From economic point of view, the optimal time to replace is when the marginal cost is equivalent to the average cost.

Special case: The trend in operating cost is linear:

$$c(t) = a + bt$$

Then:

$$\int_0^{t_{r^*}} c(t)\,dt = \int_0^{t_{r^*}} (a + bt)\,dt = at_{r^*} + \frac{bt_{r^*}^2}{2}$$

Then:

$$\frac{1}{t_{r^*}} \int_0^{t_{r^*}} (a + bt)\,dt = a + \frac{bt_{r^*}}{2}$$

And:

$$C(t_{r^*}) = a + bt_{r^*} = a + \frac{bt_{r^*}}{2} + \frac{C_r}{t_{r^*}}$$

$$\frac{bt_{r^*}}{2} = \frac{C_r}{t_{r^*}}$$

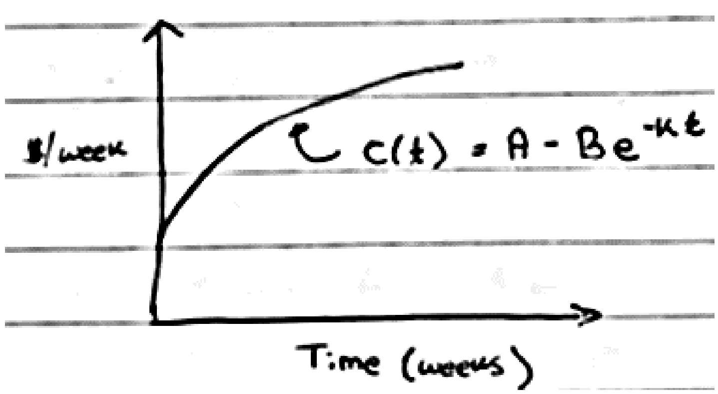
$$t_{r^*} = \sqrt{\frac{2C_r}{b}}$$

In using the derived relationship, $c(t_r) = C(t)$ for optimal replacement, there is an implicit assumption that the operating cost is increasing with time, and this is a realistic assumption.

If the trend in operating cost is not continuous, but discrete, then the optimal replacement time is when the next period's operating cost is equal to or greater than the current average cost of replacement to that time.

Example: The trend in operating cost for an item is of the form:

$$c(t) = A - Be^{-Kt}$$



Where
$A = \$100$
$B = \$80$
$K = 0.21/week$

$A - B \geq 0$ may be interpreted at the operating cost of unit time if no deterioration occurs.

K is a constant describing the rate of deterioration.

Given C_r, the total cost of replacement, is $100.

Thus:

$$C(t_r) = \frac{1}{t_r}\left[\int_0^{t_r} (100 - 80e^{-0.218})\, dt + 100\right]$$

Analytical solution (closed form)

Using the result, $c(t_r) = C(t_r)$ cannot be obtained.

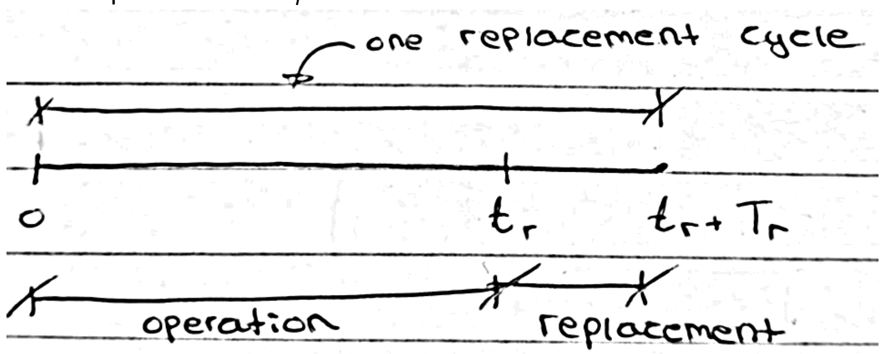
Numerical solution:

t_r	1	2	3	4	5	6	7
$C(t_r)$	127.8	84.7	74.0	70.9	70.5	71.5	72.5

So, $t_r \approx 5\ weeks$

So far, we did not consider the time required to perform a replacement.

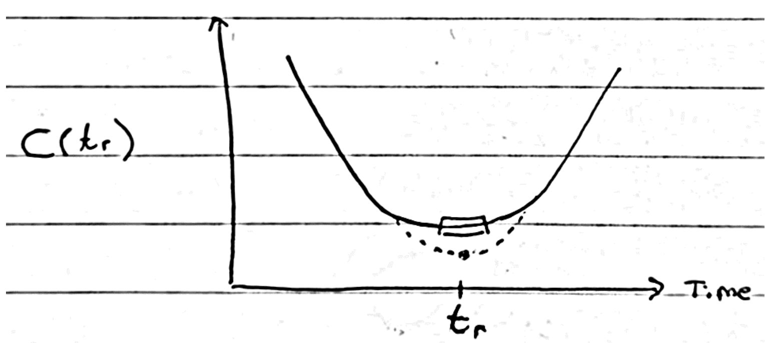
If the time to perform replacement $= T_r$



$$C(t_r) = \int_0^{t_r} \frac{c(t)dt + C_r}{t_r + T_r}$$

Any costs, such as production losses incurred due to the duration of the replacement, need to be incorporated into the cost of the replacement action.

