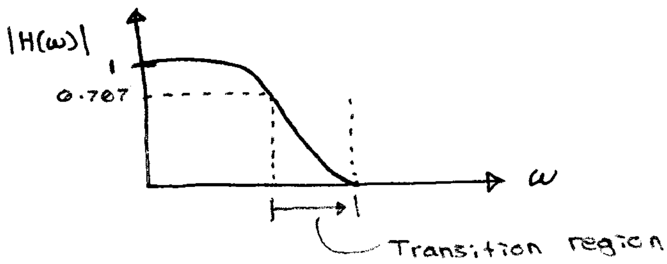
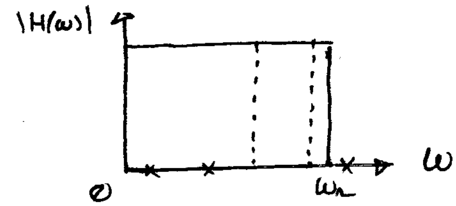


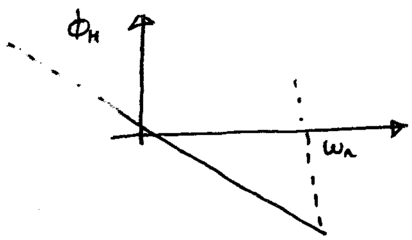
Def'n of passband of a lowpass filter.

Ideal Filter (LPF)



Freq. domain

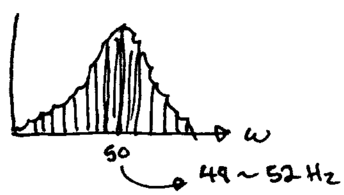
time domain

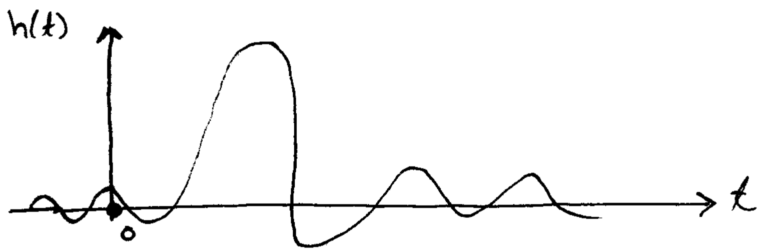


$\phi_H = -\omega t_d$

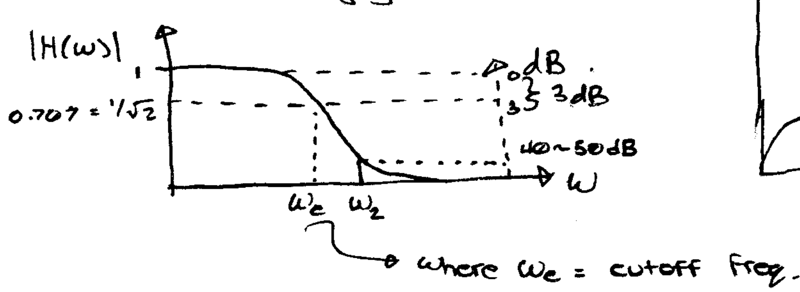
Input : $x(t) = A\cos(\omega_0 t)$

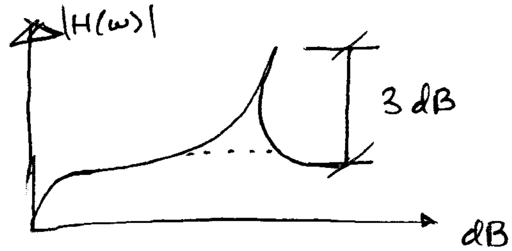
Output : $y(t) = A|H(\omega)| * \cos(\omega_0 t + \phi_H)$

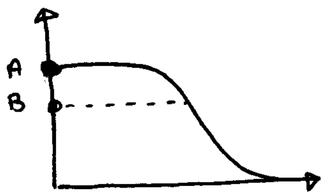




no initial energy







$$3\,dB = 20\log_{10}(A/B)$$

$B = A/\sqrt{2}$

($or \quad B = 0.707A$)

(2) Butterworth Filter

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ζ = damping ratio (internal impedance)

$s = j\omega$

$$H(\omega) = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2}$$

Let $\omega_n = 1$

Poles : $-\omega_n/\sqrt{2} + j\omega_n/\sqrt{2}$

Zeroes : none

$$H(\omega) = \frac{\omega_n^2[\qquad]}{[(\omega_n^2-\omega^2) + j(2\zeta\omega_n\omega)][(\omega_n^2-\omega^2) - j\omega]}$$

$$= Re + jIm$$

$$|H(\omega)| = \sqrt{Re^2 + Im^2} = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2) + 4\zeta^2\omega_n^2\omega}}$$

