General notes:

- Textbook: Maintenance, Replacement, and Reliability (2nd Edition)
    - Will be needed, course will follow textbook

EMEC-0538-WB: Maintenance Decision Analysis

- Course outline has already been emailed out
- First 5 weeks; tutorial class to be used as a lecture
- Group presentation (groups of 4-5), in-class presentation and report required.
    - Report is no longer required, just need to submit PowerPoint slides (last time this class is being offered)

Chapter 1: Introduction

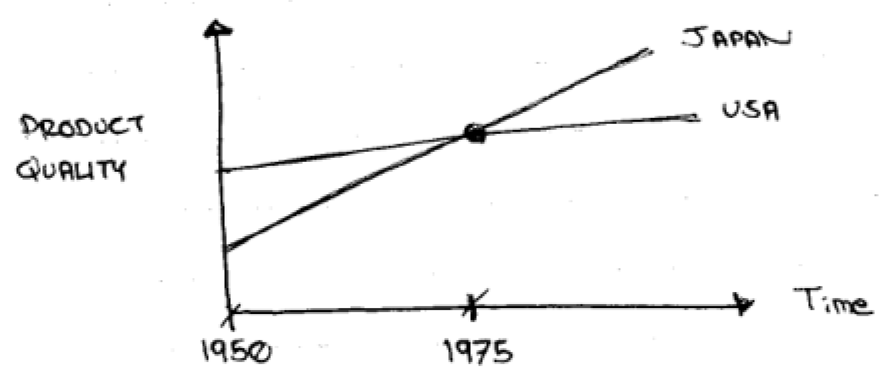
Definition: A formal definition of maintenance is that a function of manufacturing management that is concerned with day-to-day problems of keeping the physical plant in good operating condition.

But maintenance is not limited to manufacturing, so, there are many definitions:
“all actions necessary for retaining an item, or restoring it, a serviceable condition, including, servicing, repair, modification, overhaul, inspection and condition verification.”

Maintenance and reliability
“ The objective of maintenance and reliability is to maintain the capability of the system while controlling costs.”

“Maintenance is all activities involved in keeping a systems equipment in working order.”

“Reliability is the probability that a machine will function properly for a specified amount of time.”



Consider:

- Kaizen improvement
- Decline of US auto
- Steady improvement of Japanese automakers

“Maintenance is the combination of all technical, administrative, and management actions during the life cycle of an item intended to retain it in, or restore it to, a state in which it may perform the required function” (CEN 2001). CEN stands for European Committee for Standardization.

Maintenance Engineering and Maintenance
US Department of defence sees maintenance engineering as a discipline that assists in acquisition of resources in performing or accomplishing maintenance.

In contrast, maintenance activities are viewed as those that use resources in physically performing those actions and tasks attendant on the equipment maintenance function for test, servicing, repair, calibration, overhaul, modification, and so on.

Maintenance engineering is an analytical function as well as it is deliberate and methodical.

In contrast, maintenance is a function that must be performed under normally adverse circumstances and stress, and its main objective is to rapidly restore the equipment to its operational readiness state using available resources.

Objectives of maintenance engineering:

- Improves maintenance operations
- Reduce the frequency and amount of maintenance
- Reduce the effect of complexity
- Reduce the maintenance skills required
- Establish optimum frequency and extent of preventative maintenance to be carried out.
- Improve and ensure maximum utilization of maintenance facilities.
- Improve maintenance organization.

The scope of modern maintenance management covers every stage in the life cycle of technical systems (plant, machinery, equipment and facilities):

- Specification
- Acquisition
- Planning
- Operating
- Performance evaluation
- Improvement
- Disposed

When perceived in this wider context, the maintenance function is also known as Physical Asset Management (PAM). The performance demanded of PAM has become more challenging as a result of four developments these days.

1. <u>Emerging Trends of Operation Strategies</u>

- The concept of economy of scale is losing scale
- An increasing number of organizations have switched to lean manufacturing, just-in-time production, and six-sigma operations.
- Shift of emphasis from volume to quick response, elimination of waste, reduced stock holding, and defect prevention.
- Installation of the right equipment and facilities optimization of maintenance of these assets, and the effective deployment of staff to perform maintenance activities are crucial factors to support these operation strategies.

2. <u>Toughening Societal Expectations</u>

- Widespread acceptance, at least in the developed countries
- Of the need to preserve essential services
- Protect the environment
- Safeguard people's safety and health

So,

- Wide range of regulations have been enacted in these countries to control industrial pollution and prevent accidents in the workplace.

Scrap, defects, and inefficient use of materials and energy are source of pollution.

- They are often the result of operating plant and facilities under less than optimal conditions.

Keeping facilities in optimal condition and preventing critical failures are effective means of managing the risks of service interruptions, pollution, and industrial accidents.

3. <u>Technological Changes</u>
With technology changing at a breathtaking rate, the condition of equipment can be monitored continuously or intermittently while it is in operation.

This has given birth to condition-based maintenance (CBM), an alternative to the classis preventative maintenance.

The development of new technologies is instrumental to enhancing system availability, improving cost-effectiveness, and delivering better or innovative services to customers.

4. <u>Increased emphasis on sustainability</u>
Sustainability demands all developments to "meet the needs of present without compromising the ability of future generations to meet their own needs."

Total cost of ownership, life cycle performance, energy consumption, and safety are the parameters that can be effectively optimized by the application of appropriate methodologies of and tools for PAM.

Improvement in PAM can be accompanied by:
- Having a clear strategy
- The right people and systems
- Appropriate tactics
- Controlled work through planning and scheduling, maintenance optimization, and process engineering.

A survey of maintenance budgets ranged from 2% to 90% of the total planned operating budget, with average being 20.8%.

Maintenance excellence is concerned with balancing:
- Performance
- Risks
- And resource impacts to achieve optimal solutions

Structured approach to achieving & maintaining excellence:

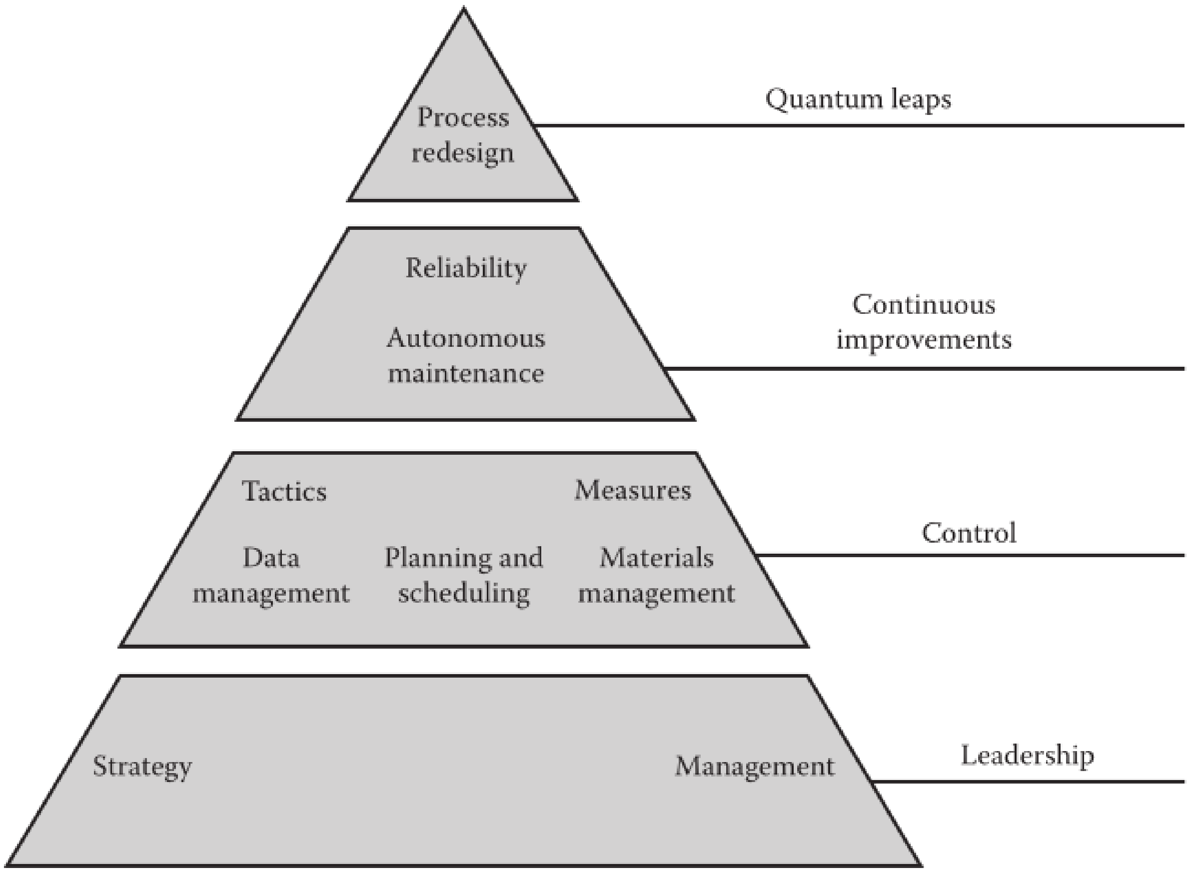


FIGURE 1.1 Structured approach to achieving maintenance excellence.

Tactical options:

- Time-based maintenance actions
- Time-based discord
- Condition-based monitoring (CBM)
- Run-to-failure
- Fault-finding tests

Continuous improvement:

- Two complementary methodologies to enhance the reliability (up time) of physical assets.
- Total productive management (TPM)
- Reliability centered management (RCM)
- TPM is people-centered
- RCM is asset-centered

<u>PAS 5S</u>: a framework for optimized management of physical assets.

PAS 5S is a publicly available specification, the status of which is between codes of standards and ISO standards.
It offers a framework for a holistic, systematic approach to optimize the management of physical assets.

PAS 5S-1 - Asset management Part I: Specification for the optimized management of physical assets.
PAS 5S-2 - Asset management Part II: Guidance for the application of PAS 5S-1.

PAS 5S is not sector specific. It is applicable to organizations with any type of distribution of physical assets and asset ownership structure.
Reliability through the operator: Total productive maintenance (TPM)

TPM is a people-centered methodology.

It has been proven to be effective for optimizing equipment effectiveness and eliminating breakdowns. It mobilizes the machine operators to play an active role in maintenance work by cultivating in these frontline workers a sense of ownership of facilities they operate, and enlarging their job facilities to include routine servicing and minor repair of these machines.

Through this type of operation participation in maintenance activities, TPM aims to eliminate the six big losses of equipment effectiveness:

- Breakdown
- Setup and adjustment
- Idling and minor stoppages
- Reduced speed
- Defects in process
- Reduced yield

To achieve zero breakdowns, hidden defects in the machine need to be expected and corrected before they have deteriorated to the extent that they will cause the machine to break down.

To do this:
- Maintain equipment in good operating condition through proper cleaning and effective lubrication
- Restore the condition of deteriorated parts
- Enhance the operation, setup, inspection and maintenance skills of operators

Traditionally, these duties fall outside the responsibility of machine operator, whose role is nothing else but operate the machine.

When it breaks down, the operator's duty is to request maintenance to fix.

Thus, TPM involves a restructuring of work relating to equipment maintenance.

Machine operator are empowered to perform routine maintenance, servicing, and minor repairs.

This concept of operator involvement is enhancing equipment wellness is known as autonomous maintenance**.**

It is cultivated through 5S and CLAIR.

5S is a tool for starting the journey towards world class competitiveness.

It is a team effort that involved everyone in the organization to create a productive workplace by keeping it safe, clean, and orderly.

5S stands for:
- Sorting
  - Separate the needed from not needed.
  - Identify items that you use frequently. Sort, tag, and dispose of the unneeded items.
- Simplifying
  - A place for everything and everything in its place.
  - Once you have determined what you need, organize it and standardize its use to increase your effectiveness.
- Systematic Cleaning:
  - Making things ready for inspection.
  - Regular cleaning helps to solve problems before they become too serious by identifying sources and root cause of potential problems.
- Standardizing:
  - Create common methods to achieve consistency.
- Sustaining:
  - Constant maintenance, improvement, and communication.

5S has become a continuous improvement process.

CLAIR – clean, lubricate, adjust, inspect, (minor) repair

- Have operators work with maintenance toward the common goals of stabilizing equipment conditions, and halting accelerated deterioration.

- The operators are empowered to perform the basic tasks of cleaning, checking lubrication, simple adjustments, inspections, and replacement of parts, minor repairs, and other simple maintenance tasks.

By providing them with training on equipment functions and functional failures, the operations will also prevent failure through early detection and treatment of abnormal conditions.

Being relieved of the routine tasks of maintenance, the experts in maintenance unit can be deployed to focus on more specialized work, such as major repairs, overhauls, tracking and improving equipment performance, and replacement or acquisition of physical assets.

Reliability by Design
Reliability Centered Maintenance (RCM)

TPM is people focused. Its emphasis is on the early detection of wear out to prevent in-service failures.

RCM is an alternative approach to enhancing asset reliability by focusing on designs.

It asks questions such as: Do we have to do maintenance at all?

Will a design chance eliminate the root cause of failure?

What kind of maintenance is most likely to meet the organizations business objectives?

RCM is a structured methodology for determining the maintenance requirement of a physical asset in its operating context.

The primary objective of RCM is to preserve system function rather than to keep on asset in service.

Application of the RCM requires a full understanding of the functions of physical assets and the nature of the functions of physical assets and the nature of failures related to those functions.

Some failures can not be prevented by overhaul or preventative maintenance.

Thus, maintenance actions that are not cost effective in preserving system functions will not be performed.

Benefits of RCM:
- Improved understanding of the equipment. How it fails and consequences of failure.
- Clarify the roles that operators and maintenance play in making equipment more reliable and less costly to operate.
- Make the equipment safer, more environmentally friendly, more productive, more maintainable, and more economical to operate.

Results of RCM applications reported in various industry sectors have been published:
- Manufacturing
- Utility
- Mining
- Military

The RCM Methodology developed the appropriate maintenance tactics using a thorough and rigorous decision process in several steps.

Step 1: Select and Prioritize Equipment

- Production and supporting processes are examined to identify key physical assets.
- These key physical assets are then prioritized according to how critical they are to operations, cost of downtime, and cost of repair.