To further assist the engineer in deciding what an appropriate replacement policy should be, it is usually useful to plot the total cost per unit time versus time curve.



It shows the form of the total cost around the optimum.

If the curve is fairly flat around the optimum, it is not really very important that the engineer should plan for the replacement to occur exactly at the optimum, thus giving some leeway in scheduling the work.

If the total cost curve is not flat around the optimum, then the optimal interval should be adhered to if at all possible.

If there is uncertainty about the value of the particular parameter required in the analysis, then evaluation of the total cost curve for various values of this uncertain parameter should be performed, the replacement policy depends upon the sustainability of the analysis with respect to the uncertain parameter.

Sensitivity checking gives guidance on what information is important from a decision-making viewpoint and, consequently, what information should be gathered in a data collection scheme.

<u>Stochastic Preventative Replacement</u>
Before developing component replacement models, it should be noted that preventative replacement actions require two necessary conditions.

1. The total cost of the replacement must be greater after the failure than before (The total cost of the replacement must be greater after failure than before (if cost is the appropriate criterion; otherwise, an appropriate criterion, such as downtime, is substituted in place of cost).

   This may be caused by a greater loss of production because replacement after failure is unplanned or failure of one piece of the plant may cause damage to other equipment.

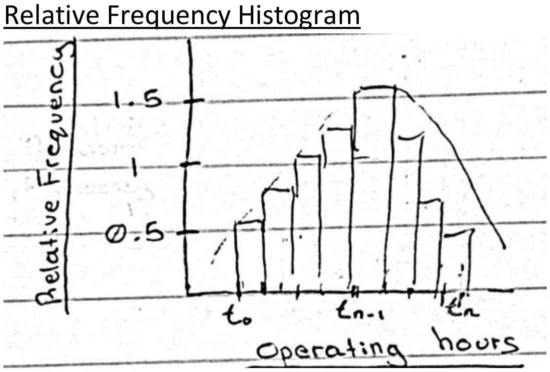
2. The hazard rate (failure rate) of the equipment must be increasing. It is not worthwhile to perform preventative replacement if hazard rate is constant or decreasing.

<u>Maintenance Mathematics</u>
Decisions relating to probabilistic maintenance problems, such as deciding when to perform maintenance on equipment that is subject to breakdown, require information about when the equipment will reach a failed state.

It is difficult to know when failure will occur, but it is possible to assign probability of failure.

For that we need knowledge of statistics.

<u>Relative Frequency Histogram</u>



Where the area under the curve is 1.

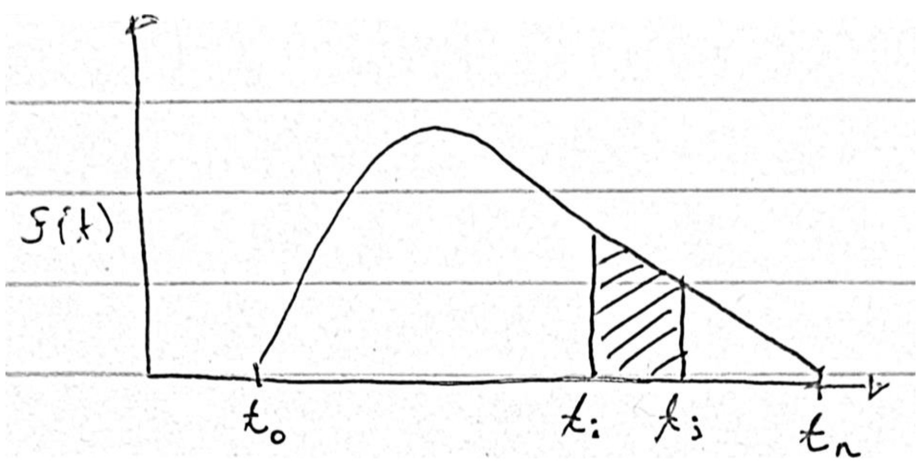
If we wish to determine the probability of failure occurring between times t_i and t_{i-1} we simply multiply the ordinate y by the interval $(t_i - t_{i-1})$.

The probability of a failure occurring between t_o and t_n, where t_o and t_n are the earliest and latest times for failure respectively, at which time all pieces have failed, is unity.

<u>Probability Density Function</u>
In maintenance studies, we tend to use probability density function rather than relative frequency histograms.

This is because:
1. The variable to be modeled, such as time to failure, is a continuous variable.
2. These functions are easier to manipulate.
3. It should give a clear understanding of the failure distribution.

The probability density functions (pdfs) are similar to RF histograms except that a continuous curve is used instead of bars.