$\zeta = 0 \sim 1$

$\zeta = 1/\sqrt{2} = 0.707 \approx 0.7$

$\Rightarrow$ PB Maximal flat

(Butterworth) BW

$$|H(\omega)| = \frac{\omega_n^2}{\sqrt{(\omega^2-\omega_n^2)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

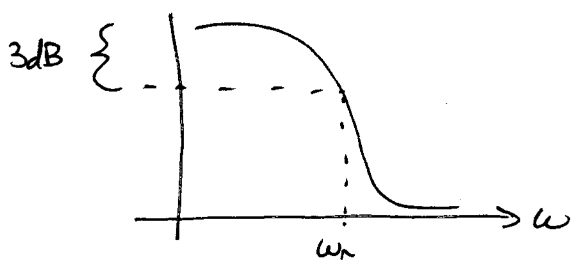
$$= \frac{\omega_n^2}{\sqrt{(\omega^2-\omega_n^2)^2 + 2\omega_n^2\omega^2}} \longrightarrow |H(\omega)| = \frac{1}{\sqrt{\frac{(\omega^2-\omega_n^2)^2}{\omega_n^4} + \frac{2\omega_n^2\omega}{\omega_n^4}}}$$

then $|H(\omega)| = \dfrac{1}{\sqrt{1+(\omega/\omega_n)^4}}$

If $\omega = \omega_n$

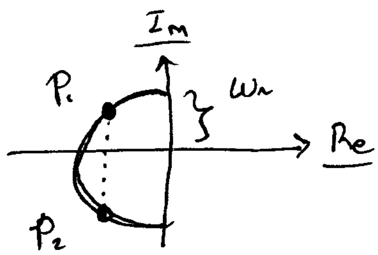
$|H(\omega)| = 1/\sqrt{2} \sim 3\,dB$

ω_n = cutoff freq. of LP BW Filter



$$-1/\sqrt{2}\,\omega_n \pm j\,1/\sqrt{2}\,\omega_n$$

$$|P_{1,2}| = \sqrt{(-1/\sqrt{2}\,\omega_n)^2 + (\omega_n/\sqrt{2})^2} = \omega_n$$



two poles

B.W. Filters

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

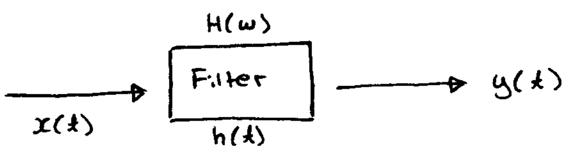
$$P_{1,2} = \frac{-\omega_n}{\sqrt{2}} \pm j\frac{\omega_n}{\sqrt{2}}$$

$s = j\omega$

$$|H(\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

$\omega = 0 \longrightarrow 10\,\omega_n$

ζ = damping ratio



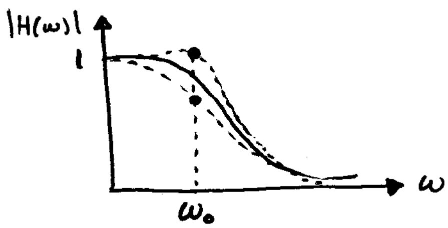
$A\cos(\omega_0 t)$

$y(t) = A\,|H(\omega)|\cos(\omega_0 t + \Phi_H)$

$\zeta = (1/\sqrt{2}) \approx 0.7$

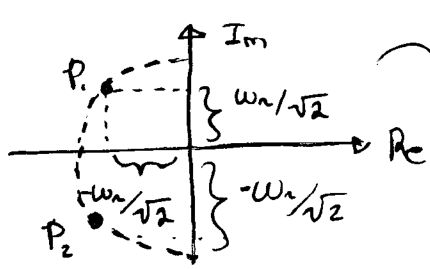
P.B. ~ maximal Flat

Butterworth



$$|P_1| = \sqrt{(-\omega_n/\sqrt{2})^2 + (\omega_n/\sqrt{2})^2} = \omega_n$$

$$|P_2| = \qquad + \qquad = \omega_n$$



semi circle w/ radium ω_n

If $\zeta = 1/\sqrt{2}$

$$|H(\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 2\omega_n^2\omega^2}}$$

$$= \frac{1}{\sqrt{1 + (\omega/\omega_n)^4}}$$

If $\omega = \omega_n$

$|H(\omega)| = 1/\sqrt{2}$, ω_n = cutoff Freq.