Step 2: Define functions and performance standards

- The functions of each system selected for risk analysis need to be defined.
- The functions of equipment are what it does.
- Some systems are dormant until some other event occurs, as in safety systems.
- Each function also has a set of operating limits.
- The parameters define the normal operation of the function under a specialized operating environment.

Step 3: Define function failures

- When the system operates outside its normal parameters, it is considered to have failed.
- Failures can be total, partial, and intermittent.

Step 4: Identify failure modes / root causes

- A failure mode is how the system fails to perform its functions.
- A cylinder may be stuck in one position because of a lack of lubrication.
- The functional failure in this case is the failure to provide linear motion, but the failure mode is the loss of lubricant properties of the hydraulic fluid.
- A failure may have more than one possible root cause.
- This step identifies the chain of events when a failure occurs.
- What conditions need to exist? What event was necessary to trigger the failure?

Step 5: Determine failure effects and consequences

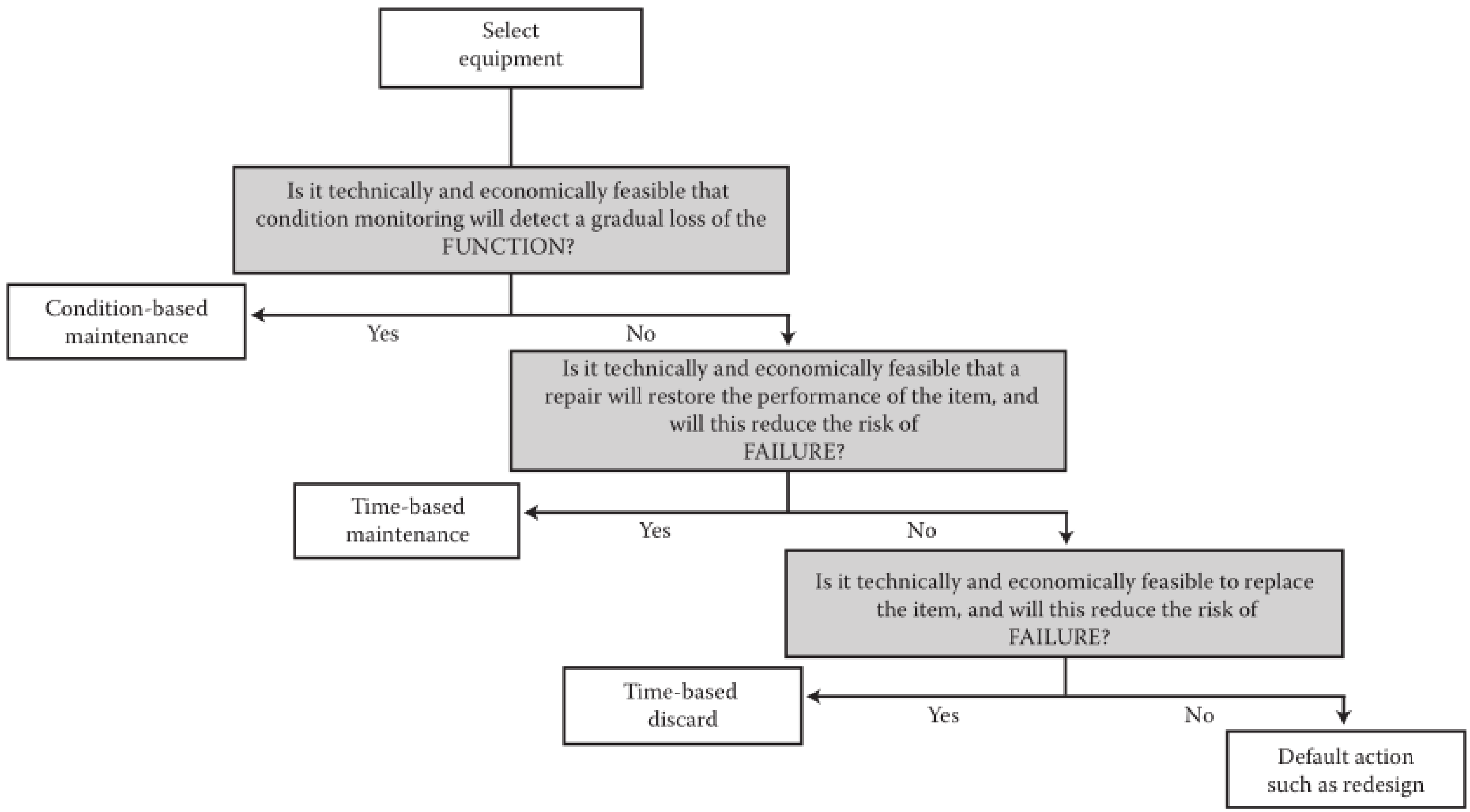
- What will happen when a functional failure occurs?
- The severity of the failures effect on safety, the environment, operation, and maintenance is assessed.

The results of analysis made in steps 2 to 5 are documented in a failure mode, effect, and criticality analysis worksheet.

Step 6: Select Maintenance Tactics

- A decision logic tree is used to select the approximate maintenance tactics for the various functional failures.

RCM Methodology Logic



If time-based maintenance, intervention or periodic inspection has been selected, the frequency of such a task needs to be determined to achieve optimal results.

Step 7: Implement and refine the maintenance plan

- Need multidisciplinary team with members knowledgeable in the day-to-day operations of the plant and equipment as well as in the details of equipment itself.

Optimizing maintenance and replacement decisions:

RCM determines the type of maintenance tactics to be applied to an asset, while it answers the question of “what type of maintenance actions need to be taken?”

The issue of when to perform the recommended maintenance actions that will produce the best results possible remain to be addressed.

The optimization of these tactical decisions is the important issues addressed

The optimization of these tactical decisions is the important issue addressed in the top of the “continuous improvement” layer of the maintenance excellence pyramid.

Traditionally, maintenance practitioners in industry were expected to cope with maintenance problems without seeking to operate in optimal manner.

Many preventative maintenance schemes are put into operation with only a slight, if any, qualitative approach to the scheme.

Asset managers who wish to optimize the life cycle value of the organizations human and physical assets must consider four key decision areas.

1. Component Replacement
2. Inspection Procedures
3. Capital Equipment Replacement
4. Resource Requirement

The course will be looking at several models for these items.

Chapter 2: Component Replacement Decisions

Two types of situations:

1. Deterministic
2. Probabilistic

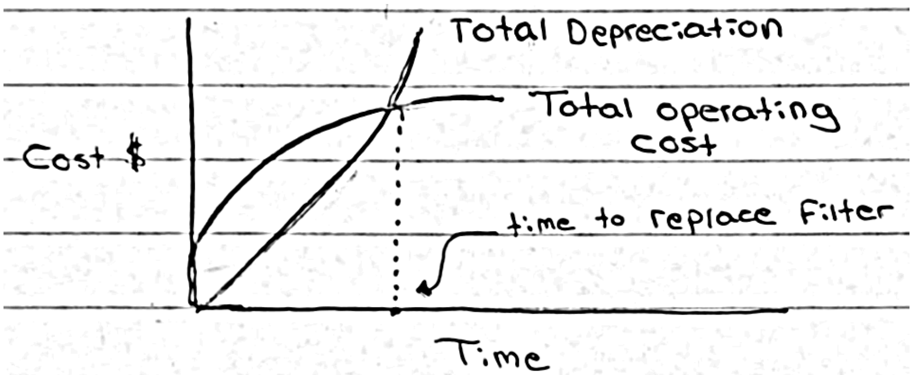
Optimal Replacement times for equipment whose operating cost increases with use:

Some equipment operates with excellent efficiency when it is new, but as it ages, its performance deteriorates.

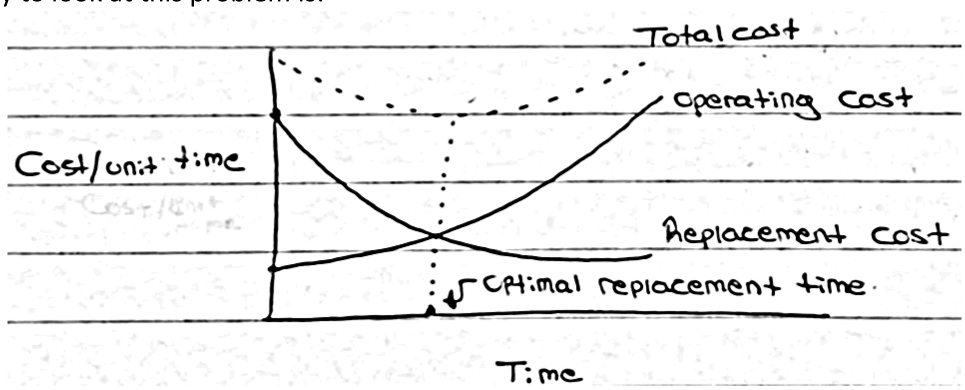
It has some resale value that keeps on decreasing with time.

The decreasing resale value results in increasing depreciation, which is the difference between the purchase price and the resale value.

The optimal replacement policy for such items is to replace the equipment at a point where the total cost curve intersects the total depreciation curve.

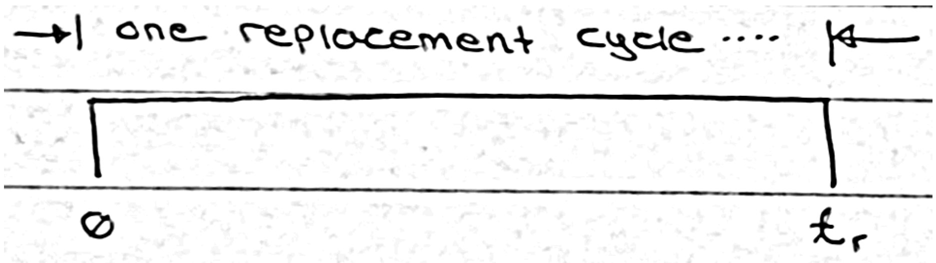


Another way to look at this problem is:



This class of problem can be called a short-term deterministic problem, for a long-term situation time value of money becomes very important and complicated.

Construction of the model:
$c(t)$ is the operating cost per unit time at time t after replacement cycle
C_r is the total cost of a replacement
The replacement policy is to perform replacements at intervals of length t_r



The objective is to determine the optimal interval between replacements to minimize the total cost of operation and replacement per unit time.

$$C(t_r) = \frac{total\ cost\ in\ interval\ (0, t_r)}{length\ of\ interval}$$

Total cost in interval $((0, t_r) = cost\ of\ operating + cost\ of\ replacement$

$$= \int_0^{t_r} c(t)dt + C_r$$

So,

$$C(t_r) = \frac{1}{t_r}\left[\int_0^{t_r} c(t)dt + C_r\right]$$

The optimal replacement time is determined from calculus.

$\frac{d}{dt_r}C(t_r) = 0$ for determining minimum

$$= -\frac{C_r}{t_r^2} - \frac{1}{t_r^2}\int_0^{t_r} c(t)dt + \frac{1}{t_r}c(t_r)$$

$$c(t_r) = \frac{C_r}{t_r} + \frac{1}{t_r}\int_0^{t_r} c(t)dt$$

$$= \frac{1}{t_r}\left[\int_0^{t_r} c(t)dt + C_r)\right]$$

$= C(t_r)$ The total cost per unit time.

So, the optimal replacement time is when current operating cost rate is equal to the average total cost per unit time.

From economic point of view, the optimal time to replace is when the marginal cost is equivalent to the average cost.

Special case: The trend in operating cost is linear:

$$c(t) = a + bt$$

Then:

$$\int_0^{t_{r^*}} c(t)\,dt = \int_0^{t_{r^*}} (a + bt)\,dt = at_{r^*} + \frac{bt_{r^*}^2}{2}$$

Then:

$$\frac{1}{t_{r^*}} \int_0^{t_{r^*}} (a + bt)\,dt = a + \frac{bt_{r^*}}{2}$$

And:

$$C(t_{r^*}) = a + bt_{r^*} = a + \frac{bt_{r^*}}{2} + \frac{C_r}{t_{r^*}}$$

$$\frac{bt_{r^*}}{2} = \frac{C_r}{t_{r^*}}$$

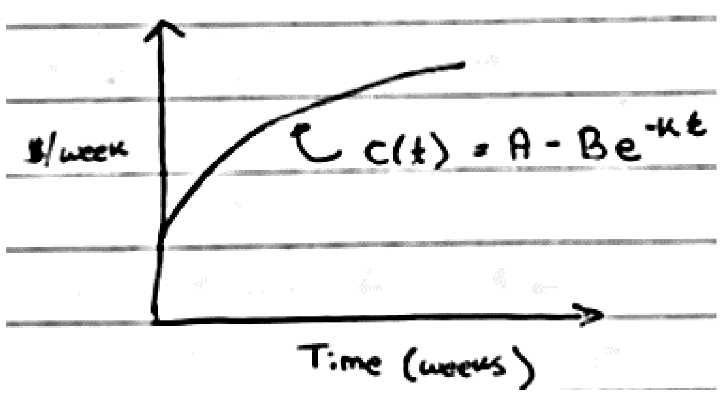
$$t_{r^*} = \sqrt{\frac{2C_r}{b}}$$

In using the derived relationship, $c(t_r) = C(t)$ for optimal replacement, there is an implicit assumption that the operating cost is increasing with time, and this is a realistic assumption.

If the trend in operating cost is not continuous, but discrete, then the optimal replacement time is when the next period's operating cost is equal to or greater than the current average cost of replacement to that time.

Example: The trend in operating cost for an item is of the form:

$$c(t) = A - Be^{-Kt}$$



Where
$A = \$100$
$B = \$80$
$K = 0.21/week$

$A - B \geq 0$ may be interpreted at the operating cost of unit time if no deterioration occurs.

K is a constant describing the rate of deterioration.

Given C_r, the total cost of replacement, is $\$100$.

Thus:

$$C(t_r) = \frac{1}{t_r}\left[\int_0^{t_r} (100 - 80e^{-0.218})\, dt + 100\right]$$

Analytical solution (closed form)

Using the result, $c(t_r) = C(t_r)$ cannot be obtained.

