Chapter 2 – Macro-Mechanical Analysis of a Lamina

2.1 Introduction

A lamina is typically in the order of $0.127\ mm$ or $0.005''$ thick and not isotropic

2.2 Review

1. Stress
2. Strain
- Normal
- Shear
3. Elastic moduli
4. Strain energy

2.3 Hooke's Law for Different Types of Materials

1. <u>Homogeneous</u> vs. heterogeneous materials

FGM – functionally graded materials (properties at different locations are different)

2. Anisotropic vs. isotropic materials

<u>Isotropic</u> materials are those whose properties are orientation independent.

For example, steel, aluminum.

They require 2 <u>independent</u> mechanical property constants. (Young's modulus, poisons ratio or Young's modulus, shear modulus)

<u>Anisotropic</u> materials are those whose properties are orientation dependent.

For example, natural wood.

They require 21 independent constants, see (Eq. 2.25 – only looking at triangular matrix, it's symmetric)

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}, \qquad (2.25)$$

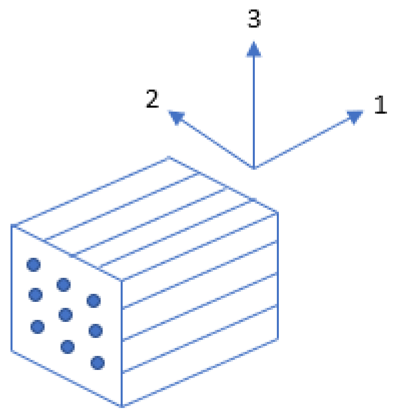
3.Special Cases of Anisotropic Materials

3.1 Orthotropic Materials

They possess 3 mutually perpendicular principal planes, or 3 principal directions mutually perpendicular to each other.

For example, unidirectional continuous fiber-reinforced composite blocks are considered orthotropic.

Orthotropic materials require 9 independent constants.



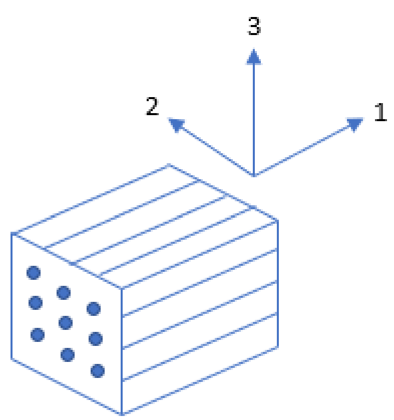
(3 axes and planes – first plane in direction of fiber)
1 – direction of fibers
2, 3 – perpendicular to fibers

E_1 E_2 E_3: Young's moduli
G_{12} G_{23} G_{13}: Shear moduli

Note:
$G_{ij} = G_{ji}$
But:
$v_{ij} \neq v_{ji}$ (but they are related)
Since:
$\frac{v_{ij}}{E_i} = \frac{v_{ji}}{E_j}$

3.2 Transversely isotropic materials
An orthotropic material is called transversely isotropic if one of its principal planes is a plane of isotropy that is, material properties are symmetric about one of its principal axes.

For example,
- Carbon/graphite and aramid fibers are transversely isotropic.
- Unidirectional continuous fiber-reinforced composites, when fibers are packed in a hexagonal way or very close to it, can be considered transversely isotropic.



Transversely isotropic materials require 5 independent constants.

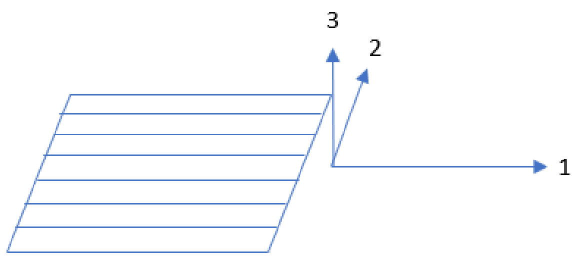
Axis 1 being the axis of symmetry
Axis 2, 3 being the plane of isotropy

E_1 $E_2 = E_3$: Young's moduli
$G_{12} = G_{31}$ G_{23} : Shear moduli
$v_{12} = v_{13}$ v_{23} : Poisson's ratio

Also, E_2 G_{23} v_{23} are related.

3. Plane stress vs. Plane Strain
Thin unidirectional continuous fiber-reinforced composite layers become a plane stress case.

4 independent mechanical properties are required.



