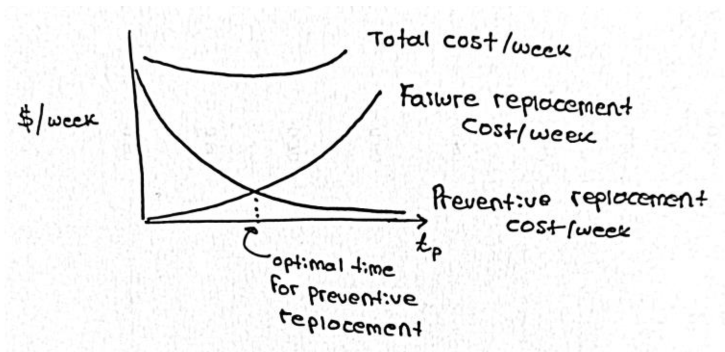
Optimal Preventive Replacement Interval of Items Subject to Breakdown (Also known as the Group or Block Policy).

An item is subject to sudden failure, and when failure occurs, the item has to be replaced.

Because failure is unexpected, a failure replacement is usually more costly than a preventive replacement.

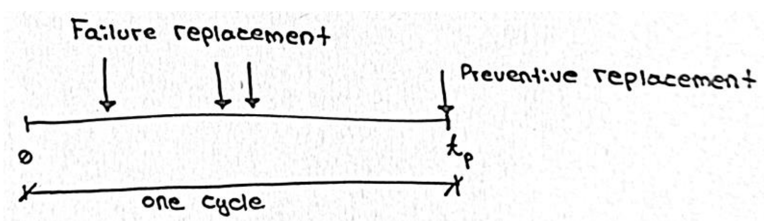
To reduce the number of failures, preventive replacements can be scheduled to occur at specified intervals.

A balance has to be made because too short an interval for preventive replacement is expensive and so is too long an interval.




The replacement policy is one in which preventive replacements occur at fixed interval of time, and failure replacements occurs whenever necessary.

Construction of a model:
$C_P$ is the total cost of a preventive replacement
$C_f$ is the total cost of failure replacement
$f(t)$ is the probability density function of the item's failure times




The objective is to determine the optimal interval between preventive replacements to minimize the total cost per unit time.

$$C(t_P) = \frac{total\ expected\ cost\ in\ interval\ (0, t_p)}{length\ of\ interval}$$

Total expected cost in interval $(0, t_p)$
$= cost\ of\ a\ preventive\ replacement + expected\ cost\ of\ failure\ replacements$
$= C_P + C_f \cdot H(t_p)$

Where $H(t_p)$ is the expected number of failure in interval $(0, t_p)$

So,

$$C(t_p) = \frac{C_P + C_f \cdot H(t_p)}{t_p}$$

Differentiate the right side of the equation with respect to $t_p$ and equate it to zero

$$-\frac{C_P}{t_r^2} - \frac{C_f \cdot H(t_p)}{t_p^2} + \frac{C_f \cdot h(t)}{t_p} = 0$$

$$C_P = t_p \cdot C_f \cdot h(t_p) - C_f \cdot H(t_p)$$

$$t_p \cdot h(t_p) - H(t_p) = \frac{C_P}{C_f}$$

($h(t_p)$ is the derivative of $H(t_p)$ and is termed renewal density]

<u>How to determine $H(t)$:</u>

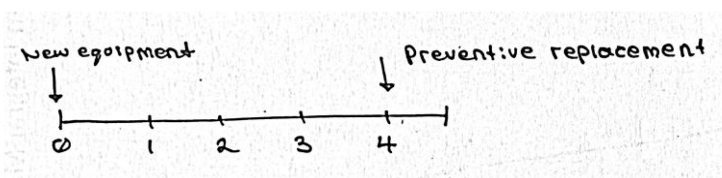
Two methods: Renewal Theory Approach is too much statistical, so we will not do it.

We will do:

Discrete Approach to establishing $H(t)$:

$H(t)$ is the expected number of failures in $(0, t)$

Consider a case in which there are 4 weeks between preventive replacements.




Then $H(t)$ is the expected number of failures in interval $(0, 4)$, starting with new equipment.

When we start at time zero, the first failure (if there is one) will occur during either the first, second, third, or fourth week of operation.