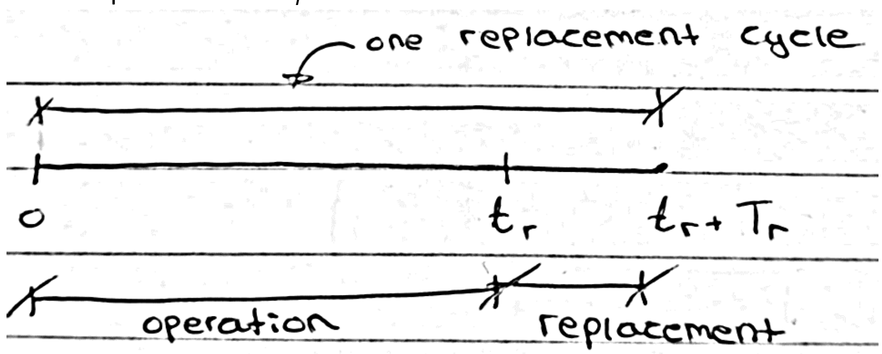
Numerical solution:

t_r	1	2	3	4	5	6	7
$C(t_r)$	127.8	84.7	74.0	70.9	70.5	71.5	72.5

So, $t_r \approx 5\ weeks$

So far, we did not consider the time required to perform a replacement.

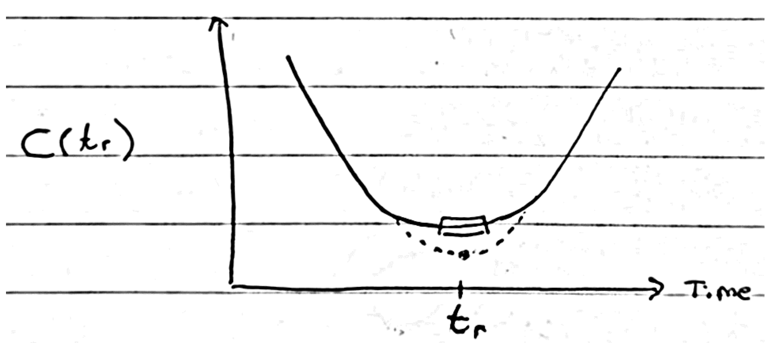
If the time to perform replacement $= T_r$



$$C(t_r) = \int_0^{t_r} \frac{c(t)dt + C_r}{t_r + T_r}$$

Any costs, such as production losses incurred due to the duration of the replacement, need to be incorporated into the cost of the replacement action.

To further assist the engineer in deciding what an appropriate replacement policy should be, it is usually useful to plot the total cost per unit time versus time curve.



It shows the form of the total cost around the optimum.

If the curve is fairly flat around the optimum, it is not really very important that the engineer should plan for the replacement to occur exactly at the optimum, thus giving some leeway in scheduling the work.

If the total cost curve is not flat around the optimum, then the optimal interval should be adhered to if at all possible.

If there is uncertainty about the value of the particular parameter required in the analysis, then evaluation of the total cost curve for various values of this uncertain parameter should be performed, the replacement policy depends upon the sustainability of the analysis with respect to the uncertain parameter.

Sensitivity checking gives guidance on what information is important from a decision-making viewpoint and, consequently, what information should be gathered in a data collection scheme.

Stochastic Preventative Replacement

Before developing component replacement models, it should be noted that preventative replacement actions require two necessary conditions.

1. The total cost of the replacement must be greater after the failure than before (The total cost of the replacement must be greater after failure than before (if cost is the appropriate criterion; otherwise, an appropriate criterion, such as downtime, is substituted in place of cost).

   This may be caused by a greater loss of production because replacement after failure is unplanned or failure of one piece of the plant may cause damage to other equipment.

2. The hazard rate (failure rate) of the equipment must be increasing. It is not worthwhile to perform preventative replacement if hazard rate is constant or decreasing.

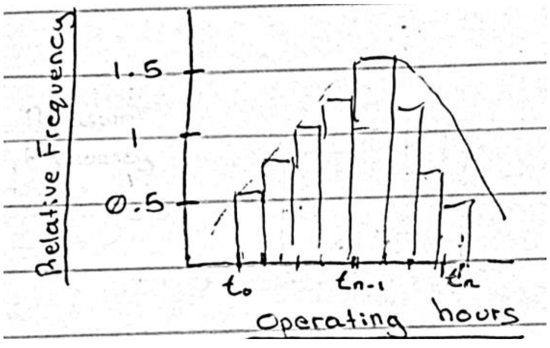
Maintenance Mathematics

Decisions relating to probabilistic maintenance problems, such as deciding when to perform maintenance on equipment that is subject to breakdown, require information about when the equipment will reach a failed state.

It is difficult to know when failure will occur, but it is possible to assign probability of failure.

For that we need knowledge of statistics.

Relative Frequency Histogram



Where the area under the curve is 1.

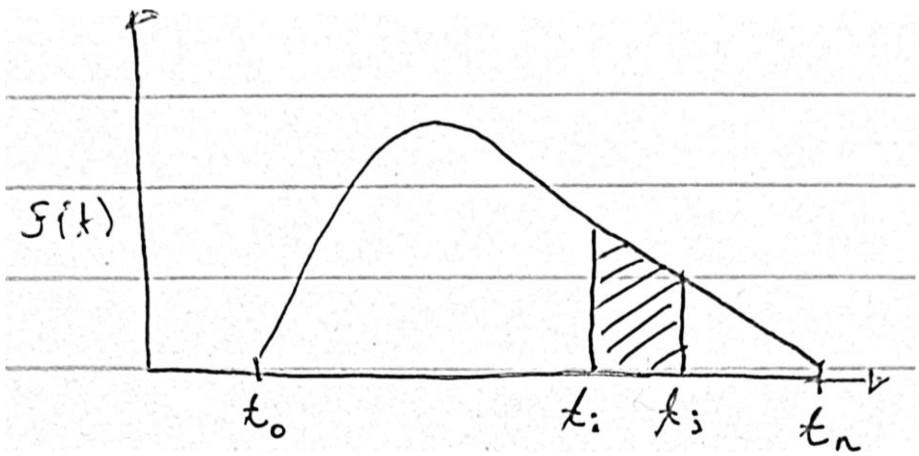
If we wish to determine the probability of failure occurring between times t_i and t_{i-1} we simply multiply the ordinate y by the interval $(t_i - t_{i-1})$.

The probability of a failure occurring between t_o and t_n, where t_o and t_n are the earliest and latest times for failure respectively, at which time all pieces have failed, is unity.

<u>Probability Density Function</u>
In maintenance studies, we tend to use probability density function rather than relative frequency histograms.

This is because:
1. The variable to be modeled, such as time to failure, is a continuous variable.
2. These functions are easier to manipulate.
3. It should give a clear understanding of the failure distribution.

The probability density functions (pdfs) are similar to RF histograms except that a continuous curve is used instead of bars.



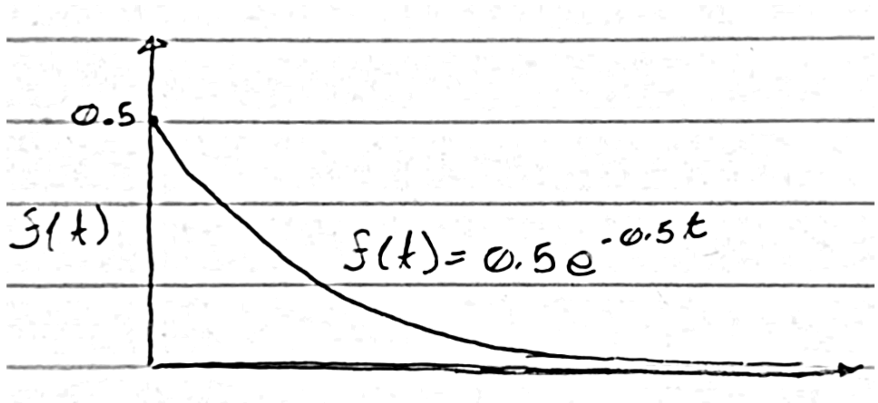
The probability (risk) of a failure occurring between t_i and t_j is the hatched area.

From calculus, this is:

$$\int_{t_i}^{t_j} f(t)dt$$

The equation of the curve of the pdf is denoted by $f(t)$.

For example, if we have $f(t) = 0.5e^{-0.5t}$, we get the following curve:



This is a pdf of an exponential distributions.

The area under the probability density curve is equivalent to j.

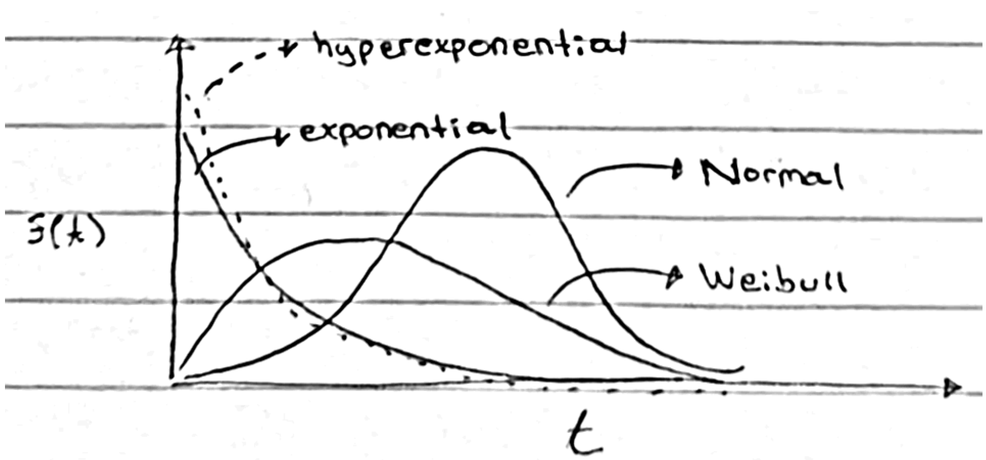
The probability of a failure occurring between times t_o and t_∞ is then:

$$\int_{t_i}^{\infty} f(t)dt = 1$$

The failure characteristics of different items of equipment is likely to be different from each other.

Even the failure characteristics of identical equipment may not be the same if they are operating in different environments.

There are a number of well-known pdfs that have been found in practice to describe the failure characteristics of equipment.



Hyperexponential Distribution

When equipment has a failure time that can be very short or very long, its failure distribution can be obtained by hyperexponential distribution.

Computers have been found to fail according to this distribution.

In the hyperexponential distribution, the short time to failure occur more often then in the negative exponential distribution, and similarly, the long times to failure occur more frequently than in the exponential case.

The density function of the hyperexponential distribution:

$$f(t) = 2x^2\lambda \exp[-2K\lambda t] + 2(1 - K^2)\lambda \exp[-2(1 - K)\lambda t]$$
$$\text{For } t \geq 0 \text{ with } 0 < K \leq 0.5$$

Where:
λ is the arrival route of breakdowns and
K is a parameter of the distribution

<u>Exponential Distribution</u>
This is one of the most widely used probability distributions in engineering, particularly in the reliability work.

It is relatively easy to handle in conducting analysis.

This arises in practice wherein failure of the equipment can be causes by failure of any one of a number of components of which the equipment is comprised.

It is also characteristic of equipment subject to failure due to random causes, such as sudden excessive loading.

This distribution is found to be typical for many electronic components and complex industrial plants.

The pdf is:

$$f(t) = \lambda\, e^{-\lambda t}$$
$$\text{For } t \geq 0 \text{ with } \lambda > 0$$

Where:
λ is the arrival rate breakdown (Called the distribution parameter)
$1/\lambda$ is the distribution parameter

In-class project:
Make groups of 4-5 for class presentation.

Group leader to email me names of group by next Thursday.

Choose any topic related to maintenance.

Main objective is to improve your communication skills.

Normal Distribution: Most widely used statistical distribution

This applies when a random outcome (such as time to failure) is the additive effect of a large number of small and independent random variables.

In practice, the lifetime of light bulbs and the time until the first failure of bus engines have been found to follow a normal distribution.

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right]$$
$$-\infty < t < \infty$$

Were μ is the mean and σ the standard deviation of the distribution.

For normal distribution:

Where: $\int_0^\infty f(t)\,dt < 1$

But: $\int_{-\infty}^\infty f(t)\,dt = 1$

In practice, however, if the mean of the normal distribution, μ, is considerably removed from the origin $t = 0$ and variance σ^2, is not too large, than it is acceptable to use the normal distribution as an approximate to the real situation.

Weibull Distribution: Fits a large number of failure characteristics of equipment.

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$
$$t \geq 0$$
$$\beta > 0$$
$$\eta > 0$$

Cumulative Distribution Function

Probability of failure before time t

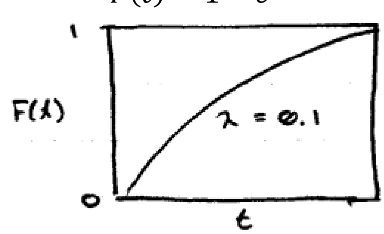
$$= \int_0^t f(t)dt$$

The integral $\int_0^t f(t)dt$ is denoted by $F(t)$ and is termed the cumulative distribution function.

As $t \to \infty, F(t) = 1$

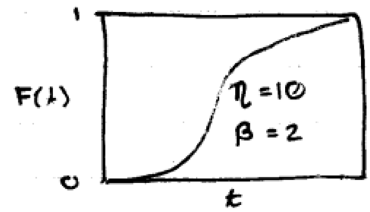
Exponential Distribution

$$F(t) = 1 - e^{\lambda t}$$



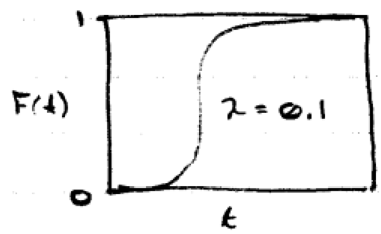
Normal Distribution

$$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{t-\mu}{2\sigma^2}\right)^2\right]$$



Weibull Distribution

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$



For this course, you should be familiar with standard normal distribution table (Appendix 9) – will give xeroxed pages of the table in exams!

The cumulative distribution function, $F(t)$, of a normal distribution with mean $= \mu$ and standard deviation $= \sigma$ can be determined from the standard table in Appendix 9.

It tabulates the value of $1 - \phi(z)$, where:

$$z = \frac{t-\mu}{\sigma}$$

is a standardized normal distribution variable and $\phi(z)$ is the cumulative distribution function of the standard normal distribution.

Thus, the table provides the probability that the standardized normal variable chosen at random is greater than a specified value of z.

The normal distribution being symmetrical about its mean, $\phi(-z) = 1 - \phi(z)$.

Thus, only the probability for $z \geq 0$ is tabulated.