E_1 E_2 G_{12} v_{12}

Chapter 3 Micromechanical Analysis of a Lamina
7 Sections in total (pp. 203-314)
3.1 Introduction
3.2 Volume and Mass Fractions, Density, and Void Content
3.3 Evaluation of Elastic Moduli (pp. 215-270)
3.4 Ultimate Strengths (pp. 271-295)
3.5 CTE
3.6 CME
3.7 Accuracy

3.1 Introduction
1. Determination of the 4 elastic moduli (3.3), 5 ultimate strengths (3.4), and 4 transport properties (3.5, 3.6) by experimental means is costly; experimental results are also limited or restricted.

2. Transport properties include, CTE in 1- and 3- directions, and CME in 1- and 2- directions, respectively.

3. Unidirectional lamina is NOT homogenous. However, it's customary to assume homogeneity once micro-analysis is completed.

4. Lamina is the building block of composites. This is true from analysis as well as physical perspectives.

3.2 Volume and Mass Fractions, Density, and Void Content
- to quantify how much fibers and matrix there are;

- by volume fractions, or by mass (or weight) fractions;

- volume fractions and mass/weight fractions are related;

- void content can't be avoided in reality, but complicates analysis;

1. Volume Fractions
Consider the case of composite having one type of fibers, one type of matrix, and some voids.

$v_{c,f,m,v}$: Volumes of composite, fibers, matrix and voids respectively (eg. in^3).

Then volume fraction of fibers is:
$V_f = v_f / v_c$

Then volume fraction of matrix is:
$V_m = v_m / v_c$

Volume fraction of voids is:
$V_v = v_v / v_c$

$\therefore V_f + V_m + V_v = 1$

(Thus, $V_f + V_m = 1$ implies zero void content.)

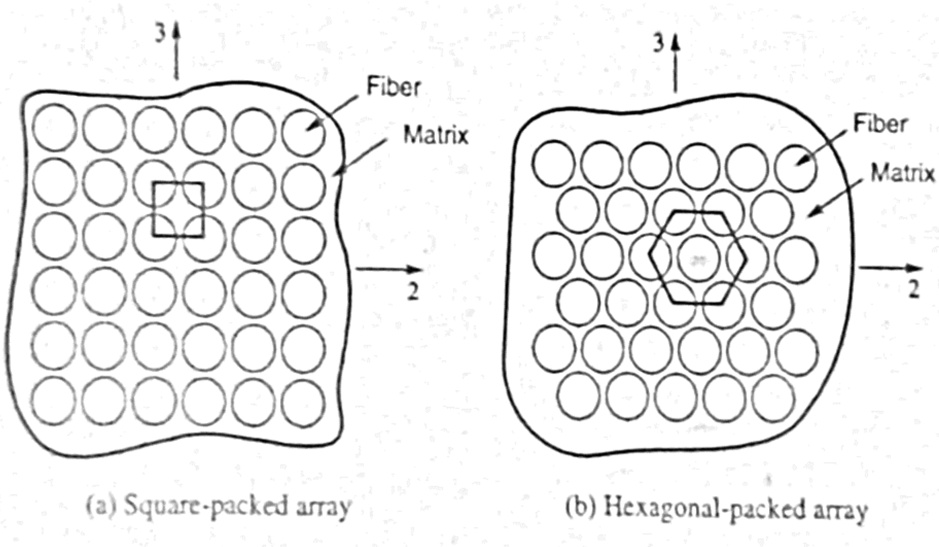
2. Limits on V_f
V_f has some upper limit, from the theoretical as well as the practical perspectives.

2.1 Theoretical Limit on v_F or v_{fmax} to be examined by RVE
RVE = Representative Volume Element

Used in determining v_{fmax} as well as elastic moduli and so on (see 3.3)

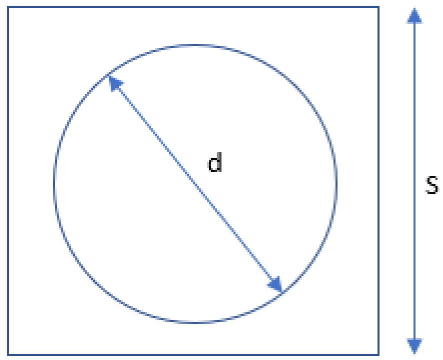
RVE is the smallest portion of material that is representative of the composite as a whole.

Here, "smallest" means smallest for analysis; it may or may not fit the strict mathematical definition.



