Normal Distribution: Most widely used statistical distribution

This applies when a random outcome (such as time to failure) is the additive effect of a large number of small and independent random variables.

In practice, the lifetime of light bulbs and the time until the first failure of bus engines have been found to follow a normal distribution.

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right]$$
$$-\infty < t < \infty$$

Were μ is the mean and σ the standard deviation of the distribution.

For normal distribution:

Where: $\int_0^\infty f(t)\, dt < 1$

But: $\int_{-\infty}^\infty f(t)\, dt = 1$

In practice, however, if the mean of the normal distribution, μ, is considerably removed from the origin $t = 0$ and variance σ^2, is not too large, than it is acceptable to use the normal distribution as an approximate to the real situation.

Weibull Distribution: Fits a large number of failure characteristics of equipment.

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^\beta\right]$$
$$t \geq 0$$
$$\beta > 0$$
$$\eta > 0$$

Cumulative Distribution Function

Probability of failure before time t

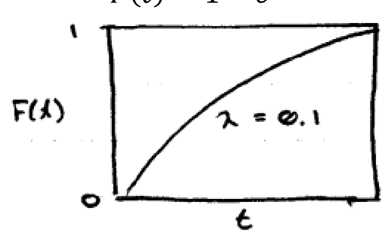
$$= \int_0^t f(t)dt$$

The integral $\int_0^t f(t)dt$ is denoted by $F(t)$ and is termed the cumulative distribution function.

As $t \to \infty, F(t) = 1$

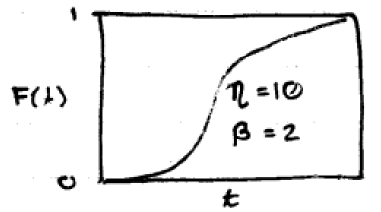
Exponential Distribution

$$F(t) = 1 - e^{\lambda t}$$



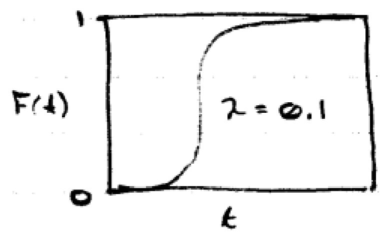
Normal Distribution

$$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{t-\mu}{2\sigma^2}\right)^2\right]$$



Weibull Distribution

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$



For this course, you should be familiar with standard normal distribution table (Appendix 9) – will give xeroxed pages of the table in exams!

The cumulative distribution function, $F(t)$, of a normal distribution with mean $= \mu$ and standard deviation $= \sigma$ can be determined from the standard table in Appendix 9.

It tabulates the value of $1 - \phi(z)$, where:

$$z = \frac{t-\mu}{\sigma}$$

is a standardized normal distribution variable and $\phi(z)$ is the cumulative distribution function of the standard normal distribution.

Thus, the table provides the probability that the standardized normal variable chosen at random is greater than a specified value of z.

The normal distribution being symmetrical about its mean, $\phi(-z) = 1 - \phi(z)$.

Thus, only the probability for $z \geq 0$ is tabulated.

Reliability Function: (also known as survival function)

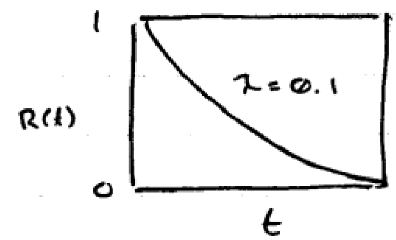
It is determined from the probability that the equipment will survive at least to some specified time t.

The reliability function is denoted by $R(t)$ and is defined as:

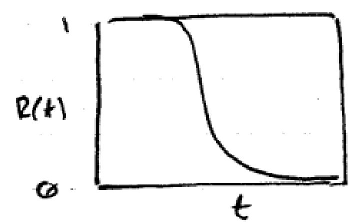
$$R(t) = \int_0^{\infty} f(t)\, dt$$

$$R(t) = 1 - F(t)$$
$$t \to \infty, \;\; R(t) = 0$$

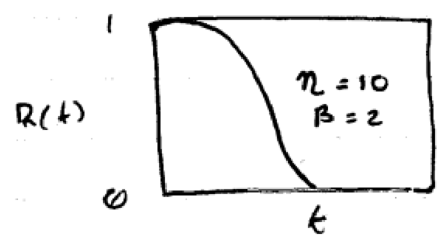
For exponential distribution:



For Normal distribution:



For Weibull distribution:



For exponential distribution:

$$R(t) = e^{-\lambda t}$$

For normal distribution:

$$R(t) = \frac{1}{\sigma\sqrt{2\pi}} = \int_0^{\infty} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$

For Weibull distribution:

$$R(t) = e^{\left(-\frac{t}{\eta}\right)^{\beta}}$$

Consider an item that is operational time t, when a mission starts.