Reliability Function: (also known as survival function)

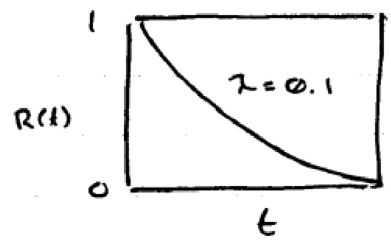
It is determined from the probability that the equipment will survive at least to some specified time t.

The reliability function is denoted by $R(t)$ and is defined as:

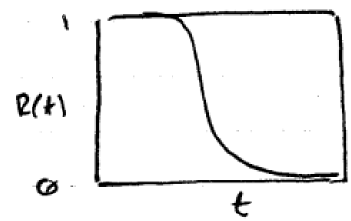
$$R(t) = \int_{0}^{\infty} f(t)\, dt$$

$$R(t) = 1 - F(t)$$
$$t \to \infty, \; R(t) = 0$$

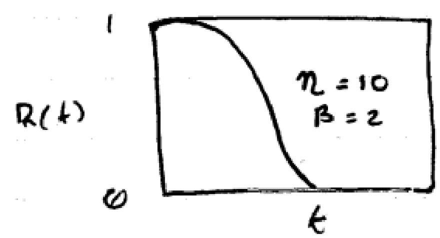
For exponential distribution:



For Normal distribution:



For Weibull distribution:



For exponential distribution:

$$R(t) = e^{-\lambda t}$$

For normal distribution:

$$R(t) = \frac{1}{\sigma\sqrt{2\pi}} = \int_0^{\infty} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$

For Weibull distribution:

$$R(t) = e^{\left(-\frac{t}{\eta}\right)^{\beta}}$$

Consider an item that is operational time t, when a mission starts.

We want to determine the probability of the item surviving the mission of duration t.

The required measure can be expressed in the usual notation of conditional probability as:

$$R(t_1 + t \mid t) = P(T \geq t_1 + t \mid T \geq t_1)$$

$$= \frac{P(T \geq t_1 + t)}{P(T \geq t_1)} = \frac{R(t_1 + t)}{R(t_1)}$$

$$= \frac{\int_{t_1+t}^{\infty} f(t)\, dt}{\int_{t_1}^{\infty} f(t)\, dt}$$

Where T is the time to failure.

If the failure time follows an exponential distribution:

$$R(t_! + t \mid t_1) = \frac{\int_{t_1+t}^{\infty} f(t)\, dt}{\int_{t_1}^{\infty} f(t)\, dt} = \frac{\int_{t_1+t}^{\infty} \lambda e^{-\lambda t}\, dt}{\int_{t_1}^{\infty} \lambda e^{-\lambda t}\, dt}$$

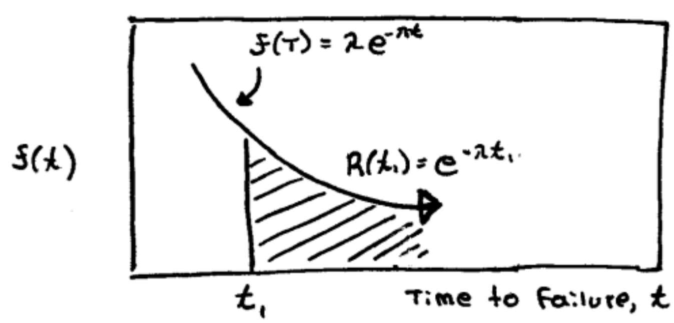
$$= \frac{e^{-\lambda(t_1+t)}}{e^{-\lambda t_1}} = e^{-\lambda t} = R(t)$$

Thus, for operational items with failure times that are exponentially distributed,

$$R(t_! + t \mid t_1) = R(t)$$

In words, their chance of survival (or conversely, their risk of failure) in the next instance is independent of their current age.

This memoryless property is unique to the exponential distribution, the only continuous distribution with this feature.



Hazard rate: The hazard rate of an item is the probability that the item will fail in the next interval of time given that it is good at the start of the interval, that is, it is a conditioned probability.

Consider a test in which a large number of identical components are put into operation and the time to failure of each component is noted.

An estimate of the hazard rate of a component at any point in tie may be thought of as the ratio of a number of items that failed in an interval of time to the number of items in the original population that were operational at the start of the interval.

Specifically, letting $h(t)\delta t$ to be the probability that an item fails during a short interval δt, given that it has survived to time t, the usual notation.

$P(A|B) =$ probability of event A occurring once it is known B has occurred.
Where A is the event "failure occurs in interval δt" and B is the event that no failure occurred up to time "t".

$P(A|B)$ is given by:

$$P(A|B) = \frac{P(A\ and\ B)}{P(B)}$$

$$P(A\ and\ B) = \int_t^{t+\delta t} f(t)\,dt$$

$$P(B) = \int_t^{\infty} f(t)\,dt$$

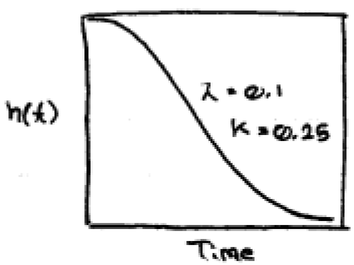
So, the hazard rate in interval δt is:

$$h(t)\delta t = \frac{\int_t^{t+\delta t} f(t)\,dt}{\int_t^{\infty} f(t)\,dt}$$

So, $h(t) = \frac{f(t)}{1-F(t)}$

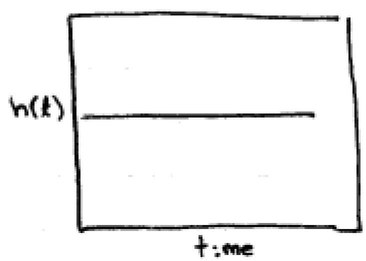
$As\ \delta t \to 0$

<u>Hyper-exponential:</u>



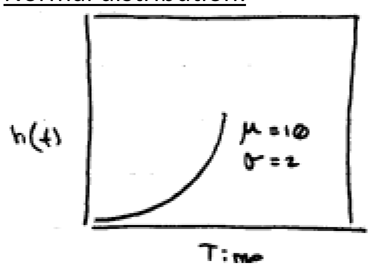
$$h(t) = \frac{2\lambda\left[k^2 + (1-k)\exp[-2(1-2k)\lambda t]\right]}{K + (1-k)\exp[-2(1-2k)\lambda t]}$$

<u>Exponential:</u>



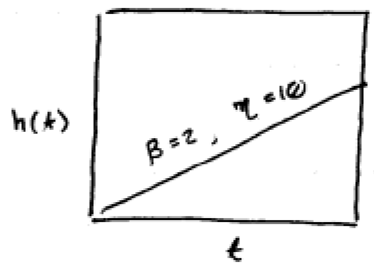
$h(t) =$

<u>Normal distribution:</u>



$$h(t) = \frac{\exp\left[-(t-\mu)^2/2\sigma^2\right]}{\int_t^{\infty} \exp\left[\frac{-(t-\mu)^2}{2\sigma^2}\right]dt}$$

Weibull distribution:



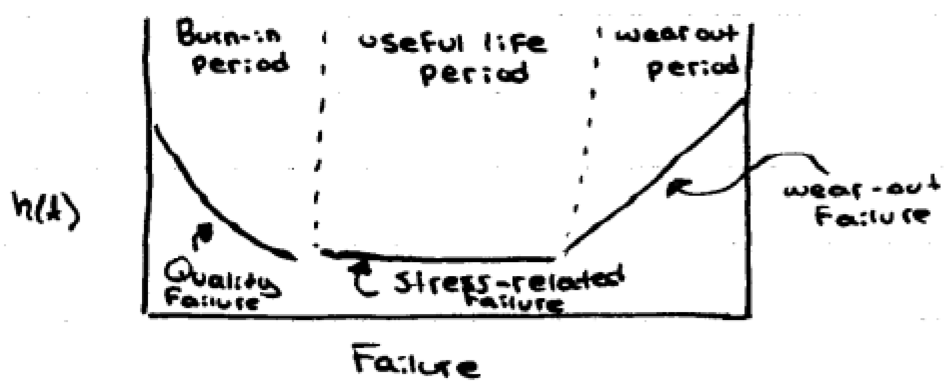
$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

When the hazard rate increases with time, such as for the normal and Weibull distribution, it indicates an aging or wear-out effect.

With the exponential distribution, the hazard rate is constant. This failure pattern can be the result of completely random events such as sudden stresses and extreme conditions.

It also applies to the steady-state condition of complex equipment which fails when any one of a number of independent constituent component breaks, or when any one of a number of failure modes occur.

For complex equipment, the hazard rate look like:



This is called the "Bathtub curve"

Bathtub curve:
$A =$ a burn in or running-in period
$B =$normal operation in which failures that occur are predominantly due to choice
$C =$ deterioration, i.e. wear-out due to age

How to determine the most appropriate policy to adopt when equipment is in one of the regions A, B, C.

If the only form of the maintenance possible is replacement, either on a preventative basis, or because of failure, then in regions A and B no preventative replacement should be applied because such replacements will not reduce the risk of equipment failures.

If preventative replacements are made in regions A and B, maintenance effort is wasted.

Unfortunately, this is often the case in practice because it is often mistakenly assumed that as equipment ages, the risk of failure will increase.

In region C, preventative replacement will reduce the risk of equipment failure in the future, and just when these preventive replacements should occur will be influenced by the relative costs or other relevant impact factors, such as downtime of preventative and failure replacement.

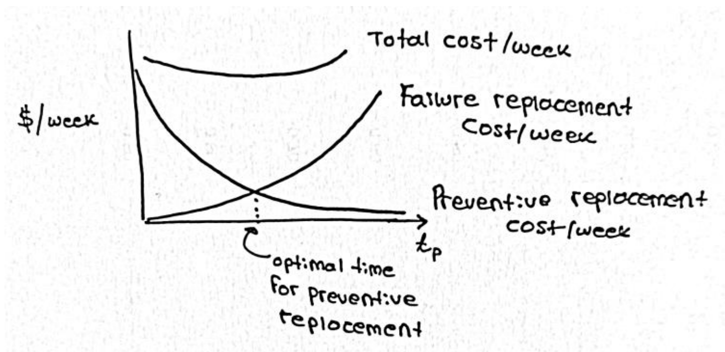
Optimal Preventive Replacement Interval of Items Subject to Breakdown (Also known as the Group or Block Policy).

An item is subject to sudden failure, and when failure occurs, the item has to be replaced.

Because failure is unexpected, a failure replacement is usually more costly than a preventive replacement.

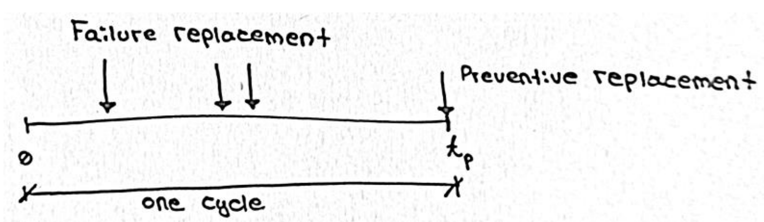
To reduce the number of failures, preventive replacements can be scheduled to occur at specified intervals.

A balance has to be made because too short an interval for preventive replacement is expensive and so is too long an interval.



The replacement policy is one in which preventive replacements occur at fixed interval of time, and failure replacements occurs whenever necessary.

Construction of a model:
C_P is the total cost of a preventive replacement
C_f is the total cost of failure replacement
$f(t)$ is the probability density function of the item's failure times



The objective is to determine the optimal interval between preventive replacements to minimize the total cost per unit time.

$$C(t_P) = \frac{total\ expected\ cost\ in\ interval\ (0, t_p)}{length\ of\ interval}$$

Total expected cost in interval $(0, t_p)$
$= cost\ of\ a\ preventive\ replacement + expected\ cost\ of\ failure\ replacements$
$= C_P + C_f \cdot H(t_p)$

Where $H(t_p)$ is the expected number of failure in interval $(0, t_p)$

So,

$$C(t_p) = \frac{C_P + C_f \cdot H(t_p)}{t_p}$$

Differentiate the right side of the equation with respect to t_p and equate it to zero

$$-\frac{C_P}{t_r^2} - \frac{C_f \cdot H(t_p)}{t_p^2} + \frac{C_f \cdot h(t)}{t_p} = 0$$

$$C_P = t_p \cdot C_f \cdot h(t_p) - C_f \cdot H(t_p)$$

$$t_p \cdot h(t_p) - H(t_p) = \frac{C_P}{C_f}$$

($h(t_p)$ is the derivative of $H(t_p)$ and is termed renewal density]

<u>How to determine $H(t)$:</u>

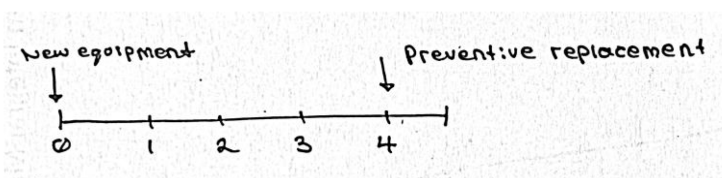
Two methods: Renewal Theory Approach is too much statistical, so we will not do it.

We will do:

Discrete Approach to establishing $H(t)$:

$H(t)$ is the expected number of failures in $(0, t)$

Consider a case in which there are 4 weeks between preventive replacements.



Then $H(t)$ is the expected number of failures in interval (0, 4), starting with new equipment.

When we start at time zero, the first failure (if there is one) will occur during either the first, second, third, or fourth week of operation.

So,
$H(4) =$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the first week · probability of the first failure occurring in the first interval $(0, 1)$
$+$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the second week · probability of the first failure occurring in the first interval $(1, 2)$
$+$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the third week · probability of the first failure occurring in the first interval $(2, 3)$
$+$ number of expected failures that occur in interval $(0, 4)$, when the first failure occurs in the fourth week · probability of the first failure occurring in the first interval $(3, 4)$

Assume that not more than one failure can occur in any weekly interval (Because the length of interval can be made short is needed).

Number of expected failures that occur in interval $(0, 4)$ when the first failure occurs in the first week

= the failure that occurred in the first week + the expected number of failure in the remaining three weeks

$= 1 + H(3)$

We use $H(3)$ because we have a new equipment as a result of replacing the failed equipment in the first week, and we have 3 weeks to go before the preventive replacement occurs.