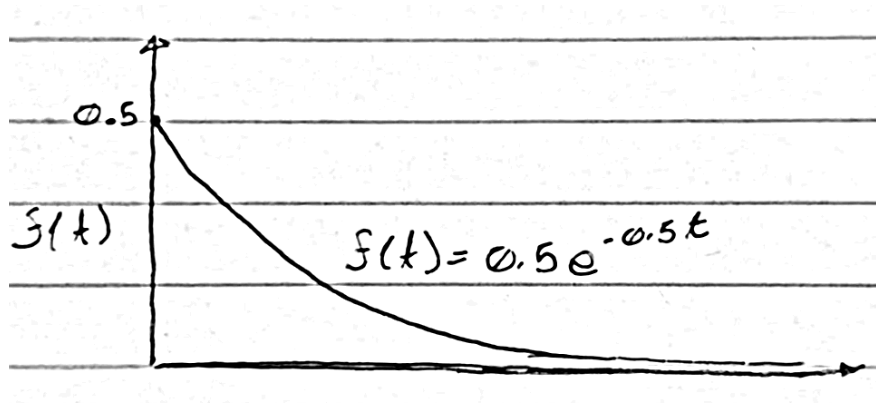
The probability (risk) of a failure occurring between t_i and t_j is the hatched area.

From calculus, this is:

$$\int_{t_i}^{t_j} f(t)dt$$

The equation of the curve of the pdf is denoted by $f(t)$.

For example, if we have $f(t) = 0.5e^{-0.5t}$, we get the following curve:



This is a pdf of an exponential distributions.

The area under the probability density curve is equivalent to j.

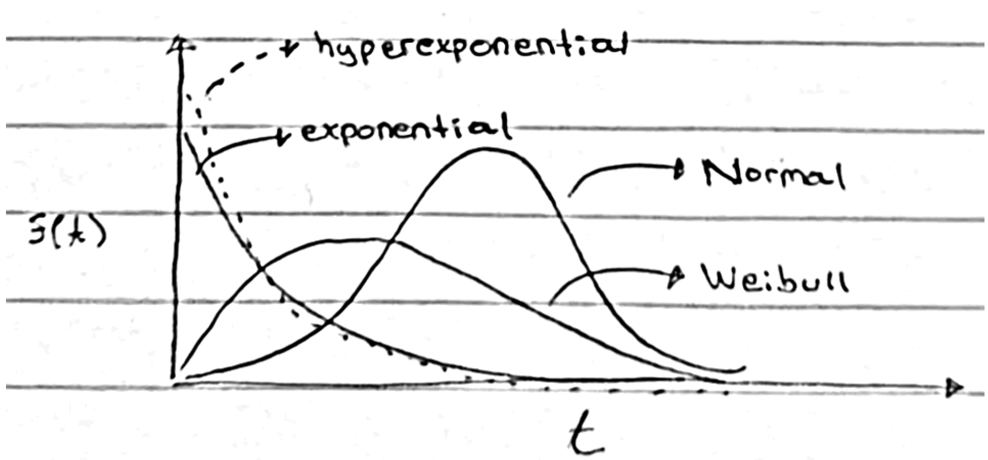
The probability of a failure occurring between times t_o and t_∞ is then:

$$\int_{t_i}^{\infty} f(t)dt = 1$$

The failure characteristics of different items of equipment is likely to be different from each other.

Even the failure characteristics of identical equipment may not be the same if they are operating in different environments.

There are a number of well-known pdfs that have been found in practice to describe the failure characteristics of equipment.



<u>Hyperexponential Distribution</u>
When equipment has a failure time that can be very short or very long, its failure distribution can be obtained by hyperexponential distribution.

Computers have been found to fail according to this distribution.

In the hyperexponential distribution, the short time to failure occur more often then in the negative exponential distribution, and similarly, the long times to failure occur more frequently than in the exponential case.

The density function of the hyperexponential distribution:

$$f(t) = 2x^2\lambda \exp[-2K\lambda t] + 2(1-K^2)\lambda \exp[-2(1-K)\lambda t]$$

For $t \geq 0$ with $0 < K \leq 0.5$

Where:
λ is the arrival route of breakdowns and
K is a parameter of the distribution

<u>Exponential Distribution</u>
This is one of the most widely used probability distributions in engineering, particularly in the reliability work.

It is relatively easy to handle in conducting analysis.

This arises in practice wherein failure of the equipment can be causes by failure of any one of a number of components of which the equipment is comprised.

It is also characteristic of equipment subject to failure due to random causes, such as sudden excessive loading.

This distribution is found to be typical for many electronic components and complex industrial plants.

The pdf is:

$$f(t) = \lambda\, e^{-\lambda t}$$

For $t \geq 0$ with $\lambda > 0$

Where:
λ is the arrival rate breakdown (Called the distribution parameter)
$1/\lambda$ is the distribution parameter

In-class project:
Make groups of 4-5 for class presentation.

Group leader to email me names of group by next Thursday.

Choose any topic related to maintenance.

Main objective is to improve your communication skills.