So,

$H(4) =$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the first week · probability of the first failure occurring in the first interval $(0, 1)$

$+$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the second week · probability of the first failure occurring in the first interval $(1, 2)$

$+$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the third week · probability of the first failure occurring in the first interval $(2, 3)$

$+$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the fourth week · probability of the first failure occurring in the first interval $(3, 4)$

Assume that not more than one failure can occur in any weekly interval (Because the length of interval can be made short is needed).

Number of expected failures that occur in interval $(0, 4)$ when the first failure occurs in the first week

$=$ the failure that occurred in the first week $+$ the expected number of failure in the remaining three weeks

$= 1 + H(3)$

We use $H(3)$ because we have a new equipment as a result of replacing the failed equipment in the first week, and we have 3 weeks to go before the preventive replacement occurs.

By definition, the expected number of failures in the remaining 3 weeks, starting with the new equipment is $H(3)$.

The probability of the first failure occurring in the first week:

$$= \int_0^1 f(t)\, dt$$

Similarly, in consequence of the of the first failure occurring in the second, third, or fourth weeks

So,

$$H(4) = [1 + H(3)] \int_0^1 f(t)\, dt + [1 + H(2)] \int_1^2 f(t)\, dt + [1 + H(1)] \int_2^3 f(t)\, dt + [1 + H(0)] \int_3^4 f(t)\, dt$$

Obviously, $H(0) = 0$ That is, with zero weeks to go, the expected number of failures is zero.

So,

$$H(4) = \sum_{i=0}^{3} [1 + H(3 - i) \int_i^{i+1} f(t)\, dt$$

$$H(0) = 0$$

In general,

$$H(T) = \sum_{i=0}^{T-1} [1 + H(T - i - 1)] \int_i^{i+1} f(t)\, dt$$

This is called a recurrence relation with $T \geq 1$ and $H(0) = 0$

Because we know $H(0) = 0$, we can get $H(1)$, $H(2)$, $H(3)$ etc.

<u>Optimal Preventive Replacement Age of an Item Subject to Breakdown:</u>

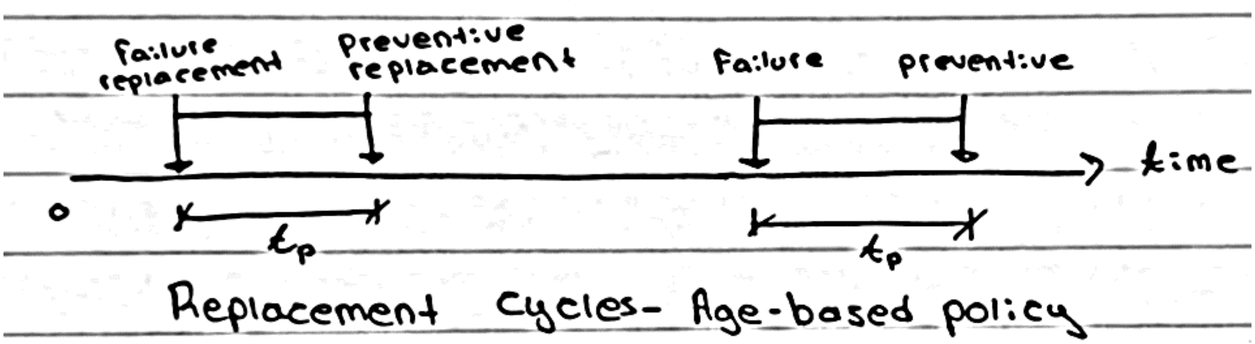
Instead of making preventive replacements at fixed intervals, with the possibility of performing a preventive replacement shortly after a failure replacement, the time at which preventive replacement occurs depends on the age of the item.

When failures occur, failure replacements are made.

When this happens, the time clock is reset to zero, and the preventive replacement occurs only when the item has been in use for a specified period.

So, the problem is to balance the cost of the preventive replacements against their benefits, and we do this by determining the optimum preventive replacement age for the item to minimize the total expected cost of replacements per unit time.