By definition, the expected number of failures in the remaining 3 weeks, starting with the new equipment is $H(3)$.

The probability of the first failure occurring in the first week:

$$= \int_0^1 f(t)\, dt$$

Similarly, in consequence of the of the first failure occurring in the second, third, or fourth weeks

So,

$$H(4) = [1 + H(3)] \int_0^1 f(t)\, dt + [1 + H(2)] \int_1^2 f(t)\, dt + [1 + H(1)] \int_2^3 f(t)\, dt + [1 + H(0)] \int_3^4 f(t)\, dt$$

Obviously, $H(0) = 0$ That is, with zero weeks to go, the expected number of failures is zero.

So,

$$H(4) = \sum_{i=0}^{3} [1 + H(3 - i) \int_i^{i+1} f(t)\, dt$$

$$H(0) = 0$$

In general,

$$H(T) = \sum_{i=0}^{T-1} [1 + H(T - i - 1)] \int_i^{i+1} f(t)\, dt$$

This is called a recurrence relation with $T \geq 1$ and $H(0) = 0$

Because we know $H(0) = 0$, we can get $H(1)$, $H(2)$, $H(3)$ etc.

<u>Optimal Preventive Replacement Age of an Item Subject to Breakdown:</u>

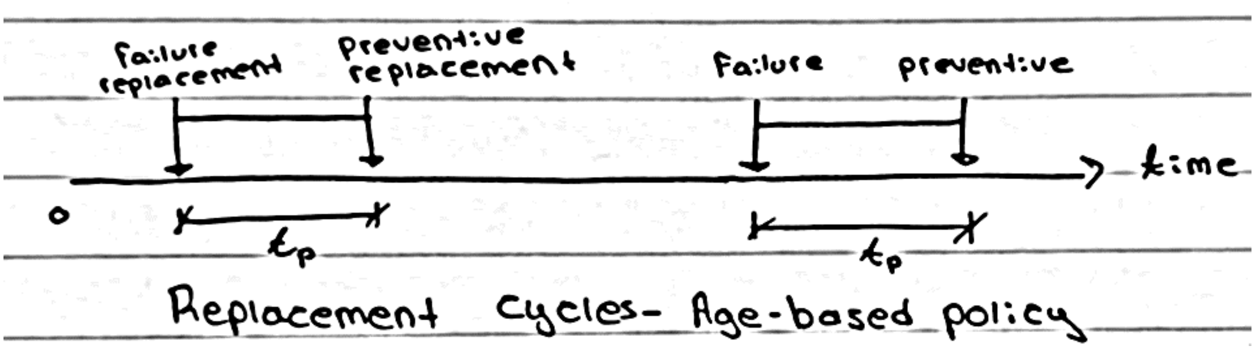
Instead of making preventive replacements at fixed intervals, with the possibility of performing a preventive replacement shortly after a failure replacement, the time at which preventive replacement occurs depends on the age of the item.

When failures occur, failure replacements are made.

When this happens, the time clock is reset to zero, and the preventive replacement occurs only when the item has been in use for a specified period.

So, the problem is to balance the cost of the preventive replacements against their benefits, and we do this by determining the optimum preventive replacement age for the item to minimize the total expected cost of replacements per unit time.

Construction of the model:

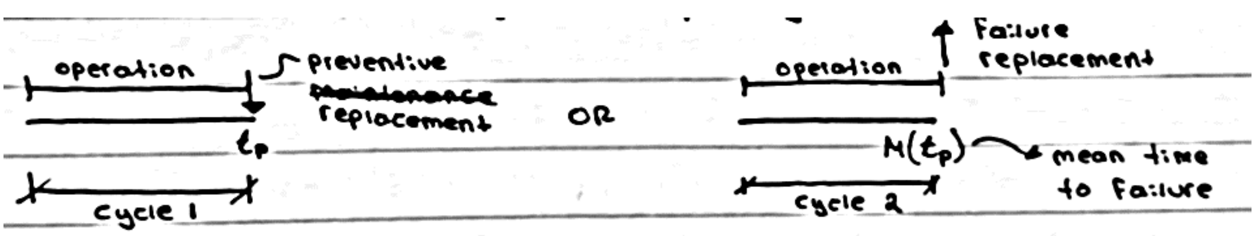


.C_p is the total cost of preventive replacement

C_f is the total cost of failure replacement
$f(t)$ is the probability density function of the failure times of the item
t_p is the time for preventive replacement (if there is failure before reaching t_p, the clock is reset for the next preventive replacement)

In this problem, there are two possible cycles of operation, one cycle being determined by the item reaching the planned replacement age t_p; and the other being determined by the equipment failing before reaching t_p.

Two possible cycles



Possible replacement cycles for Age based policy.

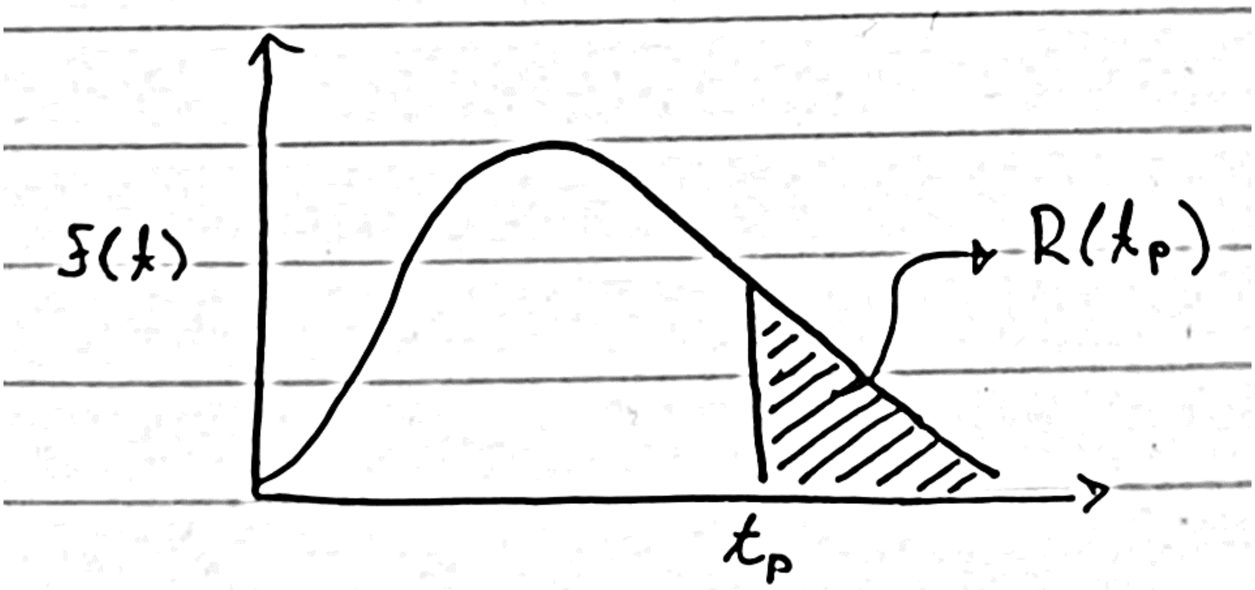
The total expected replacement cost per unit time, $C(t_p)$, is:

$$C(t_p) = \frac{Total\ expected\ replacement\ cost\ per\ cycle}{Expected\ length\ of\ cycle}$$

Total expected replacement cost per cycle
= cost of preventive cycle · probability of a preventive cycle
+ cost of failure cycle · probability of a failure cycle

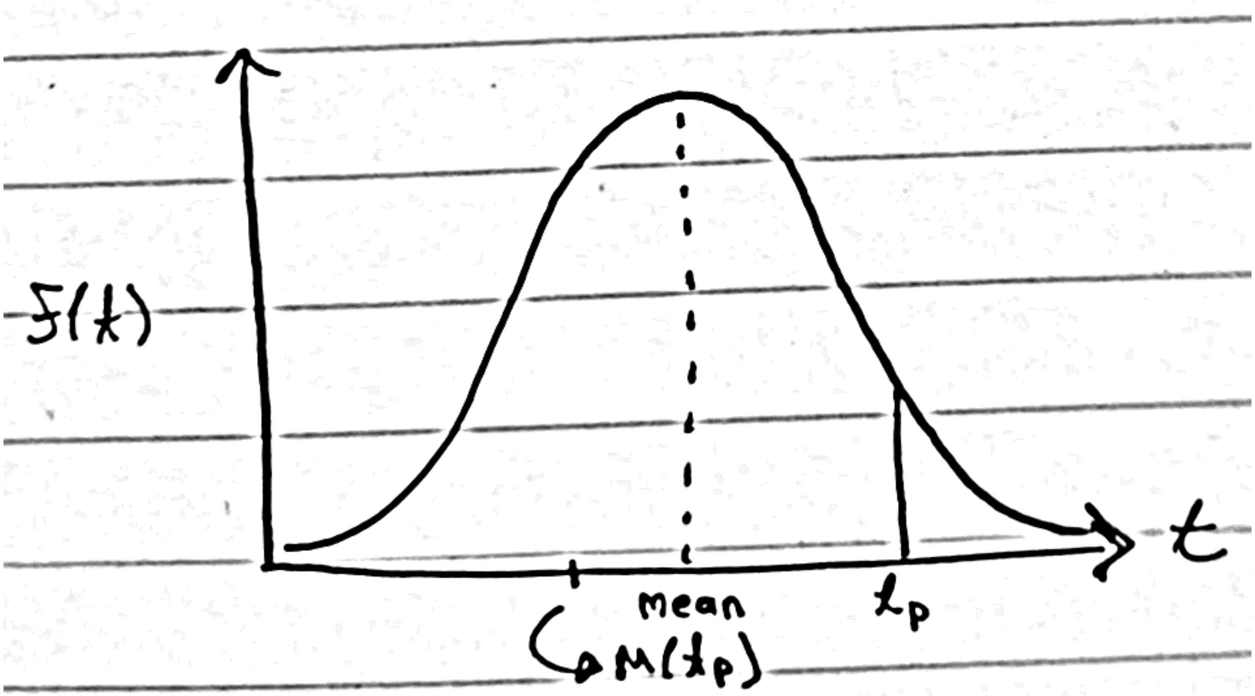
$$= C_p \cdot R(t_p) + C_f \cdot [1 - R(t_p)]$$

The probability of a preventive cycle is equivalent to the probability of failure occurring after time t_p, that is, equivalent to the shaded area, which is denoted as $R(t_p)$.



The probability of a failure cycle is the probability of failure occurring before time t_p, which is the unshaded area $[1 - R(t_p)]$

Expected cycle length
$=$ length of the preventive cycle · probability of a preventive failure
$+$expected length of a failure cycle · probability of a failure cycle



Mean time to failure: $M(t_p)$

Mathematically,

$$M(t_p) = \int_{-\infty}^{t_p} \frac{t\, f(t)\, dt}{1 - R(t_p)}$$

(Need not memorize)

$M(t_p)$ is essentially determined from the mean of area up to t_p

So,
Expected cycle length
$= t_p \cdot R(t_p) + M(t_p) \cdot [1 - R(t_p)]$

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \cdot [1 - R(t_p)]}{t_p \cdot R(t_p) + M(t_p) \cdot [1 - R(t_p)]}$$

This is a model of the problem relating the replacement age to the total expected replacement cost per unit time.

No simple solution to obtain the minimum (optimal) t_p for this problem.

Need trial values of several points to determine the optimal t_p.

(A misprint in the book)

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f \cdot [1 - R(t_p)]}{t_p \cdot R(t_p) + \int_{-\infty}^{t_p} t\, f(t)\, dt}$$

For Midterm:
1 or 2 descriptive questions on what we covered
1 or 2 derivations on what we covered
Numerical examples to be posted

Optimal Preventative Replacement Age of an item subject to breakdown, taking account of the time required to carry out failure and preventive replacements.

So far, we assumed zero time for performing replacements.

Actually, there is time required to perform replacements.

The optimal preventive replacement age of the item is again taken as that age which minimizes the total expected cost of replacement per unit time.

Construction of the model:

C_p is the total cost of a preventive replacement

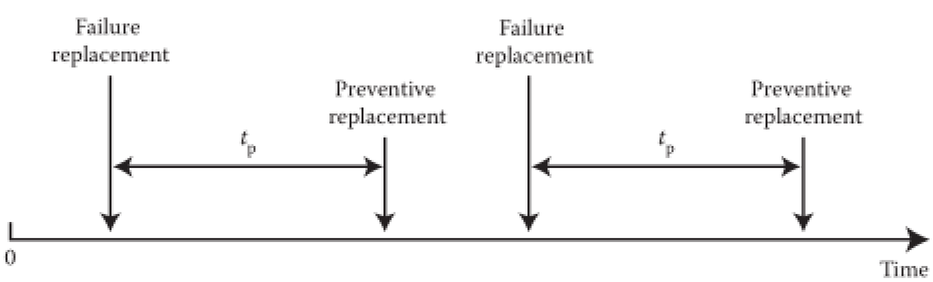
C_f is the total cost of a failure replacement

T_p is the mean time required to make a preventative replacement

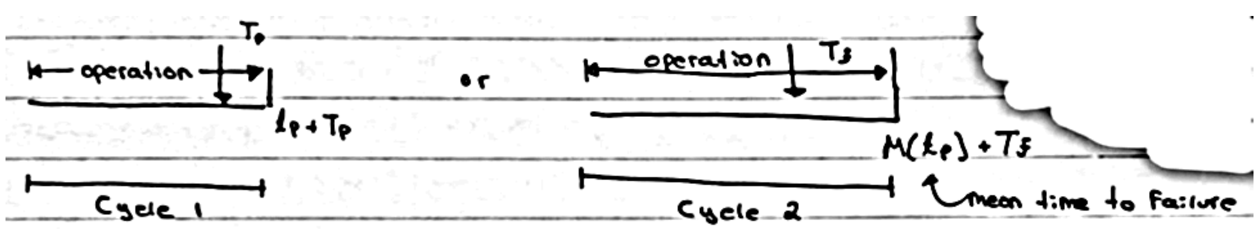
T_f is the mean time required to make a failure replacement

$f(t)$ is the probability density function of the failure times of the item

The replacement policy is to perform a preventative replacement once the item has reached a certain age, t_p, plus failure replacements as needed.