Cross section idealizations for micromechanics studies

For (a), smallest is 1/8 (half of 1/4
For (b), smallest is 1/12 (half of 1/6)



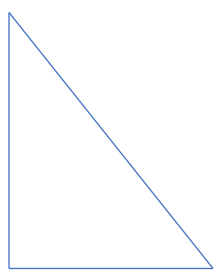
S
d
$l = length/depth\ of\ RVE$

$$v_c = S^2 l$$

$$v_f = \frac{\pi d^2}{4} l$$

$$v_f = \frac{v_f}{v_c} = \frac{\frac{\pi d^2}{4} l}{S^2 l} = \frac{\pi d^2}{4S^2}$$

$$\therefore v_{fmax} = v_f\big|_{s=d} = \frac{\pi}{4} = 0.785$$



For hexagonal packed:

$$v_f = \frac{\pi d^3}{2\sqrt{3} S^2}$$

$$v_{fmax} = 0.907$$

2.2 Practical Limit

For fiber-reinforced lamina with long and unidirectional fibers, $v_{fmax} = 60\%$

Elementary Materials Science, W.F. Hosford, ASM International, 2013 (Ch. 10, p. 117)

3. Density of the Composite

$\rho_c\ f\ m$: mass densitites (per unit volume) of composite, fibers, and matrix, respectively

$w_c\ f\ m$: masses of composite, fibers, and matrix, respectively

$\therefore w_c = \rho_c v_c \quad w_f = \rho_f v_f \quad w_m = \rho_m v_m$

And $\rho_c v_c = \rho_f v_f + \rho_m v_m$

$\therefore\ \rho_c = \rho_f V_f + \rho_m V_m$: works if there is void content

$V_f + V_m = 1$: works if there is ZERO void content

4. Mass Fractions

mass fraction of fibers is $W_f = w_f / w_c$

mass fraction of matrix is $W_m = w_m / w_c$

$\therefore W_f + W_m = 1$

5. Volume Fractions and Mass Fractions

These fractions are related

$\therefore w_c = \rho_c v_c \quad w_f = \rho_f v_f \quad w_m = \rho_m v_m$

$$\therefore W_f = \frac{w_f}{w_c} = \frac{\rho_f v_f}{\rho_c v_c} = \frac{\rho_f}{\rho_c} V_f$$

$$= \frac{\rho_f V_f}{\rho_f V_f + \rho_m V_m} \quad (Eq. 1)$$

And:

$$W_m = \frac{\rho_m V_m}{\rho_f V_f + \rho_m V_m} \quad (Eq. 2)$$

a) Knowing $V_m\ V_f\ \rho_m\ \rho_f$
   Then $W_f\ W_m$ solved by (Eq. 1 and Eq. 2)

b) Knowing $W_m\ W_f\ \rho_m\ \rho_f$
   Then $V_m\ V_f$ solved by (Eq. 1 or Eq. 2)

c) How to determine V_v or v_v?
   mostly by experiments

1) Some specifications for determining void contents

For example:

ASTM D3171-06

Standard Test Method for Constituent Content of Composite Materials

ISO-14127:2008

Composites – Determination of resin, fiber and void content of composites reinforces with carbon fiber.

2) What the text has in terms of determining void content?

pp. 212-215

By means of theoretical density of composite ρ_{ct} and experimental density of composite ρ_{ce} leading to (Eq. 3.16)

$$V_v = \frac{v_v}{v_c}$$
$$= \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}. \quad (3.16)$$

Example 3.2, which in essence is ASTM-D3171

d) Equations in 3.2 of the text (pp. 204-215)
- Some equations are valid only for the case of zero void content
- Condition under which an equation is valid isn't spelled out
- Equations (3.5a), (3.5b), (3.10)

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f,$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m. \quad (3.5a, b)$$

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}. \quad (3.10)$$

e) Corrected volume fractions

In later sections of chapter 3, V_v is assumed to be zero.
When V_v is NOT zero, corrected volume fractions are to be used
$V_f' = \dfrac{v_f}{v_f + v_m}$; $V_m' = \dfrac{v_m}{v_f + v_m}$

3.3 Evaluation of Elastic Moduli
Recalling from 2.3
Orthotropic materials: 9 independent constants
E_1 E_2 E_3: Young's moduli in the 1, 2, and 3 directions, respectively.
G_{12} G_{23} G_{13}: Shear moduli on the 1-2, 2-3, and 3-1 planes respectively.
$v_{12} v_{23}$ v_{31}: Poisson's ratio
1st subscript: strain in the loading direction
2nd subscript: lateral strain