Construction of the model:

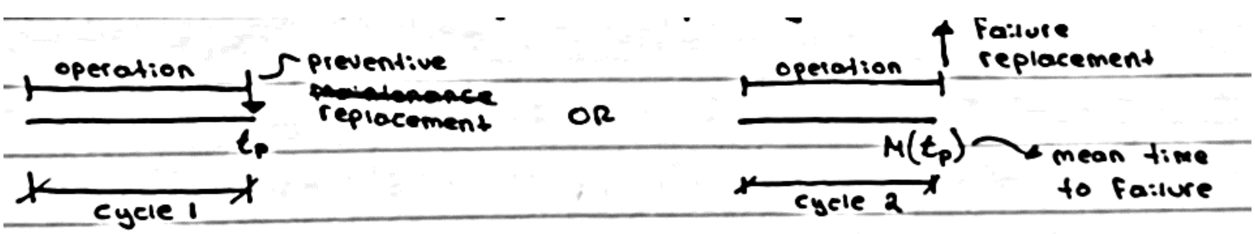



.$C_p$ is the total cost of preventive replacement

$C_f$ is the total cost of failure replacement
$f(t)$ is the probability density function of the failure times of the item
$t_p$ is the time for preventive replacement (if there is failure before reaching $t_p$, the clock is reset for the next preventive replacement)

In this problem, there are two possible cycles of operation, one cycle being determined by the item reaching the planned replacement age $t_p$; and the other being determined by the equipment failing before reaching $t_p$.

Two possible cycles




Possible replacement cycles for Age based policy.

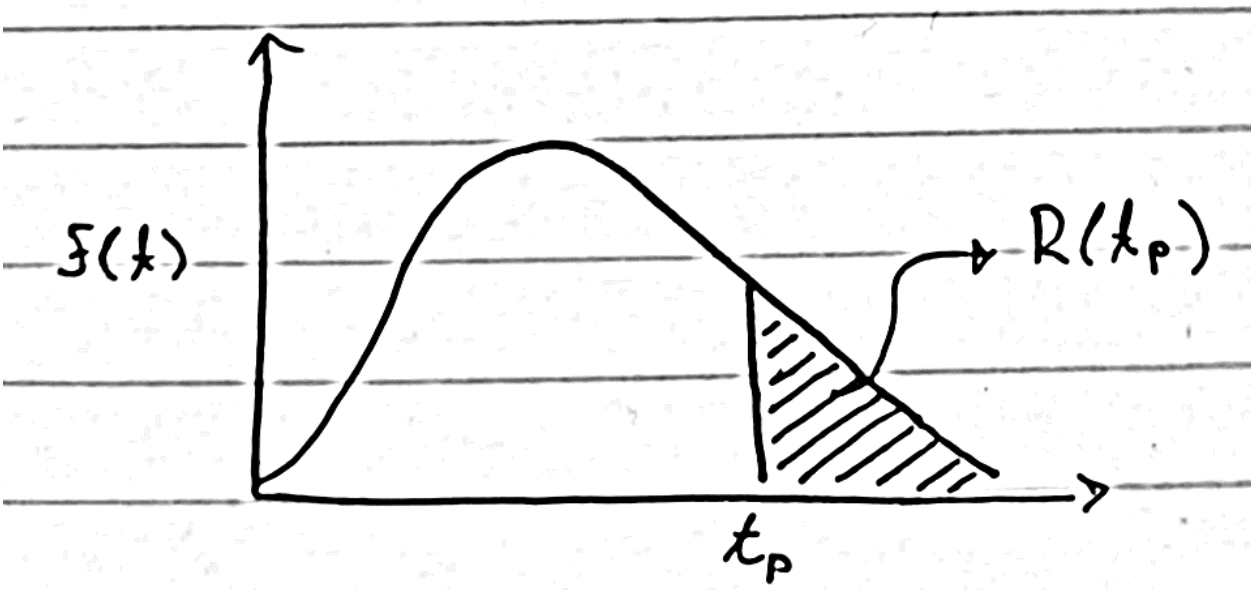
The total expected replacement cost per unit time, $C(t_p)$, is:

$$C(t_p) = \frac{Total\ expected\ replacement\ cost\ per\ cycle}{Expected\ length\ of\ cycle}$$

Total expected replacement cost per cycle
$=$ cost of preventive cycle · probability of a preventive cycle
$+$ cost of failure cycle · probability of a failure cycle