The possible cycles of operation:



The total expected replacement cost per unit time denoted $C(t_p)$, is:

$$C(t_p) = \frac{total\ expected\ replacement\ cost\ per\ cycle}{expected\ cycle\ length}$$

Total expected replacement cost per cycle

$$= C_p \cdot R(t_p) + C_f\left[1 - R(t_p)\right]$$

Expected cycle length
= length of a preventive cycle · probability of a preventive cycle
+ expected length of a failure cycle · probability of a failure cycle.
$= (t_p + T_p)R(t_p) + [M(t_p) + T_f][1 - R(t_p)]$

$$C(t_p) = \frac{C_p \cdot R(t_p) + C_f\,[1 - R(t_p)]}{(t_p + T_p)R(t_p) + [M(t_p) + T_f][1 - R(t_p)]}$$

This is a model of the problem relating preventive replacement age, t_p, to the total expected replacement cost per unit time.

Sometimes downtime is more important, so, the objective becomes minimizing the downtime.

Optimal Preventive Replacement Interval or Age of an item subject to breakdown: Minimization of downtime.

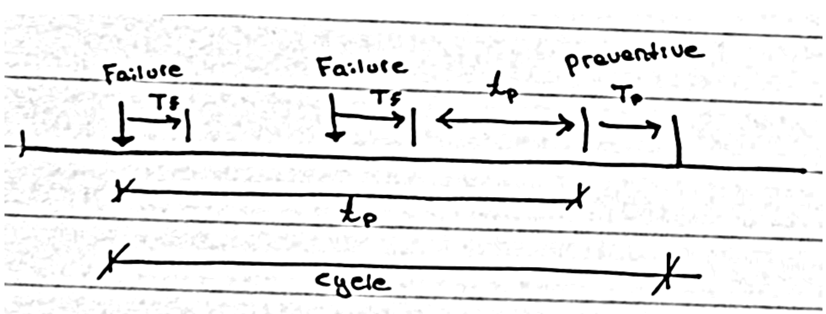
So far – minimized total cot per unit time.

In some cases, the replacement policy required may be one that minimizes total downtime per unit time or, equivalently, maximizes availability.

As the preventive replacement frequency increased, there is an increase in downtime due to these replacements, but a consequence of this is a reduction of downtime due to failure replacements, and we wish to get the best balance between them.

Construction of the model:
T_f is the mean downtime to make a failure replacement.
T_p is the time required to make a preventive replacement.
$f(t)$ is the probability density function of the failure time of the item.

Model 1: Determination of Optimal Preventive Replacement Interval



The total downtime per unit time, for preventive replacement at time t_p, denoted as $D(t_p)$

$$D(t_p) = \frac{Expected\ downtime\ due\ to\ failure + downtime\ due\ to\ preventive\ replacment}{Cycle\ length}$$

Downtime due to failures = Number of failures in interval $(0, t_p)$ x time required to make a failure replacement.

$$= H(t_p) \cdot T_f$$

Downtime due to preventive replacement $= T_p$

So,

$$D(t_p) = \frac{H(t_p)T_f + T_p}{t_p + T_p}$$

Is the model of the problem relating replacement interval t_p to total downtime $D(t_p)$

Group Replacement:

The lamp replacement problem: The cost of transporting a lighting department's maintenance staff to a single streetlight failure and discounts associated with bulk purchase of lamps, it may be economically justifiable to replace all the lamps on a street rather than only the failed ones.

Statement of the problem:

A large number of similar items are subject to failure.

Whenever an item fails, it is replaced by a new item.

We do not assume group replacement in such conditions.

There is also possibility that group replacements can be performed at fixed intervals of time.

The cost of replacing an item under group replacement conditions is assumed to be less than that for failure replacement.

A balance is needed.

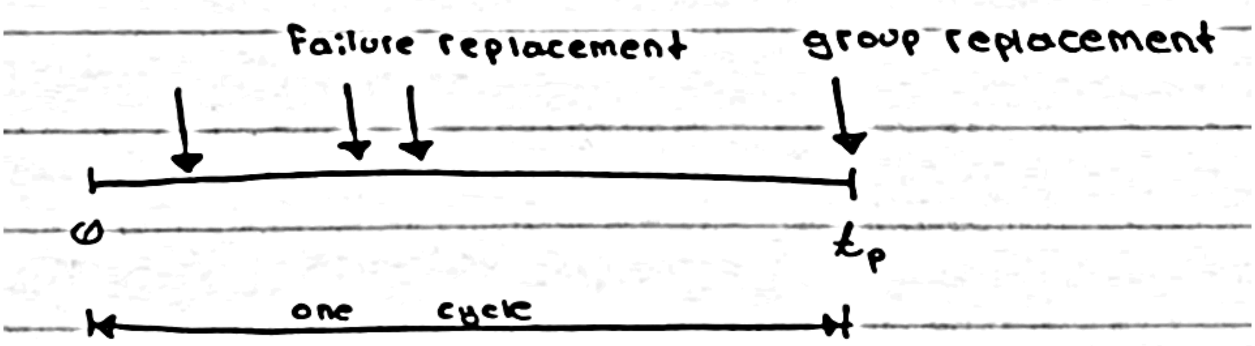
Model:

C_g – cost of replacing one item under conditions of group replacement

C_f – cost of replacing one item under failure replacement

$f(t)$ – the probability density function of the failure of the items

N – total number of items in the group



The total expected replacement cost per unit time for group replacement at time t_p,

$$C(t_p) = \frac{Total\ expected\ costin\ interval\ (0, t_p)}{Interval\ Length}$$

Total expected cost in interval $(0, t_p) =$

Cost of group replacement at time t_p + expected cost of failure replacements in interval $(0, t_p)$

So,

Total expected cost in interval $(0, t_p) = NC_g + NH(t_p)C_f$

$H(t_p)$ is the expected number of times the item fails in interval $(0, t_p)$

So the model is,

$$C(t_p) = \frac{NC_g + NH(t_p)C_f}{t_p}$$

To find the optimal time t_p, we need trial and error.

Multistage replacement:
A multistage replacement strategy may be relevant in the situation in which there is a group of similar items that can be divided into subgroups dependent on the cost of replacing an item upon its failure.

One such strategy: Assume cost of replacement in stage 1 is greater than stage 2.

In this case, the failures that occur in stage 2 are replaced by operating items in stage 1. Vacancies that occur in Stage 1, whether caused by failure or transfer of operating items in Stage 2, are replaced by new items.

(example, if a failure occurs in a rear tire in a trailer, it is replaced by a tire in the front wheel and a new tire put in the front wheel)

<u>Optimal Policies:</u>
Frequently, an item ceases to operate not because of its own failure but because there is a production stoppage for some reason.

When this happens, the maintenance specialists may have to device whether to take advantage of the downtime opportunity to perform a preventative replacement.

<u>Reparable Systems:</u>
So far, we only discussed replacement of items.

Many items are replaceable.

How best to handle the optimization of maintenance decisions associated with repairable items?

The terms minimal and general repair are frequently used.

A minimal repair can be through of as a very minor maintenance action (such as replacing a snapped fan belt on an automobile), that retains the equipment to the same state of health it was just before the minor maintenance action.

A general repair improves the system state.

Spare parts provisioning: Spare parts are required for each preventive and failure replacements.

Need models that can be useful in forecasting the inventory needed.

Construction of the model:
$t_p -$ preventive replacement time (either interval or age)
$f(t)$ −probability density function of the item's failure times
$T -$ planning horizon, typically 1 year

$EN(T, t_p)$ is the expected number of spare parts required over the planning horizon, T, when preventive replacement occurs in time t_p

$EN(T, t_p) =$ number of preventive replacement in interval $(0, t_p) +$ Expected number of failure replacement in interval $(0, T)$

$$= \frac{T}{t_p} + M(t_p)\left(\frac{T}{t_p}\right)$$

Chapter 3 -Inspection Decisions

The purpose of inspections is to determine the state of equipment.

Once indicators such as bearing wear, gage readings, and quality of the product, which are used to describe the state, have been specified, and the inspection made to determine the values of the indicators, some further maintenance action may be taken, depending on the state specified.

When, specifically, the inspection should take place ought to be influenced by the costs of the inspection.

Benefit vs. cost of equipment is important.

The primary goal is to make a system more reliable through inspection.

Optimal Inspection frequency – maximization of profit.

Equipment break down from time to time and that requires repair and leads to downtime → costs money.

To reduce the number of breakdowns, we can periodically inspect the equipment.

Inspection also shuts down the equipment, although for a shorter time.

Need an inspection policy that will give the correct balance between the number of inspections, and the resulting output, such that the profit per unit time for the equipment is maximized out over a long period.

A complex system can fail for many reasons, such as that caused by component 1, component 2, and so on.

Each of these causes of equipment failure could have its own independent failure distribution.

As the frequency or intensity of inspection increases, there is an exception that the frequency of equipment/system failure will be reduced.

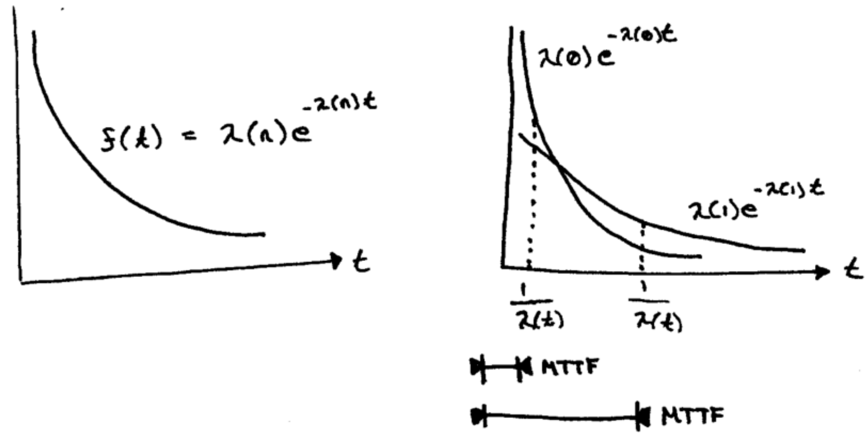
Construction of the Model –

1. Equipment failures occur according to the exponential distribution with mean time to failure (MTTF) = $1/\lambda$, where λ is the mean arrival rate of the failure.

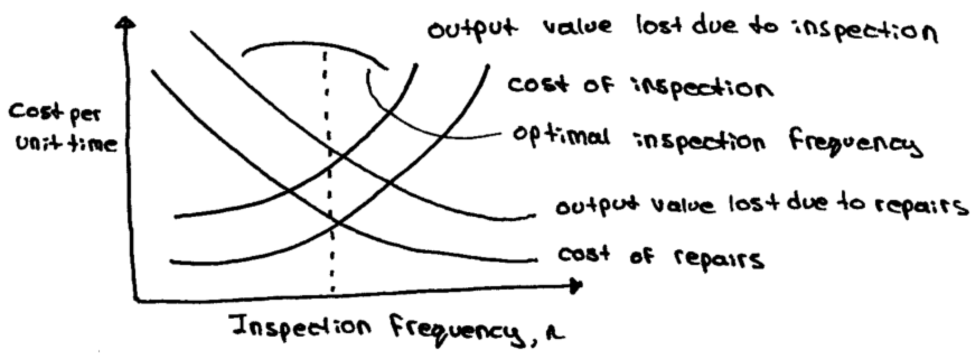
   So, if the MTTF = 0.5 years, then the mean number of failures per year is equal to $\frac{1}{0.5} = 2\ (\lambda = 2)$
2. Repair times are exponentially distributed with a mean time of $\frac{1}{\mu}$
3. The inspection policy is to perform n inspections per unit time

   Inspection times are exponentially distributed with a mean time $\frac{1}{i}$
4. The value of the output in an uninterrupted unit of time has a profit value V (e.g. selling price less material cost less production cost)

   That is, V is the profit value per unit time if there are no downtime issues.
5. The average cost of inspection per uninterrupted unit of time is I
6. The average cost of repairs per uninterrupted unit of time is R
7. The breakdown rate of equipment, λ, is a function of n, the frequency of inspection per unit time.



8. $\lambda(0)$ is the breakdown rate if no inspection is made, and $\lambda(1)$ is the breakdown rate is one inspection is made per unit time.
   So, the effect of performing inspections is to increase the $MTTF$ of the equipment



We can denote the profit per unit time by $P(n)$ ($i.e. P$ is the function of n)

$P(n) =$ Value of output per uninterrupted unit of time
− output value lost due to repairs per unit time
− output value lost due to inspections per unit time
− cost of repair per unit time
− cost of inspections per unit time

Output value lost due to repairs per unit time = value of output per uninterrupted unit of time · number of repairs per unit time · mean time to repair

$$= V\,\lambda(n)/\mu$$

Output value lost due to inspections per unit time = value of output per uninterrupted unit of time · number of inspections per unit time · mean time to inspect

$$= \frac{Vn}{i}$$

Cost of repairs per unit time =
Cost of repairs per uninterrupted unit of time · number of repairs per unit time · mean time to repair

$$= \frac{R\lambda(n)}{\mu}$$

Cost of inspections per unit time =
Cost of inspections per uninterrupted unit time · Number of inspections per unit time · mean time to inspect

$$= \frac{In}{i}$$

So,

$$P(n) = V - \frac{V\lambda(n)}{\mu} - \frac{Vn}{i} - \frac{R\lambda(n)}{\mu} - \frac{In}{i}$$

This is a model of the problem relating inspection frequency n to the profit $P(n)$

Assume $P(n)$ to be a continuous function of n, so

$$\frac{dP(n)}{dn} = -V\frac{\lambda'(n)}{\mu} - \frac{V}{i} - \frac{R\lambda'(n)}{\mu} - \frac{I}{i}$$

(where $\lambda'(n)$ is the derivation of $\lambda(n)$)

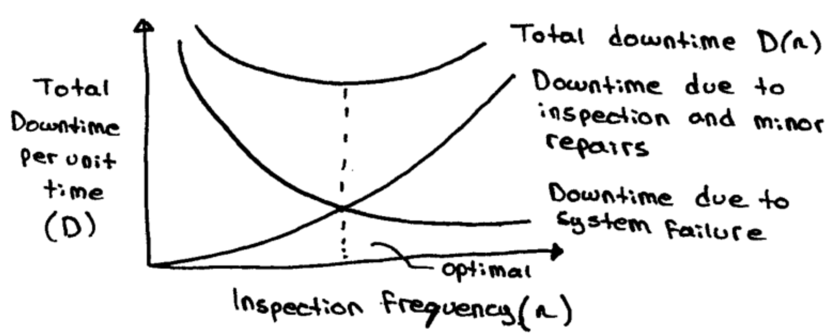
$$\frac{dP(n)}{dn} = 0 = \frac{\lambda' n}{mu}(V+R) + \frac{I}{i}(V+i)$$

$$\lambda'(n) = \frac{\mu}{i}\left[\frac{V+I}{V+R}\right]$$

If values of μ, i, V, R, I and the form of $\lambda(n)$ are known, the optimal frequency can be calculated.