Note:
$G_{ij} = G_{ji}$
But:
$v_{ij} \neq v_{ji}$ (but they are related)
Since:
$\frac{v_{ij}}{E_i} = \frac{v_{ji}}{E_j}$ (but $i \neq j$)

Lamina: 4 independent constants
E_1 E_2 G_{12} v_{12}
v_{12}: major Poisson's ratio
v_{21}: minor Poisson's ratio

Major assumptions
1. Fibers are isotropic
(this assumption is okay with G-fibers; C- and K- fibers are transversely isotropic)

2. Matrix is isotropic

3. Void content is zero
$V_f + V_m = 1$
or use V_f' and V_m'

4. Perfect bonding exists between fibers and matrix
Need to know (i.e., need to determine beforehand)
fibers: E_f v_f G_f V_f (or V_f')
matrix: E_m v_m G_m V_m (or V_m')

(only two are independent)

4 Subsections
3.3.1 Strength of Materials Approach (Mechanics of Materials)
3.3.2 Semi-Empirical Approach
3.3.3 Theory of Elasticity Approach
3.3.4 Transversely Isotropic Fibers (C- and K- fibers are transversely isotropic)

Example 1

A unidirectional fiber-reinforced composite consists of one type of fibers and one type of matrix. The weight fraction of matrix is 0.45. The specific gravity of the fibers and matrix is 2.5 and 1.3, respectively.

a) Find the specific gravity of the composite. Assume zero void content.

b) Find the specific gravity of the composite if $V_v = 5\%$

Solution

a) Given:

$\rho_f = 2.5$

$\rho_m = 1.3$

$W_m = 45\% = 0.45$

$W_f + W_m = 1$

$\therefore W_f = 0.55$

$$(Eq. 1): W_f = \frac{\rho_f V_f}{\rho_f V_f + \rho_m V_m}$$

$\therefore 0.715 V_m = 1.125 V_f$

$$(Eq. 2): W_m = \frac{\rho_m V_m}{\rho_f V_m + \rho_m V_m}$$

$\therefore 0.715 V_m = 1.125 V_f$

But we also know that:

$(Eq. 3): V_f + V_m = 1$

Solving:

$V_f = 0.3886$

$V_m = 0.6114$

Thus:

$\rho_c = \rho_f V_m + \rho_m V_m = 1.766$

b) $W_f = 0.55$

$0.715 V_m = 1.125 V_f$ (still holds true)

$V_f + V_m = 1 - V_v = 0.95$

Solving:

$V_f = 0.3692$

$V_m = 0.5808$

$\rho_c = 1.678$

Example 2 (similar to midterm question – last year)

A unidirectional fiber-reinforced composite consists of two types of fibers (fiber 1 and fiber 2) and one type of matrix. The total volume is v_c. Void content is assumed zero. The volume fractions of the fibers and the matrix are V_{f1} and V_{f2} and V_m respectively. The weight densities (per unit volume) of the fibers and matrix are ρ_{f1} ρ_{f2} and ρ_m respectively. Gravitational acceleration is g.

a) Express the total mass of the composite in terms of ρ_{f1}, ρ_{f1}, ρ_m, V_{f1}, V_{f2}, V_m, v_c, and g; and

b) Express the mass fraction of fiber 2, in terms of ρ_{f1}, ρ_{f2}, ρ_m, V_{f1}, V_{f2}, and V_m

Solution

a) $w_c = w_{f1} + w_{f2} + w_m$

$$w_c = \frac{\rho_{f1}v_{f1} + \rho_{f2}v_{f2} + \rho_m v_m}{g} \cdot \frac{v_c}{v_c}$$

$$w_c = \frac{\rho_{f1}V_{f1} + \rho_{f2}V_{f2} + \rho_m V_m}{g} \cdot v_c$$

b) $w_c = w_{f1} + w_{f2} + w_m$

Then:

$$w_{f2} = \frac{\rho_{f2}v_{f2}}{g} v_c$$

$$\therefore W_{f2} = \frac{w_{f2}}{w_c} = \frac{\rho_{f2}V_{f2}}{\rho_{f1}V_{f1} + \rho_{f2}V_{f2} + \rho_m V_m}$$