \quad z = e^{j\Omega} = e^{j\omega T} = e^{sT}$$

$$z = e^{sT}$$

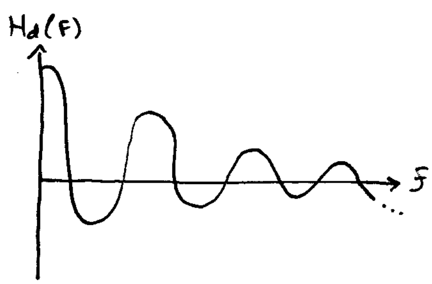
- Input impulse





Finite number of steps

FIR : Finite impulse response fi...



infinite number of steps

IIR : infinite input response

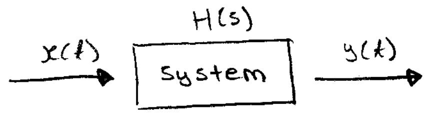
2) Design of IIR Filters

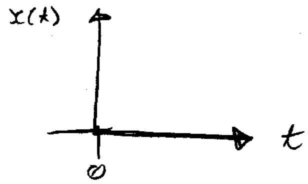
- analog filter
- digitization , $H_d(z)$

$$z = e^{sT}$$

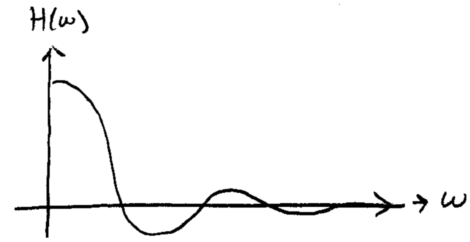
$$\ln z = sT$$

$$s = \frac{1}{T} \ln(z)$$

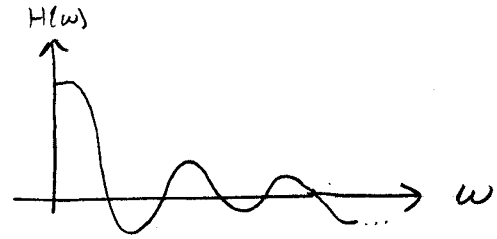




$\boxed{\text{FIR}}$



$\boxed{\text{IIR}}$



Design of IIR Filters
- analog filter prototype
  $H(s)$, $H(\omega)$
- transform analog prototype
  $\longrightarrow$ digital filter : $H_d(z)$

$S = \frac{1}{T} \ln(z)$

T = time data sample interval $1/f_s$

Taylor : $\ln z = 2\left(z + z^3/3 + z^5/5 + \dots\right)$

Bilinear transformation

$S = \frac{1}{T}\ln(z) \approx \left(\frac{2}{T}\right)\left(\frac{z-1}{z+1}\right)$

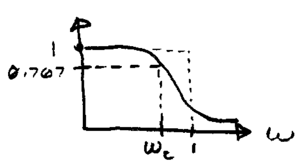
$H_d(z) = H(z) = H\left(\frac{2}{T}\,\frac{z-1}{z+1}\right)$

$H(s) \sim h(t)$ (approximate)

3) Warping Errors

$H_d(z)$

LPF with cutoff freq. ω_c



Digital Filter

$\Omega_c = \omega_c T$

$H_d(z)$ has cutoff freq.

$\Omega_c = 2\tan^{-1}\left(\omega_c T/2\right) \neq \omega_c T$ $\sim$ warping error

approx. bilinear transformation

gay

pre-warping

$\Omega_c = 2\tan^{-1}(W_cT/2)$

$\Omega_c/2 = \tan^{-1}(W_cT/2)$

$\tan(\Omega_c/2) = (W_cT/2)$

$W_c = (2/T)\tan(\Omega_c/2)$

cut-off freq. of analog prototype

$W_p - (2/T)\tan(\Omega_c/2) = (2/T)\tan(W_cT/2)$

to replace $W_c \rightarrow \Omega_c = W_cT$

$\boxed{\text{Example } 4.8}$ Consider the two-pole Butterworth Filter with transfer Function:

$$H(s) = \frac{W_c^2}{s^2 + \sqrt{2}W_c s + W_c^2}$$

Filter with $W_c = 2$, and $T = 0.2$

$\rightarrow f_s = 1/T = 5$ Hz

digital Filter

$$H_d(z) = H(s)\big|_{s=(2/T)(z-1/z+1)}$$

$$= \frac{W_c^2}{((2/T)(z-1/z+1))^2 + \sqrt{2}W_c((2/T)(\frac{z-1}{z+1})) + W_c^2}$$

$$= \frac{0.0302z^2 + 0.0605z + 0.0302}{z^2 - 1.4514z + 0.5724}$$

$\Omega_c = (2/T)\tan^{-1}(W_cT/2) = 0.3948$

Desired: $\Omega_c = W_cT = 2\times 0.2 = 0.4$

$W_p = (2/T)\tan(\Omega_c/2) = (2/0.2)\tan(0.4/2)$

$= 2.027$ rad/s

$W_p \rightarrow W_c$ (prewarping)

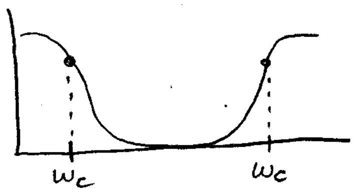
$$H_d = \frac{0.0309(z^2 + 2z + 1)}{z^2 - 1.444z + 0.5682}$$

$\Omega_c = W_cT = 0.4$

Solution $\quad H_d(z) = \Omega \quad (-\pi \sim \pi)$

Design of IIR Filters in MATLAB

- bilinear

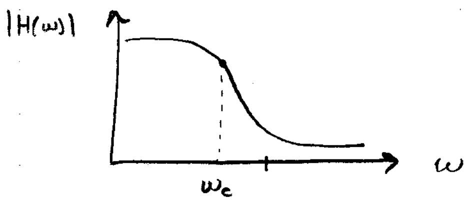


- butter (prototype, Freq. transformation, pre-warping)
- cheby ($\Omega_c = \omega_c T / \pi$
- Filter

Solution

$$x(t) = 1 + \cos t + \cos(5t)$$

$\omega = 1$ $\qquad$ $\omega = 5$



$$1 < \omega_c < 5$$