Many time maximization of profit and minimization of downtime are equivalent but not always.

Optimal inspection frequency minimization of downtime.



$f(t)$, $\lambda(n)$, n, $1/\mu$ and $1/i$ are same as before.

The objective is to determine n to minimize total downtime per unit time.

$D(n) =$ Downtime incurred due to minor repairs per unit time + downtime incurred due to inspections per unit time

$$= \frac{\lambda(n)}{\mu} + \frac{n}{i}$$

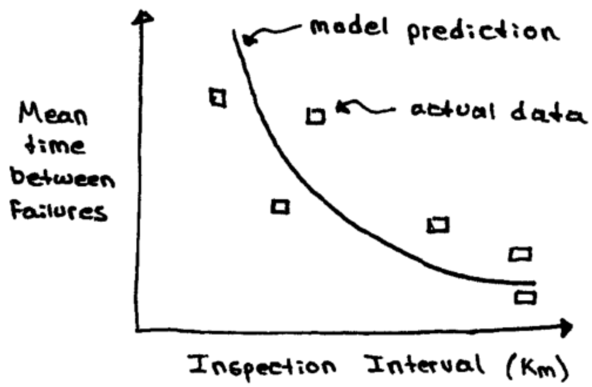
Can not find optimal solution analytically.

An application: Optimal vehicle fixed inspection schedule.

Montreal Transit = 2000 buses in fleet

Inspection policy was to inspect its buses at $5000\ km$ intervals at which an $A, B, C,$ or D depth of inspection took place.

Depth A at $5, 15, 25, 35$ (thousands of km)
Depth B at $10, 30, 50, 70$
Depth C at $20, 40, 60$
Depth D at 80



From trial and error they found $8000\ km$ was optimal for inspection interval instead of $5000\ km$.

Optimal inspection interval to maximize the availability of equipment used in emergency conditions, such as a protective device.

Equipment such as fire extinguishers and many military weapons are stored for use in an emergency.

If the equipment can deteriorate while in storage, there is a risk that it will not function when it is called into use.

To reduce the probability that equipment will be inoperable when required, inspections can be made usually called proof-checking, and if equipment is found to be in failed state, it can be repaired or replaced, this returning it to as-new conditions.

Examples of protective devices:
- Fire hydrants on city streets
- Standby diesel generators for running lights
- Full-face oxygen masks in aircraft
- Automatic transfer switches for power supply
- Methane gas detectors in underground coal mine
- Protective relays in electrical distribution
- Fire suppression equipment on vehicles
- Eyewash station in chemical plant
- Life rafts on ship

We need to establish the optimal inspection interval for protective devices, and this interval is called the failure-finding interval (FFI)

The reliability centered maintenance (RCM) methodology addresses this form of maintenance.

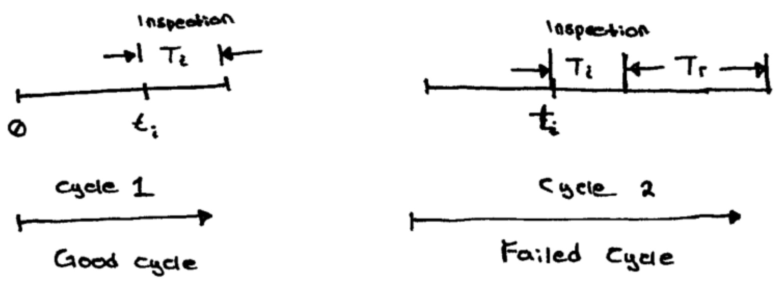
Failure-finding applies only to hidden or unrevealed failures.

Hidden failures in turn only affect protective devices.

Sometimes availability is of importance (which essentially is a function 1-downtime)

For analysis of availability:

1. $f(t)$ is the probability density function of the time to failure of the equipment.
2. T_i is the time required to carry out an inspection. It is assumed that after the inspection, if no major faults are found, the equipment is in the as-new state.
3. T_r is the time required to make a repair or replacement. After the repair or replacement, it is assumed that the equipment is in the as-new state.
4. The objective is to determine the interval t_i between inspections to maximize availability per unit time.



The availability per unit time will be a function of the inspection interval t_i

$$A(t_i) = \frac{Expected\ availability\ per\ cycle}{Expected\ cycle\ length}$$

The expected uptime (the expected availability) per cycle is:

$$t_i \cdot R(t_i) + \frac{\int_o^{t_i} t\, f(t)\, dt}{1 - R(t_i)}[1 - R(t_i)]$$

$$= t_i \cdot R(t_i) + \int_o^{t_i} t\, f(t)\, dt$$

The expected cycle length is:

$$= (t_i + T_c)R(t_i) + (t_i + T_i + T_r)[1 - R(t_i)]$$

$$A(t_i) = \frac{t_i \cdot R(t_i) + \int_o^{t_i} t\, f(t)\, dt}{t_i + T_i + T_r[1 - R(t_i)]}$$

Chapter 4 – Capital Equipment Replacement Decisions

Definitions of Capital Items:

Capital items are of considerable value and durability and are used to provide service or to make, market, keep, or transport periods (businessdictionary.com).

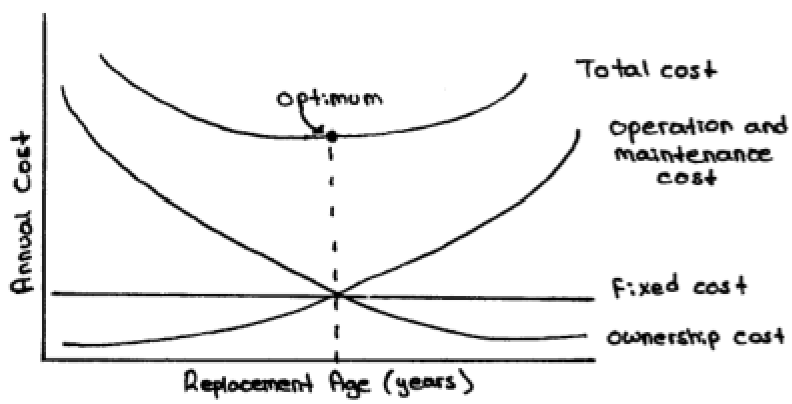
Capital equipment present tangible and intangible goods that are preserved by organizations and that present the technical prerequisites for the production of goods and services. One characteristic of capital equipment is that performance of use with the possible inclusion of services of provision, maintenance and repair; another characteristic is the high value of an individual object compared with the material used (Hoffman et al, 2012).

Most businesses establish criteria for designating acquired items as either capital or noncapital items.

These criteria are based partly on local tax items, but they also represent accounting policy choices by management.

The criteria usually specify that capital items must have a minimum useful life (for example, one year or more), have an acquisition cost above a certain threshold (e.g. $5000 or more), and contribute value to the business.

Capital equipment problems tend to be treated deterministically:



There may be additional costs incurred, associated with utilization of an item, that are independent of the age at which the asset is replaced. These are identified in fixed unit.

Fixed costs do not affect the economic life decisions.

Optimal Replacement Interval for Capital Equipment: Minimization of Total Cases

Objective is to determine an optimum replacement policy that minimizes that total discounted costs derived from operating, maintaining, and disposing of the equipment over a long period.

It will be assumed that equipment is replaced by an identical item, this returning the equipment to the as new condition after replacement.

This restriction is not valid when technological improvement is considered.

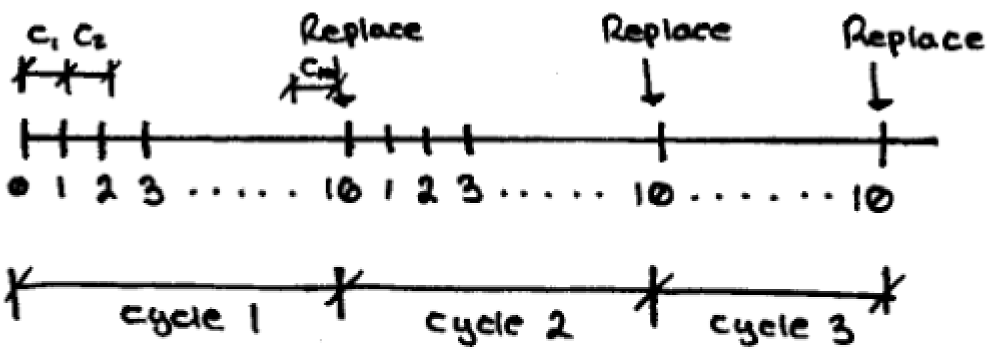
It is assumed that the trends in operating and maintenance costs after each replacement will remain identical.

Because the equipment is being operated over a long period, the replacement policy will be periodic, so we will determine the optimal replacement interval.

Construction of the model:

1. A – is the acquisition cost of the capital equipment
2. C_i – is the operation and maintenance in the i^{th} period from new, assumed to be paid at the end of the period, $(i = 1, 2, 3, \dots n)$
3. S_i – is the resale value of the equipment at the end of the i^{th} period of operation, $(i = 1, 2, 3, \dots n)$
4. r – is the discount factor $\left(r = \frac{1}{1+\frac{i}{100}}\right)$

   (i is the interest rate)
5. n is the age in periods (such as years) if the equipment when replaced.
6. $C(n)$ is the total discounted cost of operating, maintaining, and replacing the equipment (with identical equipment) over a long period, with replacement occurring at intervals of n periods.
7. The objective is to determine the optimal interval between replacements to minimize the total discounted costs, $C(n)$



Consider the first cycle of operation:

The total discounted up to the end of the first cycle of operation, with equipment already purchased and installed, is:

$$C_1(n) = C_1r^1 + C_2r^2 + C_3r^3 + \dots + C_nr^n + Ar^n - S_nr^n$$

$$= \sum_{i=1}^{n} c_ir^i + r^n(A - S_n)$$

For the second cycle, the total cost discounted from the start of the second cycle is:

$$C_2(n) = \sum_{i=1}^{n} c_ir^i + r^n(A - S_n)$$

Similarly, the total cost o the third cycle, fourth cycle, and so forth discounted back to the start of their respective cycles, is of similar format.

The total discounted costs, when discounting is calculated at the start of the operation time zero, is

$$C(n) = C_1(n) + C_2(n)r^n + C_3(n)r^{(2n)} + \cdots + C_n(n)r^{(n-1)n}$$

Between $C_1(n) = C_2(n) = \cdots = C_n(n)$

We end up with a geometric progression that gives over an infinite period:

$$C(n) = \frac{C_1(n)}{1 - r^n} = \frac{\sum_{i=1}^{n} c_i r^i + r^n (A - S_n)}{1 - r^n}$$

This is the model of the problem that relates the replacement interval to the total costs.

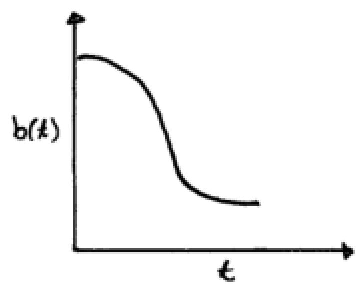
Optimal Replacement Interval for Capital Equipment: Maximization of Discounted Benefits

Similar problem as the last one except that (1) the objective is to determine the replacement interval that maximizes the total discounted net benefits derived from operating equipment over a long period, and (2) the trend in cost is taken to be continued, rather than discrete.

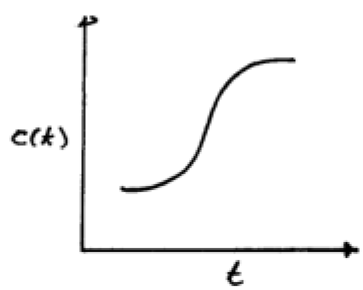
Construction of the model:

(1) $b(t)$ is the net benefit obtained from the equipment at time t

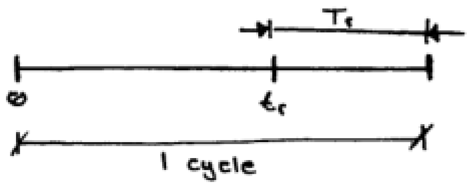
This will be the revenue derived from operating the equipment minus the operating costs, which may include maintenance costs, fuel costs, and so on



(2) $c(t)$ is the net cost of replacing equipment at age t. Replacing the equipment includes purchase price plus installation cost, and may also include cost for loss of production due to the time required to replace the equipment. These costs are often partially offset by the salvage value of the used equipment, which usually depends on the age of the capital equipment when it is replaced.

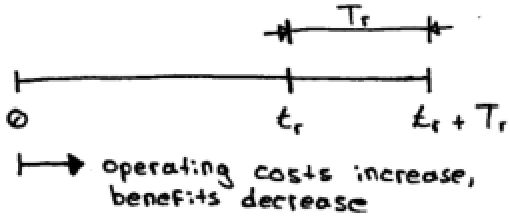


(3) T_r is the time required to replace the equipment
(4) t_r is the age of equipment
(5) $t_r + T_r$ is the replacement cycle



(6) $B(t_r)$ is the total discounted net benefit derived from operating the equipment for periods of length t_r over a long time.

(7) The objective is to determine the optimal interval between replacements to maximize the total discounted net benefits derived from operating and maintaining the equipment over a long period.