We want to determine the probability of the item surviving the mission of duration t.

The required measure can be expressed in the usual notation of conditional probability as:

$$R(t_1 + t \mid t) = P(T \geq t_1 + t \mid T \geq t_1)$$

$$= \frac{P(T \geq t_1 + t)}{P(T \geq t_1)} = \frac{R(t_1 + t)}{R(t_1)}$$

$$= \frac{\int_{t_1+t}^{\infty} f(t)\, dt}{\int_{t_1}^{\infty} f(t)\, dt}$$

Where T is the time to failure.

If the failure time follows an exponential distribution:

$$R(t_! + t \mid t_1) = \frac{\int_{t_1+t}^{\infty} f(t)\, dt}{\int_{t_1}^{\infty} f(t)\, dt} = \frac{\int_{t_1+t}^{\infty} \lambda e^{-\lambda t}\, dt}{\int_{t_1}^{\infty} \lambda e^{-\lambda t}\, dt}$$

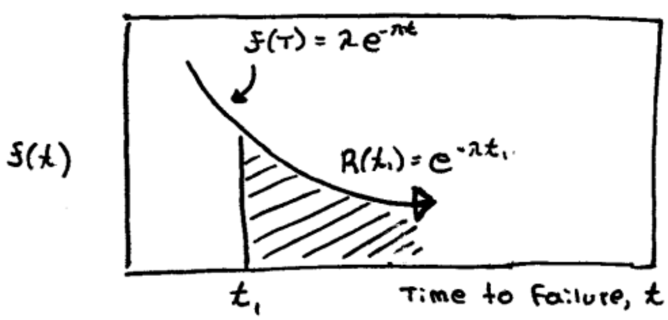
$$= \frac{e^{-\lambda(t_1+t)}}{e^{-\lambda t_1}} = e^{-\lambda t} = R(t)$$

Thus, for operational items with failure times that are exponentially distributed,

$$R(t_! + t \mid t_1) = R(t)$$

In words, their chance of survival (or conversely, their risk of failure) in the next instance is independent of their current age.

This memoryless property is unique to the exponential distribution, the only continuous distribution with this feature.



<u>Hazard rate:</u> The hazard rate of an item is the probability that the item will fail in the next interval of time given that it is good at the start of the interval, that is, it is a conditioned probability.

Consider a test in which a large number of identical components are put into operation and the time to failure of each component is noted.

An estimate of the hazard rate of a component at any point in tie may be thought of as the ratio of a number of items that failed in an interval of time to the number of items in the original population that were operational at the start of the interval.

Specifically, letting $h(t)\delta t$ to be the probability that an item fails during a short interval δt, given that it has survived to time t, the usual notation.

$P(A|B) =$ probability of event A occurring once it is known B has occurred.
Where A is the event "failure occurs in interval δt" and B is the event that no failure occurred up to time "t".

$P(A|B)$ is given by:

$$P(A|B) = \frac{P(A \; and \; B)}{P(B)}$$

$$P(A \; and \; B) = \int_t^{t+\delta t} f(t)\, dt$$

$$P(B) = \int_t^{\infty} f(t)\, dt$$

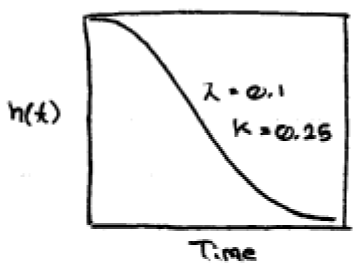
So, the hazard rate in interval δt is:

$$h(t)\delta t = \frac{\int_t^{t+\delta t} f(t)\, dt}{\int_t^{\infty} f(t)\, dt}$$