3) N^{th} order BW Filters

3^{rd} order BW:

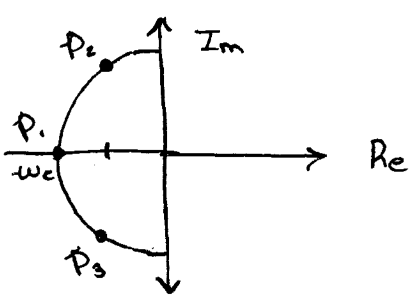
$$H(s) = \frac{\omega_c^3}{(s+\omega_c)(s^2+\omega_c s+\omega_c^2)}$$

no zeroes

$P_1 = -\omega_c$

$P_{2,3} = -\omega_c/2 \pm j\sqrt{3}/2\ \omega_c$

$|P_2| = \sqrt{(-\omega_c/2)^2 + (\sqrt{3}/2\,\omega_c)^2} = \omega_c$



$s = j\omega$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_n)^{2\times 3}}}$$

N-pole BW Filter

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_n)^{2N}}}$$

ω_n = cutoff Freq.

MATLAB:

$\omega_n = 1$ rad/s

buttap()

→ Running BW-1

$a = [1 \quad 1.4142 \quad 1]$

$b = [0 \quad 0 \quad 1]$

$$H(s) = \frac{0s^2 + 0s + 1}{s^2 + 1.4142 s + 1}$$

; H = gain

op. amp

(*) Assignment 4 due next Tuesday (Nov. 12)

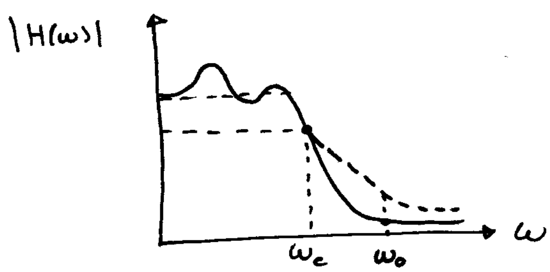
(4) Chebyshev Filters

BW : monotonic Fxn
transition bend is wide

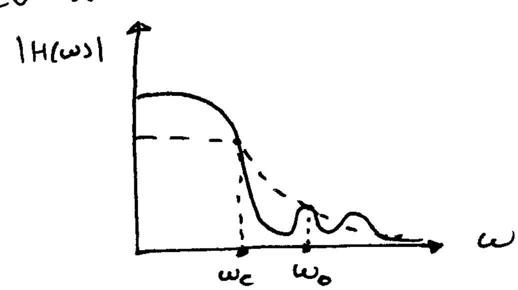
CV : ~ not monotonic
narrow transition

CV - type I filter

CV - 1



CV - 2



N - Pole CV-1

$$|H(\omega)| = \frac{1}{\sqrt{1 + \xi^2 T_N(\omega/\omega_1)}}$$

$T_N(x) = 2x\,T_{N-1} - T_{N-2}(x)$

$T_0 = 1 \;, \; T_1(x) = x$

$T_2 = 2x\,T_1 - T_0$

$= 2x^2 - 1 \quad \ldots$

$$T_3 = 2xT_2 - T_1$$
$$= 2x(2x^2 - 1) - x = 4x^3 - 3x$$

$$T_4 = 2xT_3 - T_2$$
$$= 2x(4x^3 - 3x) - (2x^2 - 1)$$
$$= 8x^4 - 8x^2 + 1$$

$N = 2$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 T^2}} = \frac{1}{\sqrt{1 + \varepsilon^2 [2(\omega/\omega_1)^2 - 1]^2}}$$

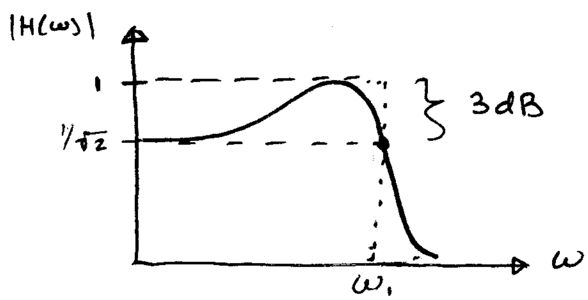
$\omega = 0 \sim \omega_1$

$$|H(\omega)| = \begin{cases} \dfrac{1}{\sqrt{1+\varepsilon^2}} & : \quad \omega = 0 \\ \dfrac{1}{\sqrt{1+\varepsilon^2}} & : \quad \omega = \omega_1 \\ 1 & : \quad \text{if } (\omega/\omega_1)^2 = 1/2 \end{cases}$$

If $\varepsilon = 1$; if $\omega = \omega_1$

if $\omega = 0$, $|H(\omega)| = 1/\sqrt{2}$

if $\omega = \omega_1$, $|H(\omega)| = 1/\sqrt{2}$



ω_1 = cutoff freq.

