Chapter 4 – Capital Equipment Replacement Decisions

Definitions of Capital Items:

Capital items are of considerable value and durability and are used to provide service or to make, market, keep, or transport periods (businessdictionary.com).

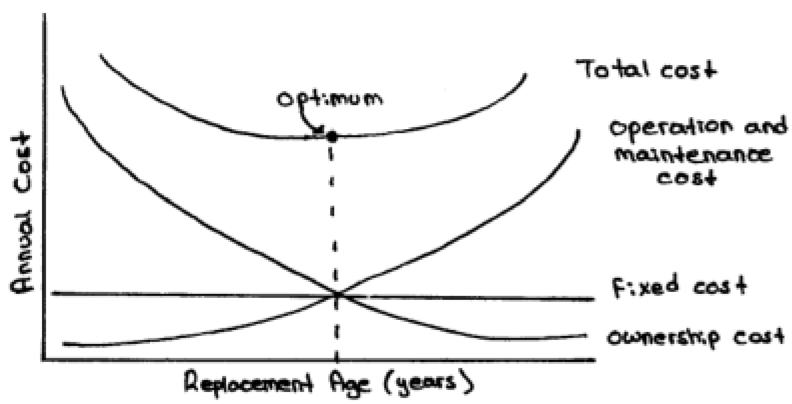
Capital equipment present tangible and intangible goods that are preserved by organizations and that present the technical prerequisites for the production of goods and services. One characteristic of capital equipment is that performance of use with the possible inclusion of services of provision, maintenance and repair; another characteristic is the high value of an individual object compared with the material used (Hoffman et al, 2012).

Most businesses establish criteria for designating acquired items as either capital or noncapital items.

These criteria are based partly on local tax items, but they also represent accounting policy choices by management.

The criteria usually specify that capital items must have a minimum useful life (for example, one year or more), have an acquisition cost above a certain threshold (e.g. $5000 or more), and contribute value to the business.

Capital equipment problems tend to be treated deterministically:



There may be additional costs incurred, associated with utilization of an item, that are independent of the age at which the asset is replaced. These are identified in fixed unit.

Fixed costs do not affect the economic life decisions.

Optimal Replacement Interval for Capital Equipment: Minimization of Total Cases

Objective is to determine an optimum replacement policy that minimizes that total discounted costs derived from operating, maintaining, and disposing of the equipment over a long period.

It will be assumed that equipment is replaced by an identical item, this returning the equipment to the as new condition after replacement.

This restriction is not valid when technological improvement is considered.

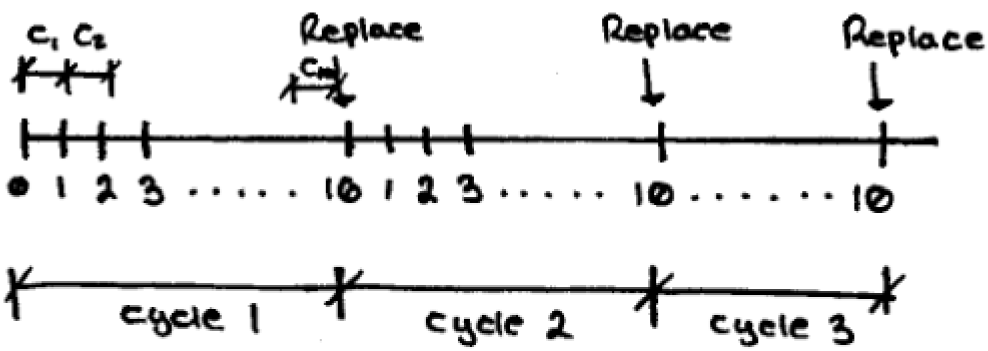
It is assumed that the trends in operating and maintenance costs after each replacement will remain identical.

Because the equipment is being operated over a long period, the replacement policy will be periodic, so we will determine the optimal replacement interval.