If we define $B_1\,(t_r + T_r)$ as the total net benefits derived from replacing the equipment at age t_r, discounted back to the present-day value at the start of the first cycle,

$B_1(t_r + T_r) =$ Benefits received in interval $(0, t_r)$ discounted to their present value
$-$ the cost of replacing equipment of age t_r discounted to its present-day value

Discounted benefits over the first cycle

$$= \int_0^{t_r} b(t)\, e^{-it}\, dt$$

i is the relevant interest rate for discounting

Discounted replacing cost = $c(t_r)e^{-i\,t_r}$

So,

$$B_1(t_r + T_r) = \int_0^{t_r} b(t)\, e^{-i\,t}\, dt - c(t_r)\, e^{-i\,t_r}$$

Similarly, second cycle of operation:

$$B_2(t_r + T_r) = \int_0^{t_r} b(t)\, e^{-i\,t}\, dt - c(t_r)\, e^{-i\,t_r}$$

Discounting $B_2(t_r + T_r)$ back to the start of the first cycle, it becomes:

$$B_2(t_r + T_r)e^{-i(t_r+T_r)}$$

So, n^{th} cycle of operation:

$$B_n(t_r + T_r) = \int_0^{t_r} b(t)\, e^{-i\,t}\, dt - c(t_r)\, e^{-i\,t_r}$$

Which discounted to the start of first cycle:

$$B_n(t_r + T_r)\, e^{-i(n-1)(t_r+T_r)}$$

Thus, the total discounted net benefits over a long period, with replacement at age t_r, is:

$$B(t_r) = B_1(t_r + T_r) + B_2(t_r + T_r)e^{-i(t_r+T_r)} + \cdots + B_n(t_r + T_r)e^{-i(n-1)(t_r+T_r)}$$

Because $B_1(t_r + T_r) = B_2(t_r + T_r) = \cdots = B_n(t_r + T_r)$

So (the sum of geometric series),

$$B(t_r) = \frac{B_1(t_r + T_r)}{1 - e^{-i(t_r+T_r)}}$$

Or,

$$B(t_r) = \frac{\int_0^{t_r} b(t)e^{-i\,t}dt - C(t_r)e^{-i\,t_r}}{1 - e^{-i(t_r+T_r)}}$$

This is a model of the replacement problem for total discounted net benefits.

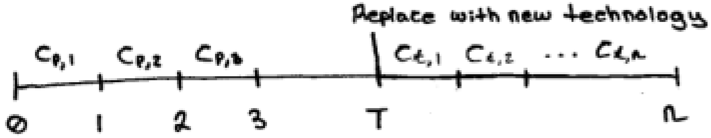
Optimal replacement policy for capital equipment considering technological improvements, finite planning horizon.

When determining a replacement policy, there may be equipment on the market that is in some way a technological improvement over the equipment currently being used.

For example:
- Maintenance and operating costs may be lower
- Throughput may be greater
- Quality of output product may be better

How to determine when, if at all, to take advantage of the technologically superior equipment.

Construction of the model:
(1) n is the number of operating periods during which equipment will be required
(2) C_p, i is the operation and maintenance cost of the prescribed equipment in the i^{th} period, from now payable at times i (where $i = 1, 2, 3, \dots, n$)
(3) S_p, i is the resale value of the present equipment at the end of the i^{th} period from now
(4) A is the acquisition cost of the technologically superior equipment
(5) $C_{t,J}$ is the operating and maintenance cost of the technologically superior equipment in the j^{th} period after its installation and payable at time J ($J = 1, 2, 3, \dots, n$)
(6) $S_{t,J}$ is the resale value of the technologically superior equipment at the end of J^{th} period of operation ($J = 0, 1, 2, \dots, n$)
$J = 0$ is included so that we can define $S_{t,0} = A$. This enables $A\,r^0$ in the model to be canceled if no changes exist.
(7) r is the discount factor
(8) The objective is to determine the value of T at which replacement should take place with the new and better equipment



The total discounted cost over n periods with replacement occurring at the end of the T^{th} period.

$C(T) =$ discounted maintenance costs for percent equipment over period $(0, T)$
$+$ discounted maintenance costs for technologically superior equipment over period (T, n)
$+$ discounted acquisition cost of the new equipment
$-$ discounted resale value of the previous equipment at the end of the i^{th} period
$-$ discounted resale value of technologically superior equipment at the end of the n^{th} period

$$= \left(C_{p,1}r^1 + C_{p,2}r^2 + \cdots + C_{p,T}r^T\right)$$
$$+ (C_{t,1}r^{T+1} + C_{t,2}r^{T+2} + \cdots + C_{t,n-1}r^n$$
$$+ Ar^T - (\mathrm{S_{p,T}r^T + S_{t,n-T}r^n})$$
$$= \sum_{i=1}^{T} C_{p,i}r^i + \sum_{J=1}^{n-T} C_{t,J}r^{T+J} + Ar^T - (S_{p,T}r^T + S_{t,n-T}r^n)$$

Maintenance Management and Control

The management and control of maintenance activities are equally important to performing maintenance.

"Maintenance may be described as the function of providing policy guidance for maintenance activities, in addition to exercising technical and management control of maintenance programs."
– Department of Defence, USA

Generally, as the size of the maintenance activity and group increases, the need for better management and control becomes essential.

In the past, the typical size of a maintenance group in a manufacturing establishment varied from 5 to 10% of the operation force (Neibel 1994).

These days, the proportional size of the maintenance effort compared to the operating group has increased significantly.

The prime factor behind this trend is the tendency in the industry to increase the mechanization and automation of many processes.

Consequently, this means lesser need for operators but greater requirement for maintenance personnel.

<u>Maintenance department functions and organization:</u>

A maintenance department is expected to perform a wide variety of functions including:

1. Planning and repairing equipment / facilities to acceptable standards.
2. Performing preventive maintenance; more specifically, developing and implementing a regularly scheduled work program for the purpose of maintaining satisfactory equipment / facility operation as well as preventing major problems.
3. Preparing realistic budgets that detail maintenance personnel and material needs.
4. Managing inventory to ensure that parts / materials necessary to conduct maintenance tasks are readily available.
5. Keeping records on equipment, services, etc.
6. Developing effective approaches to monitor the activities of maintenance staff.
7. Developing effective techniques for keeping operations personnel, upper-level management, and other concerned groups aware of maintenance activities.
8. Training maintenance staff and other concerned individuals to improve their skills and perform effectively.
9. Reviewing plans for new facilities, installation of new equipment, etc.
10. Implementing methods to improve workplace safety and developing safety education-related programs for maintenance staff.
11. Develop contract specifications and inspecting work performed contractors to ensure compliance with contractual requirements.

Many factors determine the place of maintenance in the plant organization including size, complexity, and product produced.

Some guidelines useful in planning a maintenance organization are:

- Establish reasonably clear division of authority with minimum overlap.
- Optimize number of persons reporting to an individual.
- Fit the organization to the personalities involved.
- Keep vertical lines of authority and responsibility as short as possible.

Centralized or decentralized maintenance function?

One of the first consideration in planning a maintenance organization is to decide whether it is advantageous to have a centralized or decentralized function.

Generally, centralized maintenance serves well in small and medium sized enterprises housed in one structure, or service buildings located in an immediate geographic area.

Some of the benefits and drawbacks of centralized maintenance are as follow:
Benefits:

- More efficient compared to decentralized maintenance
- Fewer maintenance personnel required
- More effective line supervision
- Greater use of special equipment and specialized maintenance persons
- Permits procurement of more modern facilities
- Generally allows for effective on-the-job training

Drawbacks:

- Requires more time to and from the work area or job
- No one individual becomes totally familiar with complex hardware or equipment
- Higher transportation cost due to remote maintenance work

Past experience indicates that in large plants, a combination of centralized and decentralized maintenance normally works out.

Managing a maintenance program effectively.

Improving a maintenance management program is a continuous process that requires progressive attitudes and active involvement. A nine-step approach for managing a maintenance program effectively is presented below:

1. Identify existing deficiencies. This can be achieved through interviews with maintenance personnel and by examining in-house performance indicators.
2. Set maintenance goals. These goals take into account existing deficiencies and identify targets for improvement.
3. Establish priorities. List maintenance projects in order of saving or merit
4. Establish performance measurement parameters. Develop a quantifiable measurement for each set goal. For example, number of jobs completed per week and percentage of cost on repair.
5. Establish short- and long-range plans. The short-range plans focus on high-priority goals, usually within a one-year period. The long-range plan is more strategic in nature and identifies important goals to be reached within three to five years.
6. Document both long- and short-range plans and forward copies to all concerned individuals.
7. Implement the plan
8. Report status. Preparing a brief report periodically and forward it to all involved individuals.
9. Examine progress annually. Review progress at the end of each year with respect to stated goals. Develop a new short-range plan for the following year by considering the goals identified in the long-range plan and adjustments made to the previous year’s planned schedule, resources, costs, and so on.

Self-evaluation of maintenance effort:

The U.S. Energy Research and Development Administration (USRDA) conducted a study on maintenance management related matters and formulated the following ten questions for maintenance managers to self-evaluate their maintenance effort:

1. Are you aware of how your craftpersons spend their time: i.e. travel, delays, etc.
2. Are you aware of what facility / equipment and activity consume most of the maintenance money?
3. Are you aware if the craftpersons use proper tools and methods to perform their tasks?
4. Have you balanced your spare parts inventory with respect to carrying cost vs. anticipated downtime losses?
5. With respect to job costs, are you in a position to compare the “should” with the “what” ?
6. Do you inspire that maintainability factors are considered properly during the design of new or modified facilities / equipment?
7. Are you aware of how much time your foreman spends at the desk at the job site?
8. Do you have an effective base to perform productivity measurements, and is productivities improving?
9. Are you aware of whether safety practises are being followed?
10. Are you providing the craftsperson with correct quality and quantity of material when and where they need it?

If an unqualified “yes” is the answer to each of the above questions, then your maintenance program is on a sound footing to meet organizational objectives. Otherwise, appropriate corrective measures are required.

Elements of Effective Maintenance Management

There are many elements of affective maintenance management whose effectiveness is the key to the overall success of the maintenance activity. Some of these elements are described here.

Maintenance Policy: maintenance policy is one of the most important elements of effective maintenance management. It is essential for continuity of operation and a clear understanding of the maintenance management program, regardless of the size of a company.

Usually, maintenance organizations have manuals containing items such as policies, programs, objectives, responsibilities and authorities for all levels of supervision, reporting requirements, useful methods and techniques, and performance measurement indices.

Work order system: A work order authorizes and directs an individual or group to perform a given task, a well-defined work order system should cover all the maintenance jobs requested and accomplished, whether repetitive or a one-time job.

The work order system is useful for management in controlling costs and evaluating job performance. The work order should at least contain information such as requested and planned completion dates, work descriptions and its reasons, planned start date, labor and material costs, work categories, and appropriate approval signatures.

Job planning: Job planning is an essential element of the effective maintenance management. A number of tasks may have to be performed prior to commencement of a maintenance job; for example, procurement of parts, tools, and materials, identification of methods and sequencing, coordination with other departments, and securing safety permits. Past experience indicates that on average one planner is required for every 20 craftspersons.

Human errors in maintenance:
Humans play an important role during the equipment lifecycle in the design, production, and operation and maintenance phases.

Human error may be defined as the failure to perform a specified task (or the performance of a forbidden action) that could lead to disruption of scheduled operations or result in damage to property and equipment.

Some of the causes of human error include:

- poor equipment design
- poor work environment
- poor work layout
- Improper work tools
- inadequate training
- poorly written equipment maintenance and operating procedures

Guidelines for reducing human error and maintenance:
over the years considerable effort has been made to develop guidelines to reduce human error in airline maintenance.

Many of these guidelines can also be used in other areas of maintenance.

The guidelines covered 10 areas:
- procedures
- human error risk management
- tools and equipment
- design
- supervision
- communication
- Shift handover
- towing aircraft
- maintenance incident feedback

Procedures are covered by 4 guidelines:
1. ensure as much as possible that standard work practices are followed all across maintenance operations.
2. Periodically review documented maintenance procedures and practices to ensure they are all accessible, realistic and consistent.
3. Periodically examine work practices to ensure that they do not differ from formal procedures.
4. Evaluate the ability of checklists to assist maintenance persons in performing routine operations such as preparing an aircraft for towing, activating hydraulics, or moving flight services .

There are three guidelines concerning human error risk management:
1. carefully considered the need to disturb normally operating systems to perform nonessential periodic maintenance inspections, as there is a risk of maintenance error occurrence associated with a disturbance.
2. Formally review the adequacy of defenses such as engine runs designed into the system for detecting maintenance errors.
3. Avoid as much as possible the simultaneous performance of the same maintenance task unsimilar redundant systems.

The following guidelines are associated with training:

- Consider introducing crew resource management for maintenance professionals and others, i.e. persons interacting with the maintenance professionals.
- Offer periodic refresher training to maintenance professionals with emphasis on company procedures:

Important guidelines concerning design:

- Ensure that manufacturers give proper attention to maintenance of human factors during the design process and actively seek information on the errors occurring during the maintenance phase for the input in the design phase.

A guideline in the area of supervision and management oversight needs to be strengthened particularly in the final hours at each shift as the occurrence of errors becomes more likely.

In the area of communication, ensure that satisfactory systems are in place to disseminate important information to all maintenance staff so that changing procedures or repeated errors are considered in an effective manner. Shift handover can be a factor in maintenance error.

In the area of towing aircraft or other equipments, review the procedures and equipment used for towing to and from maintenance facilities.

Maintenance incident feedback is covered by the following guidelines:

- ensure that management receives regular and structured feedback on maintenance incidents with particular consideration to the underlying conditions or latent failures that help promote such incidents.
- Ensure that engineering training schools receive feedback on recurring maintenance incidents so that effective corrective measures for these problems are targeted.

Quality of Safety in Maintenance:

Good quality maintenance work leads to good results - reduction or elimination of unexpected failures, lower costs, better safety, increase confidence in work performance, etc.

Good quality maintenance work in only be measured accurately after the specification of expectations. Once the aim of maintenance work is clearly identified, steps such as those listed below can be useful in producing good quality maintenance work.

Steps to producing good quality maintenance work:

- limit perplexity. Often the request for maintenance is incomplete and inaccurate.
- Define goals. Goals should be set by the maintenance team and its supervisors. Ensure that the team clearly understands the objectives for the maintenance work prior to start.
- Avoid unsafe practices. Do not permit temptation to minimize maintenance time by shortcutting prescribed safety procedures
- Do not overlook secondary damage. Ensure that less dramatic secondary problems are not overlooked. Otherwise they could be costly at a later stage.
- Report as the maintenance work progress is. Report all relevant information that could be useful for performing similar tasks in the future.
- Do not use second-hand parts. Ensure that failed parts are replaced by new ones.
- Reinstall with extra care period sometimes excessive force use and re installation can damage parts. Avoid introducing new failures while correcting old ones.
- Test the repaired items prior to its hand back.
- Complete all appropriate job records. Tasks such as maintenance planning comma and failure analysis rely heavily on an effective maintenance history.