So, $h(t) = \frac{f(t)}{1-F(t)}$

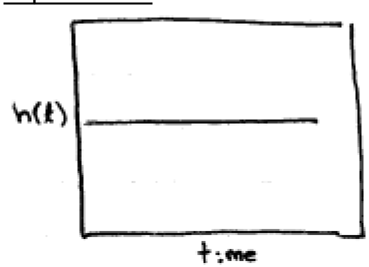
$As \; \delta t \to 0$

<u>Hyper-exponential:</u>



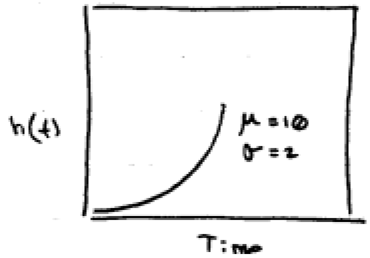
$$h(t) = \frac{2\lambda\left[k^2 + (1-k)\exp[-2(1-2k)\lambda t]\right]}{K + (1-k)\exp[-2(1-2k)\lambda t]}$$

<u>Exponential:</u>



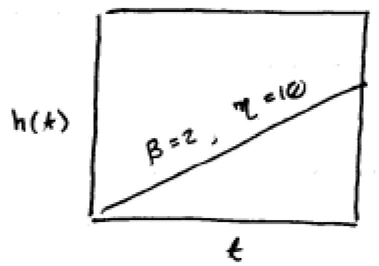
$h(t) =$

<u>Normal distribution:</u>



$$h(t) = \frac{\exp\left[-(t-\mu)^2/2\sigma^2\right]}{\int_t^{\infty} \exp\left[\frac{-(t-\mu)^2}{2\sigma^2}\right] dt}$$

Weibull distribution:



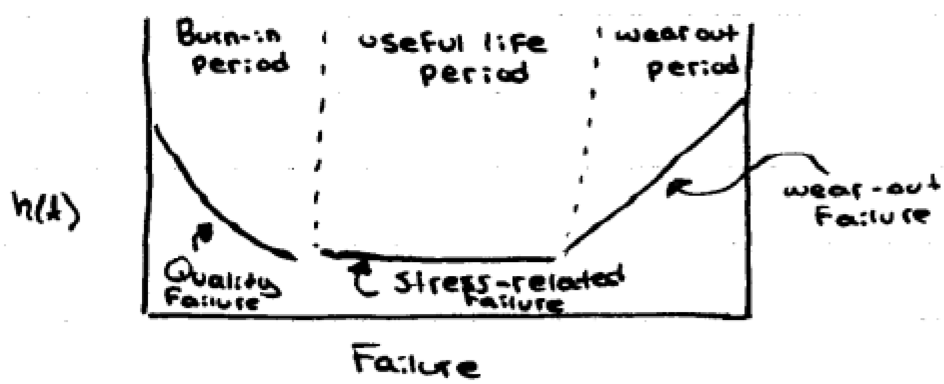
$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

When the hazard rate increases with time, such as for the normal and Weibull distribution, it indicates an aging or wear-out effect.

With the exponential distribution, the hazard rate is constant. This failure pattern can be the result of completely random events such as sudden stresses and extreme conditions.

It also applies to the steady-state condition of complex equipment which fails when any one of a number of independent constituent component breaks, or when any one of a number of failure modes occur.

For complex equipment, the hazard rate look like:



This is called the “Bathtub curve”

Bathtub curve:
$A =$ a burn in or running-in period
$B =$normal operation in which failures that occur are predominantly due to choice
$C =$ deterioration, i.e. wear-out due to age

How to determine the most appropriate policy to adopt when equipment is in one of the regions A, B, C.

If the only form of the maintenance possible is replacement, either on a preventative basis, or because of failure, then in regions A and B no preventative replacement should be applied because such replacements will not reduce the risk of equipment failures.

If preventative replacements are made in regions A and B, maintenance effort is wasted.

Unfortunately, this is often the case in practice because it is often mistakenly assumed that as equipment ages, the risk of failure will increase.

In region C, preventative replacement will reduce the risk of equipment failure in the future, and just when these preventive replacements should occur will be influenced by the relative costs or other relevant impact factors, such as downtime of preventative and failure replacement.