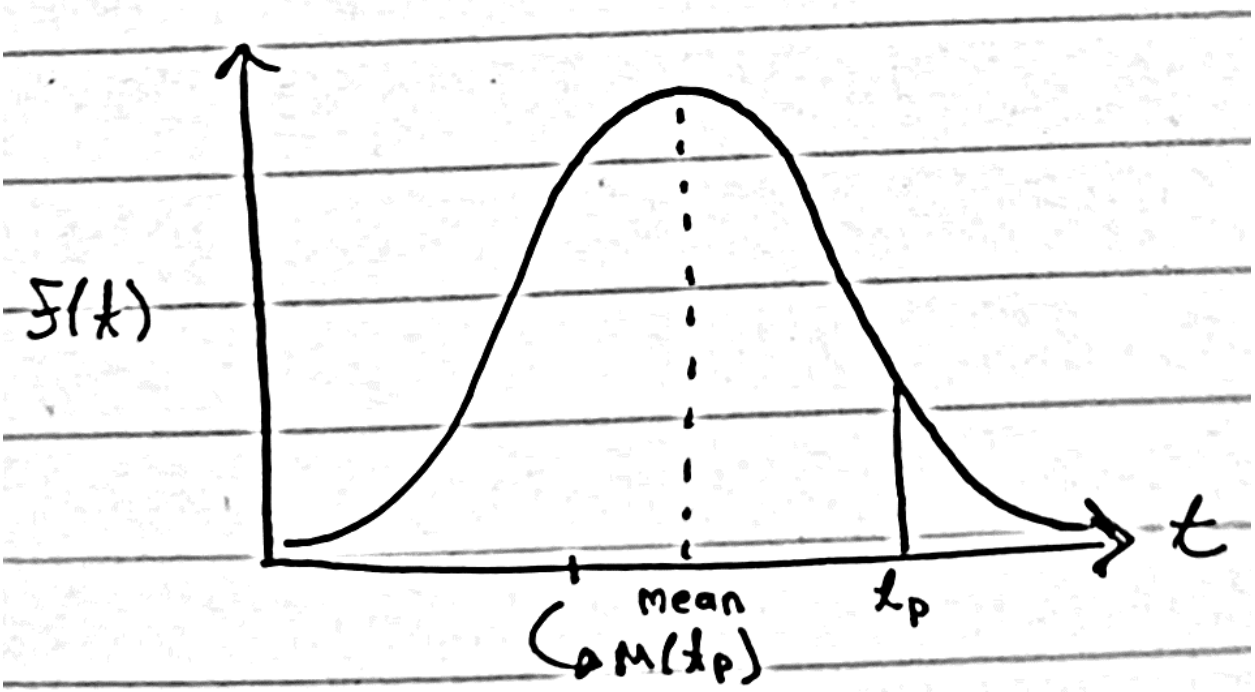
$$= C_p \cdot R(t_p) + C_f \cdot [1 - R(t_p)]$$

The probability of a preventive cycle is equivalent to the probability of failure occurring after time $t_p$, that is, equivalent to the shaded area, which is denoted as $R(t_p)$.




The probability of a failure cycle is the probability of failure occurring before time $t_p$, which is the unshaded area $[1 - R(t_p)]$

Expected cycle length
$=$ length of the preventive cycle $\cdot$ probability of a preventive failure
$+$expected length of a failure cycle $\cdot$ probability of a failure cycle



Mean time to failure: $M(t_p)$

Mathematically,

$$M(t_p) = \int_{-\infty}^{t_p} \frac{t\, f(t)\, dt}{1 - R(t_p)}$$

(Need not memorize)

$M(t_p)$ is essentially determined from the mean of area up to $t_p$

So,
Expected cycle length
$= t_p \cdot R(t_p) + M(t_p) \cdot [1 - R(t_p)]$

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \cdot [1 - R(t_p)]}{t_p \cdot R(t_p) + M(t_p) \cdot [1 - R(t_p)]}$$

This is a model of the problem relating the replacement age to the total expected replacement cost per unit time.

No simple solution to obtain the minimum (optimal) $t_p$ for this problem.

Need trial values of several points to determine the optimal $t_p$.

(A misprint in the book)

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \cdot [1 - R(t_p)]}{t_p \cdot R(t_p) + \int_{-\infty}^{t_p} t\, f(t)\, dt}$$

**For Midterm:**
1 or 2 descriptive questions on what we covered
1 or 2 derivations on what we covered
Numerical examples to be posted