Construction of the model:

1. A – is the acquisition cost of the capital equipment
2. C_i – is the operation and maintenance in the i^{th} period from new, assumed to be paid at the end of the period, $(i = 1, 2, 3, \ldots n)$
3. S_i – is the resale value of the equipment at the end of the i^{th} period of operation, $(i = 1, 2, 3, \ldots n)$
4. r – is the discount factor $\left(r = \frac{1}{1+\frac{i}{100}}\right)$

   (i is the interest rate)
5. n is the age in periods (such as years) if the equipment when replaced.
6. $C(n)$ is the total discounted cost of operating, maintaining, and replacing the equipment (with identical equipment) over a long period, with replacement occurring at intervals of n periods.
7. The objective is to determine the optimal interval between replacements to minimize the total discounted costs, $C(n)$



Consider the first cycle of operation:

The total discounted up to the end of the first cycle of operation, with equipment already purchased and installed, is:

$$C_1(n) = C_1 r^1 + C_2 r^2 + C_3 r^3 + \cdots + C_n r^n + A r^n - S_n r^n$$

$$= \sum_{i=1}^{n} c_i r^i + r^n (A - S_n)$$

For the second cycle, the total cost discounted from the start of the second cycle is:

$$C_2(n) = \sum_{i=1}^{n} c_i r^i + r^n (A - S_n)$$

Similarly, the total cost o the third cycle, fourth cycle, and so forth discounted back to the start of their respective cycles, is of similar format.

The total discounted costs, when discounting is calculated at the start of the operation time zero, is

$$C(n) = C_1(n) + C_2(n)r^n + C_3(n)r^{(2n)} + \cdots + C_n(n)r^{(n-1)n}$$

Between $C_1(n) = C_2(n) = \cdots = C_n(n)$

We end up with a geometric progression that gives over an infinite period:

$$C(n) = \frac{C_1(n)}{1 - r^n} = \frac{\sum_{i=1}^{n} c_i r^i + r^n(A - S_n)}{1 - r^n}$$

This is the model of the problem that relates the replacement interval to the total costs.

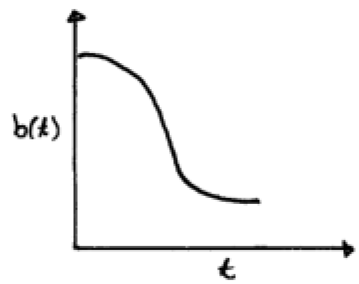
Optimal Replacement Interval for Capital Equipment: Maximization of Discounted Benefits

Similar problem as the last one except that (1) the objective is to determine the replacement interval that maximizes the total discounted net benefits derived from operating equipment over a long period, and (2) the trend in cost is taken to be continued, rather than discrete.

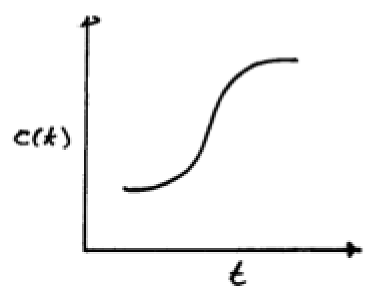
Construction of the model:

(1) $b(t)$ is the net benefit obtained from the equipment at time t

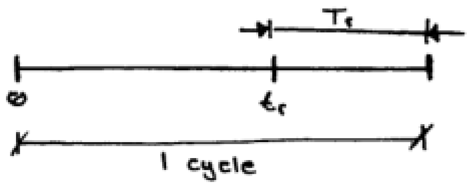
This will be the revenue derived from operating the equipment minus the operating costs, which may include maintenance costs, fuel costs, and so on



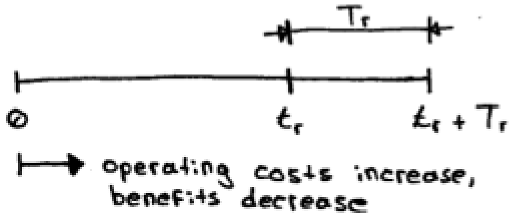
(2) $c(t)$ is the net cost of replacing equipment at age t. Replacing the equipment includes purchase price plus installation cost, and may also include cost for loss of production due to the time required to replace the equipment. These costs are often partially offset by the salvage value of the used equipment, which usually depends on the age of the capital equipment when it is replaced.



(3) T_r is the time required to replace the equipment
(4) t_r is the age of equipment
(5) $t_r + T_r$ is the replacement cycle



(6) $B(t_r)$ is the total discounted net benefit derived from operating the equipment for periods of length t_r over a long time.
(7) The objective is to determine the optimal interval between replacements to maximize the total discounted net benefits derived from operating and maintaining the equipment over a long period.