Nov. 7/19

Lab on Monday: (Monday @ 10:30)

Given 3 vectors $\left\{ \begin{array}{l} \text{healthy bearing} \quad (f_N) \\ \text{outer raceway damage} \quad (f_{od}) \\ \text{inner raceway damage} \quad (f_{id}) \end{array} \right.$

→ FFT (amplitude) → windowing function (hanning, hamming)

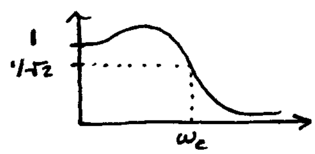
→ Kurtosis

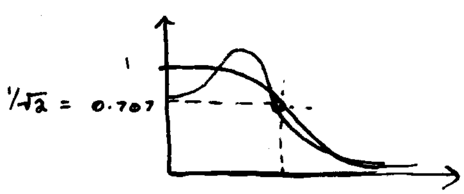
→ Characteristic Freq. (f_N)

4) CV Filter

CV-1 ~ ripples in the pass band

CV-2 ~ ripples in the stop band





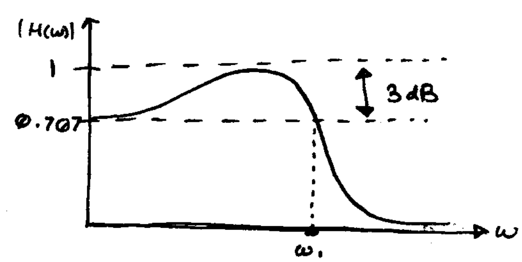
$$|H(\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 T_N^2(\omega/\omega_1)}}$$

$T_0 = 1$, $T_1 = x$

$N = 2$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 [2(\omega/\omega_1)^2 - 1]^2}}$$

$$|H(\omega)| \left\{ \begin{array}{lll} 1/\sqrt{2} & ; & \omega = 0 \\ 1/\sqrt{2} & ; & \omega = \omega_1 \\ 1 & ; & 2(\omega/\omega_1)^2 - 1 = \pm 1 \end{array} \right. \Rightarrow$$



If $\varepsilon = 1$; 3 dB ripple in pass band.

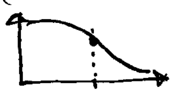
$$H(s) = \frac{0.251\,\omega_1^3}{s^3 + 0.597\omega_1 s^2 + 0.928\omega_1^2 s + 0.251\omega_1^2}$$

$$H(s) = \frac{(s - z_1)}{(s - p_1)(s - p_2)(s - p_3)}$$

LP $\omega_c = 1$ rad/s

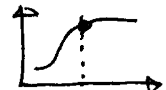
5) Freq. transformation
LP, (BW, CV ~ 3dB)
with $H(s)$, and cutoff frequency ω_1

→ LP with cutoff freq of ω_2
$(s\,\omega_1/\omega_2)$



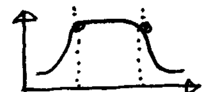
ω_2

HP with cutoff Freq of ω_2
$(\omega_1\omega_2/s)$



ω_2

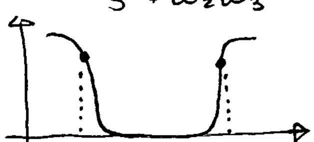
BP with BP of $\omega_2 \sim \omega_3$
$\omega_1\left(\frac{s^2+\omega_2\omega_3}{s(\omega_3-\omega_2)}\right)$



ω_2 ω_3

→ BS with SB $\omega_2 - \omega_3$

$$\omega_1 \frac{s(\omega_3-\omega_2)}{s^2+\omega_2\omega_3}$$



ω_2 ω_3

$\boxed{\text{Example 2}}$

3-pole BW Filter

$$H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3}$$

ω_c = cutoff freq., $\omega_1 = \omega_c$

BP : $\omega_2 = 3$, $\omega_3 = 5$ rad/s

$$s = \omega_c \frac{s^2 + 3\times 5}{s(5-3)} = \omega_c\left(\frac{s^2+15}{2s}\right)$$

$$H(s) = \frac{\omega_c^3}{\left(\omega_c \frac{s^2+15}{2s}\right)^3 + 2\omega_c\left(\omega_c\frac{s^2+15}{2s}\right)^2 + 2\omega_c^2\left[\omega_c\frac{s^2+15}{2s}\right] + \omega_c^3}$$

$$= \frac{8s}{s^6 + 4s^5 + 53s^4 + 128s^3 + 795s^2 + 900s + 3375}$$

$\boxed{\text{Example 4.2}}$