If we define $B_1\ (t_r + T_r)$ as the total net benefits derived from replacing the equipment at age t_r, discounted back to the present-day value at the start of the first cycle,

$B_1(t_r + T_r) =$ Benefits received in interval $(0, t_r)$ discounted to their present value
$-$ the cost of replacing equipment of age t_r discounted to its present-day value

Discounted benefits over the first cycle

$$= \int_0^{t_r} b(t)\, e^{-it}\, dt$$

i is the relevant interest rate for discounting

Discounted replacing cost = $c(t_r)e^{-i\,t_r}$

So,

$$B_1(t_r + T_r) = \int_0^{t_r} b(t)\, e^{-i\,t}\, dt - c(t_r)\, e^{-i\,t_r}$$

Similarly, second cycle of operation:

$$B_2(t_r + T_r) = \int_0^{t_r} b(t)\, e^{-i\,t}\, dt - c(t_r)\, e^{-i\,t_r}$$

Discounting $B_2(t_r + T_r)$ back to the start of the first cycle, it becomes:

$$B_2(t_r + T_r)e^{-i(t_r+T_r)}$$

So, n^{th} cycle of operation:

$$B_n(t_r + T_r) = \int_0^{t_r} b(t)\, e^{-i\,t}\, dt - c(t_r)\, e^{-i\,t_r}$$

Which discounted to the start of first cycle:

$$B_n(t_r + T_r)\, e^{-i(n-1)(t_r+T_r)}$$

Thus, the total discounted net benefits over a long period, with replacement at age t_r, is:

$$B(t_r) = B_1(t_r + T_r) + B_2(t_r + T_r)e^{-i(t_r+T_r)} + \cdots + B_n(t_r + T_r)e^{-i(n-1)(t_r+T_r)}$$

Because $B_1(t_r + T_r) = B_2(t_r + T_r) = \cdots = B_n(t_r + T_r)$

So (the sum of geometric series),

$$B(t_r) = \frac{B_1(t_r + T_r)}{1 - e^{-i(t_r+T_r)}}$$

Or,

$$B(t_r) = \frac{\int_0^{t_r} b(t)e^{-i\,t}dt - C(t_r)e^{-i\,t_r}}{1 - e^{-i(t_r+T_r)}}$$

This is a model of the replacement problem for total discounted net benefits.

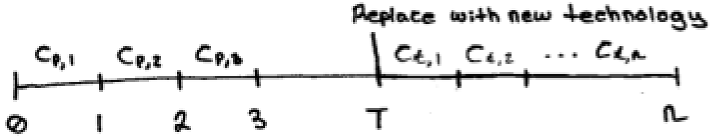
Optimal replacement policy for capital equipment considering technological improvements, finite planning horizon.

When determining a replacement policy, there may be equipment on the market that is in some way a technological improvement over the equipment currently being used.

For example:
- Maintenance and operating costs may be lower
- Throughput may be greater
- Quality of output product may be better

How to determine when, if at all, to take advantage of the technologically superior equipment.

Construction of the model:
(1) n is the number of operating periods during which equipment will be required
(2) C_p, i is the operation and maintenance cost of the prescribed equipment in the i^{th} period, from now payable at times i (where $i = 1, 2, 3, \dots, n$)
(3) S_p, i is the resale value of the present equipment at the end of the i^{th} period from now
(4) A is the acquisition cost of the technologically superior equipment
(5) $C_{t,J}$ is the operating and maintenance cost of the technologically superior equipment in the j^{th} period after its installation and payable at time J ($J = 1, 2, 3, \dots, n$)
(6) $S_{t,J}$ is the resale value of the technologically superior equipment at the end of J^{th} period of operation ($J = 0, 1, 2, \dots, n$)
$J = 0$ is included so that we can define $S_{t,0} = A$. This enables $A\,r^0$ in the model to be canceled if no changes exist.
(7) r is the discount factor
(8) The objective is to determine the value of T at which replacement should take place with the new and better equipment



The total discounted cost over n periods with replacement occurring at the end of the T^{th} period.

$C(T) =$ discounted maintenance costs for percent equipment over period $(0, T)$
$+$ discounted maintenance costs for technologically superior equipment over period (T, n)
$+$ discounted acquisition cost of the new equipment
$-$ discounted resale value of the previous equipment at the end of the i^{th} period
$-$ discounted resale value of technologically superior equipment at the end of the n^{th} period

$$= \left(C_{p,1}r^1 + C_{p,2}r^2 + \cdots + C_{p,T}r^T\right)$$
$$+ (C_{t,1}r^{T+1} + C_{t,2}r^{T+2} + \cdots + C_{t,n-1}r^n$$
$$+ Ar^T - (\mathrm{S_{p,T}r^T} + \mathrm{S_{t,n-T}r^n})$$
$$= \sum_{i=1}^{T} C_{p,i}r^i + \sum_{J=1}^{n-T} C_{t,J}r^{T+J} + Ar^T - (S_{p,T}r^T + S_{t,n-T}r^n)$$